SOFT FEEBLY SEPARATION SPACES AND SOFT FEEBLY CONTINUOUS FUNCTIONS

A.P.DHANA BALAN, C.SANTHI, RM.SIVAGAMA SUNDARI

Abstract: The aim of this paper is to introduce soft feebly open sets, soft feebly continuous functions and study some of their properties and characterizations. We also investigate the concept of soft feebly separation spaces.

Key Words: Soft set, soft topology, soft feebly open, soft feebly continuous function, soft feebly regular, soft feebly normal.

Introduction: Some theories such as the theory of fuzzy sets[10] and theory of rough sets[7]can be considered as mathematical tools for dealing with uncertainties. These theories have their inherent difficulties as pointed out in [6].The reason for those difficulties is possibly, the inadequacy of the parameterization tool of the theories. In 1999, Molodstov[6] initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties. A soft set is a collection of approximate descriptions of an object. The soft set theory has been applied to many different fields like game theory, Riemann integration, O.R, probability, Perron integration and theory of measurement. In recent years, development in the fields of soft set theory and its application has been taking in a rapid place. This is because of the general nature of parameterization expressed by a soft set. Recently Shabir and Naz[9] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently they defined the basic notions of soft topological spaces such as soft subspace, soft closure, soft neighbourhood of a point, soft separation axioms like soft regular space, soft normal space. Soft topology is a relatively new and promising domain which can lead to the development of new mathematical models and innovative approaches that will significantly contribute to the solution of complex problems in natural sciences. In [11], Zorlutuna et.al showed that a fuzzy topological space is a special case of the soft topological space.

In this paper, we introduce some new concepts in soft topological spaces such as soft feebly open set, soft feebly continuous functions. Moreover some properties and relations of the soft feebly separation axioms are discussed.

Preliminaries:

Definition 2.1[6]: Let X be the initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) is called a *soft set* over X, where F is a mapping given by $F : A \to P(X)$.

Definition2.2[3]: The *complement* of a soft set (F,A),denoted by $(F,A)^c$,is defined by $(F,A)^c = (F^c,A)$,where $F^c : A \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for each $e \in A$. F^c is called the soft complement function of F. Clearly,$(F^c)^c$ is the same as F and$((F,A)^c)^c = (F,A)$.

Definition2.3[5]: Let X be an initial universe and (F,A),(G,B) be any two soft subsets of X. (i) (F,A) is said to be a *null soft set*,denoted by ϕ,if for all $e \in A, F(e)=\phi$.
(ii) (F,A) is said to be an *absolute soft set*, denoted by X_A, if $e \in A$ and $F(e) = X$.
(iii) (F,A) is a *soft subset* of (G,B), denoted by $(F,A) \tilde{\subseteq} (G,B)$, if $A \subset B$ and $e \in A$, $F(e) \subseteq G(e)$. (iv) (F,A) is said to be a *soft superset* of (G,B), denoted by $(F,A) \tilde{\supseteq} (G,B)$, if $(G,B) \tilde{\subseteq} (F,A)$. (v) (F,A) and (G,B) are said to be *soft equal*, if $(F,A) \tilde{\subseteq} (G,B)$ and $(F,A) \tilde{\supseteq} (G,B)$.

Definition 2.4[5]: The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set $(H,C)=(F,A) \tilde{\cup} (G,B)$,where $C=A\cup B$ for all $e \in C$, $H(e) = \begin{cases} F(e), & \text{if } e \in A-B, \\ G(e), & \text{if } e \in B-A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$

Definition 2.5[8]: The intersection of two soft sets (F,A) and (G,B) over the common universe X is the soft set $(H,C) = (F,A)\tilde{\cap}(G,B)$, where $C=A\cap B$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$. The family of these soft sets are denoted by $SS(X)_E$.

