DOMINATOR COLORING OF VERTEX SWITCHING OF GRAPHS

K. KAVITHA, N.G. DAVID

Abstract: Let *G = (V, E)* be a graph. A *proper coloring* of *G* is an assignment of colors to its vertices in such a way that no two adjacent vertices receive the same color. A proper coloring of *G* partitions V into color classes. Dominator coloring of *G* is a proper coloring in which every vertex of *G* dominates every vertex of at least one color class.
In this paper, dominator coloring of vertex switching of classes of graphs is consider for our discussion and also the corresponding chromatic number of vertex switching of various classes of graphs are obtained.

Keywords: coloring, domination and dominator coloring.

1. Introduction and Motivation: Let G = (V, E) be a simple, finite, undirected and connected graph. For graph theoretic terminology we refer to Harary et al [2, 5].

Graph coloring and domination are two major areas in graph theory that have been well studied. An excellent treatment of domination is given in the book by Haynes et al. [6] and survey papers on several advanced topics on domination are given in the book edited by Haynes et al. [7]. Several variations of coloring have been introduced and studied by many researchers. The book by Jenson and Toft [9] gives an extensive survey of various graph coloring. The applications of graph coloring are from such diverse areas as time-tabling, scheduling, frequency assignment, register allocations, coding theory and resource allocation, etc. There are several variants of graph colorings. List coloring, b-coloring, harmonious coloring, total coloring, sum coloring, rank coloring, complete coloring, rainbow coloring are some of the variants of graph coloring. We are interested in dominator coloring.

The open neighborhood of $v \in V$ is denoted and defined by $N(v) = \{u \in V : uv \in E\}$ and the closed neighborhood of $v \in V$ is $N[v] = N(v) \cup \{v\}$. The degree of a vertex $v \in V$ is $\deg(v) = |N(v)|$. A vertex of degree zero in G is called an isolated vertex and a vertex of degree one is a pendant vertex or a leaf of G. The vertex which is adjacent to a pendant vertex is called a support vertex and the edge incident to a pendant vertex is called a pendant edge. For any set $S \subseteq V$, the induced subgraph $\langle S \rangle$ is the maximal subgraph of G with vertex set S. Thus two vertices of S are adjacent in $\langle S \rangle$ if and only if they are adjacent in G. A vertex $u \in V$ dominates a vertex $v \in V$ if $uv \in E$. A vertex $v \in V$ dominates a set $S \subseteq V$ if v dominates every vertex in S. A subset D of V is called a dominating set of G if every vertex in V − D is dominated by at least one vertex in S [11].

A proper coloring of G is an assignment of colors to the vertices of G in such a way that adjacent vertices receive different colors. A color class is the set of vertices, having the same color. The color class corresponding to a color i is denoted by V_i. Note that every such color class is an independent set. A proper coloring of G partitions V into color classes. A dominator coloring of G is a proper coloring in which every vertex of G dominates every vertex of at least one color class. The dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G. The concepts of dominator coloring of a graph was introduced by Hedetniemi et al. [8] and studied further by Gera et al. [1, 3, 4].

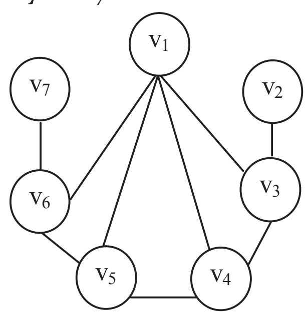
The following definition which is needed for the subsequent section.

Definition 1.2 [12]

Let *G = (V, E)* be a graph. A vertex switching G_v or $\hat{G}$ of G at a specified vertex v is obtained by removing all the edges incident with *v* and adding edges joining *v* to every vertex which are not adjacent to *v* in *G*.

Example 1.3

The following graph is obtained by switching the vertex v_1 in cycle C_7.



Vertex switching in C_7

2. Dominator Coloring of Vertex Switching of graphs : In this section, dominator coloring on vertex switching of classes of graphs is consider for our discussion and also the corresponding chromatic number of vertex switching of classes of graphs are obtained.