Definition 2.6[9]: Let τ be a collection of soft sets over a universe X with a fixed set E of parameters, then $\tau \tilde{\subseteq} SS(X)_E$ is called a *soft topology* on X with a fixed set E if

(i) ϕ_E, X_E belongs to τ.
(ii) the union of any number of soft sets in τ belongs to τ.
(iii) the intersection of any two soft sets in τ belongs to τ. The triplet (X,τ,E) is called a soft topological space over X. For simplicity, we can take the soft topological space (X,τ,E) as X unless otherwise explicitly stated. Then every member of τ is called a *soft open* set in X. A soft set (F,E) is called *soft closed* in X if $(F,E)^c \in \tau$. The end or omission of a proof will be denoted by □.

Definition 2.7: Let X be a soft topological space and (F,E) be a soft set over X.

(i) *The soft closure*[9] of (F,E),denoted by $\widetilde{cl}(F,E)$,is the intersection of all soft closed sets containing (F,E).
(ii) *The soft interior*[11] of (F,E),denoted by $\widetilde{int}(F,E)$,is the union of all soft open sets contained in (F,E).

Definition 2.8: A soft subset (A,E) of X is called a *soft semi open*[2] if $(A,E) \tilde{\subset} \widetilde{cl}(\widetilde{int}(A,E))$. The complement of the soft semi open set is *soft semi closed*.

Definition 2.9:[4] The *soft semi closure* of (A,E) is the intersection of all soft semi closed sets containing (A,E).It is denoted by $\widetilde{scl}(A,E)$.The *soft semi interior* of (A,E) is the union of all soft semi open sets contained in (A,E).It is denoted by $\widetilde{sint}(A,E)$.

Definition 2.10[9]: (i) A space X is *soft regular* if for every soft closed set (F,E) and every point $x \notin (F,E)$ there exist disjoint soft open sets (U,E) and (V,E) such that $x \in (U,E)$ and $(F,E) \widetilde{\subseteq} (V,E)$. (ii) A space X is *soft normal* if for every pair of disjoint soft closed sets are separated by disjoint soft open sets.

Soft feebly open:

Definition 3.1: A soft subset (A,E) of a soft topological space X is said to be *soft feebly open* if there exist a soft open set (S,E) such that $(S,E) \subset (A,E) \subset scl(S,E)$. The complement of soft feebly open set is *soft feebly closed*. The *soft feebly closure* of (A,E), denoted by $\widetilde{fcl}(A,E)$, is the intersection of all soft feebly closed sets containing (A,E).The *soft feebly interior* of (A,E),denoted by $\widetilde{fint}(A,E)$ is the union of all soft feebly open sets contained in (A,E).

Result 3.2: (i) Every soft open set is soft feebly open. (ii) Every soft closed set is soft feebly closed.

Lemma 3.3: Let X be a soft topological space. If (A,B) is soft open in X. and (B,E) is soft feebly open in X, then $(A,E) \widetilde{\cap} (B,E)$ is soft feebly open in X and A.

Proof: Let (B,E) be soft feebly open in X. Then there exists a soft open set (U,E) in X such that $(U,E) \widetilde{\subseteq} (B,E) \widetilde{\subseteq} \widetilde{scl}(U,E)$. This implies that $(U,E) \widetilde{\cap} (A,E) \widetilde{\subseteq} (B,E) \widetilde{\cap} (A,E) \widetilde{\subseteq} \widetilde{scl}(U,E) \widetilde{\cap} (A,E)$. But $\widetilde{scl}(U,E) \widetilde{\cap} (A,E) \widetilde{\subseteq} \widetilde{scl}((U,E) \widetilde{\cap} (A,E))$ and so $(U,E) \widetilde{\cap} (A,E) \widetilde{\subseteq} (B,E) \widetilde{\cap} (A,E) \widetilde{\subseteq} \widetilde{scl}((U,E) \widetilde{\cap} (A,E))$. Hence $(A,E) \widetilde{\cap} (B,E)$ is soft feebly open in X and A.

Lemma 3.4: Let Y be a subspace of a soft topological space X. Then every soft feebly open set (A,E) of Y is soft feebly open in X.