Theorem 2.1

Let G be a graph G of order $n \geq 6$ with $\delta(G) = 1$,

$\chi(G) \le \chi_d(\hat{G}) \le \chi(G)+2$, when switch a pendant vertex.

Proof: Let $\hat{G}$ be the graph obtained from G by switching a pendant vertex v and let vv′ ∈ E(G). As dominator coloring is a proper coloring, for any graph G we have that $\chi(G) \le \chi_d(G)$. This implies that $\chi(G) \le \chi_d(\hat{G})$.

For the upper bound, let c be a proper coloring of G with $\chi(G)$ - colors. Reassign colors to v and v′ by assigning $\chi(G)$ +1 to v and $\chi(G)$ + 2 to v′. This proper coloring of $\hat{G}$ is also a dominator coloring of $\hat{G}$, as the vertices v and v' dominate its own color class and the remaining vertices dominate the color class

$\chi(\hat{G})$ +1. Hence $\chi(G) \le \chi_d(\hat{G}) \le \chi(G)+2$.

The lower bound is sharp for the star graph $K_{1,r}$, r ≥ 2. The upper bound is sharp for the path graph P_n, n ≥ 8.

Proposition 2.2

For the cycle graph C_n of order $n \ge 6$, $\chi_d(\hat{C}_n) = 5$.

Proof

Let C_n : $v_1, v_2, \ldots, v_n$ be the cycle graph on *n* vertices and let us assume without loss of generality that $\hat{C}_n$ be the graph obtained from C_n by switching the vertex v_1.

Consider a proper coloring c of $\hat{C}_n$ with vertices v_1, v_2 and v_{n-1} are assigned colors 1, 2 and 3 and vertices $v_3, v_4, \ldots, v_{n-2}$ and v_n are alternately assigned colors 4 and 5. This coloring c results in a dominator coloring of $\hat{C}_n$, as the vertices v_1, v_2 and v_{n-1} dominate themselves, v_n dominates the color class 3 and the remaining vertices dominate the color class 1. Hence $\chi_d(\hat{C}_n) \le 5$.

On the other hand, if any one of the colors 2 (or 3) is reused at the any of the vertices $v_3, v_4, \ldots, v_{n-2}$ or v_n, then the vertex v_2 (or v_{n-1}) does not dominate any color class, implying that $\chi_d(\hat{C}_n) \nless 5$. Hence $\chi_d(\hat{C}_n) = 5$.

Note: The following particular cases are observed.

(i) $\chi_d(\hat{C}_4) = 2$ and (ii) $\chi_d(\hat{C}_5) = 3$.

Proposition 2.3

For the path graph P_n, $n \ge 8$,

(i) $\chi_d(\hat{P}_n) = 4$, when switch a pendant vertex.

(ii) $\chi_d(\hat{P}_n) = 5$, when switch a non-pendant and non-support vertex.

Proof: Let the path graph be P_n: $v_1, v_2, \ldots, v_n$, n ≥ 8.

(i) Without loss of generality let us assume that $\hat{P}_n$ is obtained from P_n by switching vertex v_1. The argument is analogous when the other pendant vertex is switched. Consider a proper coloring *c* of $\hat{P}_n$ with v_1 colored by 1 and v_2 by 2 and the remaining vertices by 3 and 4 alternately. This is a dominator coloring, as vertices $v_3, \ldots, v_n$ dominate the color class 1 and vertices v_1 and v_2 dominate themselves. Hence $\chi_d(\hat{P}_n) \le 4$.

On the other hand, if color 1 is reused at v_i, i = 3, …, *n*, *c* is not a proper coloring and if color 2 is reused, *c* is not a dominator coloring, implying that $\chi_d(\hat{P}_n) \nless$ 4. Hence $\chi_d(\hat{P}_n) = 4$.

Let us assume that $\hat{P}_n$ is obtained from P_n by switching v_i for some i, 3 ≤ i ≤ n-2. Consider a proper coloring *c* of $\hat{P}_n$ in which v_i is colored by color 1, v_{i-1} by 2 and v_{i+1} by 3. The remaining vertices are colored by colors 4 and 5 alternately. This is a dominator coloring, as each vertex colored by 4 or 5 dominates the color class 1 and vertices v_{i-1}, v_i, v_{i+1} dominate themselves. Hence $\chi_d(\hat{P}_n) \le 5$.

On the other hand, if any one of the colors 2 or 3 is reused at v_n, then *c* is not a dominator coloring, implying that $\chi_d(\hat{P}_n) \nless 5$. Hence $\chi_d(\hat{P}_n) = 5$.

Note: The following particular cases are observed.

(i) When switch a pendant vertex, $\chi_d(\hat{P}_3) = 2$, $\chi_d(\hat{P}_4) = 3$ and $\chi_d(\hat{P}_n) = 4$ for n = 5, 6, 7.

(ii) When switch a non-pendant and non-support vertex, $\chi_d(\hat{P}_5) = 3$ and $\chi_d(\hat{P}_n) = 4$, for n = 6 or 7.

(ii) Switching a support vertex, results in a disconnected graph.

Proposition 2.4: For the wheel graph $W_{1,r}$, r ≥ 5 with n = r+1, $\chi_d(\hat{W}_{1,r}) = 4$, when switch a vertex at the rim.

Proof: Let the vertices of $W_{1,r}$ be $v_0, v_1, v_2, \ldots, v_r$ with the centre labeled by v_0 and let $\hat{W}_{1,r}$ be the graph obtained from $W_{1,r}$ by switching v_r. A dominator coloring c of $\hat{W}_{1,r}$ is obtained by coloring v_0 by 1 and $v_1, v_2, \ldots, v_{r-1}$ by 2 and 3 alternately and v_r by 4. Each vertex colored by 1 and 4 dominate themselves and the remaining vertices dominate the color class 1. Therefore, $\chi_d(\hat{W}_{1,r}) \le 4$.

On the other hand, if the color 1 is reused at the vertex v_r, then the vertex v_r does not dominate any other color class, implying that $\chi_d(\hat{W}_{1,r}) \nless 4$. Hence $\chi_d(\hat{W}_{1,r}) = 4$.

Note:
The following observations are also made.
(i) $\chi_d(\hat{W}_{1,4}) = 3$, When switch a vertex at the rim.
(ii) Switching the central vertex, results in a disconnected graph.

Proposition 2.5: For the star graph $K_{1,r}$, $r \geq 2$ with $n = r+1$, $\chi_d(\hat{K}_{1,r}) = 2$, when switch a pendant vertex.

Proof: The vertices of $K_{1,r}$ with $n = r + 1$ are labeled by $v_0, v_1, v_2, \ldots, v_r$ with the centre labeled by v_0 and let $\hat{K}_{1,r}$ be the graph obtained from $K_{1,r}$ by switching a pendant vertex, say v_1.

Consider a proper coloring c of $\hat{K}_{1,r}$ in which v_0 is assigned color 1, switched vertex v_i by 1 and the remaining vertices by color 2. This is a dominator coloring, because each vertex colored by 1 dominates the color class 2 and each vertex colored by 2 dominates the color class 1 and 2 is the minimum number of colors required. Hence $\chi_d(\hat{K}_{1,r}) = 2$.

Note: Switched the central vertex results in a disconnected graph.

Proposition 2.6
For complete bipartite graph $K_{r,s}$, $r, s \geq 2$ with $n = r + s$, $\chi_d(\hat{K}_{r,s}) = 2$.

Proof: Let $V(K_{r,s}) = A \cup B$, where $A = \{v_i\}$, $1 \leq i \leq r$ and $B = \{v_j\}$, $r + 1 \leq j \leq r + s$ and let $\hat{K}_{r,s}$ be the graph obtained from $K_{r,s}$ by switching the vertex v_i, $1 \leq i \leq r + s$.

Consider a proper coloring c of $\hat{K}_{r,s}$ by coloring the switched vertex v_i by 1, the neighbors of v_i by 2 and the remaining vertices by 1. This is a dominator coloring, as the vertex v_i dominates the color class 2 and each vertex colored by 1 dominates the color class 2 and 2 is the minimum colors required. Hence $\chi_d(\hat{K}_{r,s}) = 2$.

Proposition 2.7
For the bistar graph $B_{r,s}$, $r, s \geq 2$ with $n = r + s$, $\chi_d(\hat{B}_{r,s}) = 3$, when switch a pendant vertex.

Proof
Let $V(B_{r,s}) = A \cup B \cup C$, where $A = \{u_1, u_2\}$, $B = \{v_i\}$, $1 \leq i \leq r$ and $C = \{v_j\}$, $r + 1 \leq j \leq r + s$, where $B \cup C$ contains all pendant vertices, $E(B_{r,s}) = \{u_1 u_2\} \cup \{u_1 v_i \,/\, 1 \leq i \leq r\} \cup \{u_2 v_j \,/\, r + 1 \leq j \leq r + s\}$ and let $\hat{B}_{r,s}$ be obtained from $B_{r,s}$ by switching a pendant vertex. Without loss of generality let us assume that the pendant vertex v_1 in $B_{r,s}$ is switched.

Consider a proper coloring c of $\hat{B}_{r,s}$ with color classes $V_1 = \{v_1, u_1\}$, $V_2 = \{v_i \,/\, 2 \leq i \leq r+s\}$ and $V_3 = \{u_2\}$. This is a dominator coloring, as each vertex colored 2 in B dominates the color class 1, each vertex colored 2 in C dominates the color class 3, u_1 dominates the color class 3 and u_2 dominates itself. Hence $\chi_d(\hat{B}_{r,s}) \leq 3$. As $\chi_d(\hat{B}_{r,s})$ contains K_3, $\chi_d(\hat{B}_{r,s}) \nless 3$. Hence $\chi_d(\hat{B}_{r,s}) = 3$.

Proposition 2.8: For the helm graph H_r, $r \geq 4$ with $n = 2r+1$ vertices

i) $\chi_d(\hat{H}_r) = \chi_d(C_r) + 2$, when switch the central vertex.

ii) $\chi_d(\hat{H}_r) = \chi(C_r) + 2$, when switch a pendant vertex.

Proof: Let H_r, $r \geq 4$ be the helm graph on $2r+1$ vertices with the centre labeled by v_1, vertices adjacent to v_1 (which induce a cycle C_r of length r) are labeled by $v_2, v_3, \ldots, v_{r+1}$ and their corresponding pendant vertices by $v_{r+2}, \ldots, v_{2r+1}$.

(i)Consider a proper coloring c of $\hat{H}_r$ in which vertices v_i, $2 \leq i \leq r+1$ in induced subgraph C_r are colored using $\chi_d(C_r)$- colors, the vertex v_1 by $\chi_d(C_r)+1$ and the vertices v_i, $r+2 \leq i \leq 2r+1$ are all colored by $\chi_d(C_r)+2$. This is a dominator coloring, as vertices colored 1 or 2 dominate some uniquely colored neighbors, each vertex colored k for $3 \leq k \leq \chi_d(C_r)+1$ dominates its own color class and vertices colored by $\chi_d(C_r)+2$ dominate the color class of v_1. Hence $\chi_d(\hat{H}_r) \leq \chi_d(C_r) + 2$.

At the same time, if one of the colors i, $1 \leq i \leq \chi_d(C_r)$ is reused at v_1, the vertices $\{v_i \,/\, r+2 \leq i \leq 2r+1\}$ do not dominate any color class and reusing color $\chi_d(C_r)+1$ and $\chi_d(C_r)+2$ destroys proper coloring properly, implying that $\chi_d(\hat{H}_r) \nless \chi_d(\hat{C}_r) + 2$. Hence $\chi_d(\hat{H}_r) = \chi_d(C_r) + 2$.

(ii)Let $\hat{H}_r$ is obtained from H_r by switching a pendant vertex v_k, $r+2 \leq k \leq 2r+1$. Consider a proper coloring c of $\hat{H}_r$ with $c(v_k) = 1$, $c(v_1) = 2$ and vertices in C_r are colored using $\chi(C_r)$ colors. The vertices $\{v_i \,/\, r+2 \leq i \leq 2r+1\} - \{v_k\}$ are colored using the colors used for coloring C_r. Therefore $\chi_d(\hat{H}_r) \leq \chi(C_r) + 2$.

On the other hand, if the color used at the central vertex v_1 is reused at the switched vertex v_k, then v_1 does not dominate any color class, implying that

$$\chi_d(\hat{H}_r) \nless \chi(C_r)+2=\begin{cases}2+2 & \text{if r is even}\\ 3+2 & \text{if r is odd.}\end{cases}$$

Hence $\chi_d(\hat{H}_r)=\begin{cases}4 & \text{when r is even}\\ 5 & \text{when r is odd.}\end{cases}$

Note: When switch a pendant or the central vertex, $\chi_d(\hat{H}_3) = 4.$

Proposition 2.9: For the flower graph Fl_r, r ≥ 3 with n = 2r+1 vertices, $\chi_d(\hat{Fl}_r)=4$, when switch a vertex other than the central vertex.

Proof: The vertices of the flower graph Fl_r, where n = 2r+1 vertices are labeled by v_1, ..., v_{2r+1}, where v_1 is the central vertex, v_2, ..., v_{r+1} are the vertices of degree 4 and v_{r+2} to v_{2r+1} are respectively the vertices of degree 2 adjacent to v_2 to v_{r+1}.

Case (i) When switch a vertex v_i, for a particular i from 2 to r+1.

Consider a proper coloring c of $\hat{Fl}_r$ in which color 1 is assigned to the central vertex v_1, color 2 to the switched vertex v_i and assign colors 3 and 4 to the remaining vertices from v_2 to v_{2r+1}. This is a dominator coloring, as vertices colored 1 or 2 dominate themselves and the remaining vertices dominate the color class 1. Hence $\chi_d(\hat{Fl}_r)\le 4$.

On the other hand, if color 2 is reused, c is not a proper coloring, implying that $\chi_d(\hat{Fl}_r) \nless 4$.

Case (ii)When switch a vertex v_i, for a particular i from r+2 to 2r+1.

Without loss of generality, let us assume that vertex v_{r+2} (which is adjacent to v_2) is switched. Consider a proper coloring c of $\hat{Fl}_r$ in which color 1 is assigned to v_1 and v_{r+2}, color 2 is assigned to v_2 and colors 3 and 4 are assigned properly to the remaining vertices. This is a dominator coloring, as vertex v_1 dominates the color class 2, vertex v_2 dominates itself, vertex v_{r+2} dominates color classes 3 and 4 and remaining vertices dominate color class 1. Hence $\chi_d(\hat{Fl}_r)\le 4$.

At the same time, if color 2 is reused, the vertex v_2 does not dominate any color class, implying that $\chi_d(\hat{Fl}_r) \nless 4$. Hence $\chi_d(\hat{Fl}_r)=4$.

Note:

Switching the central vertex in Fl_r, results in a disconnected graph.

Proposition 2.10

For the sunflower graph Sf_r, r ≥ 3 with n = 3r+1,

(i) $\chi_d(\hat{Sf}_r)=4$, when switch a vertex of degree 2 or 4.

(ii) $\chi_d(\hat{Sf}_r)=\begin{cases}3 & \text{when r is even}\\ 4 & \text{when r is odd}\end{cases}$, when switch a pendant vertex.

Proof

The vertices of the sunflower graph Sf_r, where n = 3r+1 are labeled by v_1, v_2, ..., v_{3r+1}, where v_1 is a central vertex, v_2, ..., v_{r+1} are the vertices of degree 4, v_{r+2}, ..., v_{2r+1} are the vertices of degree 2 and v_{2r+2}, ..., v_{3r+1} are the pendant vertices.

Case (i) When switch the vertex v_i, for some i, 2 ≤ i ≤ 2r+1.

The proof is similar to that of theorem 2.9.

Case (ii) When switch the vertex v_i, for some i, 2r+2 ≤ i ≤ 3r+1.

As the argument is same for switching a vertex v_i, for any i, 2r+2 ≤ i ≤ 3r+1, without loss of generality let us assume that vertex v_{2r+2} is switched.

When r is even, consider a proper coloring c of $\hat{Sf}_r$ in which color 1 is assigned to vertices v_1 and v_{2r+2} and colors 2 and 3 are assigned to the remaining vertices properly. This is a dominator coloring, as each vertex colored 1 dominates the color class 3 and each vertex colored 2 or 3 dominates the color class 1. As $\hat{Sf}_r$ contains K_3, we have $\chi_d(\hat{Sf}_r)=3$.

When r is odd, consider a proper coloring c of $\hat{Sf}_r$ in which color 1 is assigned to vertices v_1 and v_{2r+2} and colors 2, 3 and 4 are assigned to the remaining vertices (3 additional colors are needed as it contains an odd cycle of length r). This is a dominator coloring, as each vertex colored 1 dominates the color class 4 and each vertex colored 2, 3 or 4 dominates the color class 1. Hence

$$\chi_d(\hat{Sf}_r)=\begin{cases}3 & \text{when r is even}\\ 4 & \text{when r is odd.}\end{cases}$$

Proposition 2.11

For Gear graph G_r, r ≥ 3 with n = 2r+1 vertices,

(i) $\chi_d(\hat{G}_r)=\chi_d(G_r)=\lceil 2r/3\rceil+2$, when switch the central vertex.

(ii) $\chi_d(\hat{G}_r)=\begin{cases}4 & \text{when } r=3\\ 5 & \text{otherwise}\end{cases}$, when switch a vertex of degree 3.

(iii) $\chi_d(\hat{G}_r)=4$, when switch a vertex of degree 2.

Proof

Let $V(G_r)$ = {v_i / 1 ≤ i ≤ 2r+1}, where v_1 is the central vertex.

(i) When switch the central vertex in G_r, we get back the same gear graph G_r. Hence by [10],

$$\chi_d(\hat{G}_r) = \chi_d(G_r) = \lceil 2r/3 \rceil + 2.$$

(ii) When switch a vertex of degree 3.

When r ≥ 4, consider a proper coloring c of $\hat{G}_r$ in which assign respectively colors 1, 2 and 3 to vertices v_2, v_3 and v_{2r+1}. Assign colors 4 and 5 to the remaining vertices alternately. This is a dominator coloring, as vertices v_2, v_3 and v_{2r+1} dominate themselves, each vertex colored 4 or 5 dominates any one of the color classes 1, 2 or 3 and the central vertex v_1 dominate either color class 4 or color class 5. Hence $\chi_d(\hat{G}_r) \le 5$.

On the other hand, if one of the colors 1, 2 or 3 is reused at the central vertex v_1, the vertex v_1 does not dominate any other color classes, implying that $\chi_d(\hat{G}_r) \not< 5$. It can be easily verified that $\chi_d(\hat{G}_3) = 4$. Hence

$$\chi_d(\hat{G}_r) = \begin{cases} 4 & \text{when } r = 3 \\ 5 & \text{otherwise.} \end{cases}$$

(iii) When Switch the vertex of degree 2

Without loss of generality, let us assume that vertex v_3 is switched. Consider a proper coloring c of $\hat{G}_r$ in which color 1 is assigned to the central vertex, color 2 to the switched vertex v_3 and colors 3 and 4 are assigned to the remaining vertices. This is a dominator coloring, because each vertex colored 1 or 2 dominate themselves and the remaining vertices dominate one of the color classes 1 or 2. Hence $\chi_d(\hat{G}_r) \le 4$.

On the other hand, if the color 2 is reused at the vertex v_{2r+1}, vertices v_2 and v_3 do not dominate any color class, implying that $\chi_d(\hat{G}_r) \not< 4$. Hence $\chi_d(\hat{G}_r) = 4$.

Further Research : In this section, we pose some problems for further investigation based on the above propositions.

1) Characterize graphs for which $\chi_d(\hat{G}) = \chi_d(G)$.

2) Characterize graphs G for which $\chi_d(\hat{G}) = \chi(G)$ or $\chi_d(\hat{G}) = \gamma(G) + 1$.

Determine bounds for the dominator chromatic number of vertex switching in several other families of graphs, such as tori, d-dimensional grids, graphs with bounded tree width, planar graphs and hyper cubes.

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Department of Mathematics
Madras Christian College, Chennai - 600 059
kavitha.matha@yahoo.com and ngdmcc@gmail.com