Proof: Let (A,E) be soft feebly open in Y. Then there exists a soft open set (U,E) in Y such that $(U,E) \widetilde{\subseteq} (A,E) \widetilde{\subseteq} \widetilde{scl}(U,E)$. Since (U,E) is soft open in Y and Y is a subspace of X, then $(U,E) = (V,E) \widetilde{\cap} Y$ where (V,E) is soft open in X. By hypothesis, Y is soft open in X and $(V,E) \widetilde{\cap} Y$ is soft open in X. Thus (U,E) is soft open in X. Thus (U,E) is soft open in X, then (A,E) is soft feebly open in X. □

Result 3.5: Let X be a soft topological space, $(A,E) \widetilde{\subseteq} X$ and $(B,E) \widetilde{\subseteq} X$ and $(A,E) \widetilde{\subseteq} (B,E)$ then $\widetilde{fcl}(A,E) \widetilde{\subseteq} \widetilde{fcl}(B,E)$.

Soft Feebly Continuous Functions:

Definition 4.1: A mapping $f : X \rightarrow Y$ is said to be *soft mapping*[12] if X and Y are soft topological spaces and u: $X \rightarrow Y$ and p: $E \rightarrow K$ are mappings, where E and K are parameters in X and Y respectively.

Definition 4.2: A soft mapping $f : X \rightarrow Y$ is (i) *soft continuous*[1] if the inverse image of each soft open subset of Y is soft open in X. (ii) *soft feebly continuous* if the inverse image of each soft open subset of Y is soft feebly open in X. Throughout this paper, f and f^{-1} denotes soft functions.

Theorem 4.3: Let $f : X \rightarrow Y$ be a soft feebly continuous and g: $Y \rightarrow Z$ soft continuous. Then $g \circ f : X \rightarrow Z$ is soft feebly continuous where E,K,T are parameters of X,Y and Z respectively.

Proof: Let (C,T) be a soft open set in Z. Since g is soft continuous, $g^{-1}(C,T)$ is soft open in Y where $(B,K) = g^{-1}(C,T)$ for some soft open set (C,T) in Z. Since f is soft feebly continuous, $f^{-1}(B,K))$ is soft feebly open in X where $(A,E) = f^{-1}(B,K)$ for some soft open set (B,K) in Y. It follows that (A,E) is soft feebly open in X and hence $(g \circ f)^{-1}(C,T) = f^{-1}(g^{-1}(C,T))$ is soft feebly open in X. Therefore $g \circ f$ is soft feebly continuous.□

Theorem 4.4: Every soft continuous mapping is soft feebly continuous .

Proof: Let $f : X \rightarrow Y$ be a soft continuous mapping. To show f is a soft feebly continuous mapping. Let (H,E) be any soft open subset of Y.Since f is soft continuous, $f^{-1}(H,E)$ is soft open in X, then by Result 3.2, $f^{-1}(H,E)$ is soft feebly open in X. Hence f is soft feebly continuous mapping.

Theorem 4.5: Let $f : X \rightarrow Y$ be a soft feebly continuous and (A,E) be soft open set of X.

Then $f|_A : (A,E) \rightarrow f(A)$ defined by $f|_A(x) = f(x)$ for all $x \in (A,E)$ is soft feebly continuous.

Proof: Let (V,E) be any soft open set in $f(A,E)$. Then there exists a soft open set (U,E) in Y such that $(V,E) = f(A,E) \widetilde{\cap} (U,E)$. Clearly, $f^{-1}|_A(V,E) = (A,E) \widetilde{\cap} f^{-1}(V,E) = (A,E) \widetilde{\cap} f^{-1}(f(A,E) \widetilde{\cap} (U,E))$. From this we have $f^{-1}|_A(V,E) = (A,E) \widetilde{\cap} f^{-1}(U,E)$.Since f is soft feebly continuous then $f^{-1}(U,E)$ is soft feebly open in X, then by Lemma 3.3, $(A,E) \widetilde{\cap} f^{-1}(U,E)$ is soft feebly open in X and A. Thus $f^{-1}|_A(V,E)$ is soft feebly open in X. □

Theorem 4.6: A function $f : X \rightarrow Y$ is soft feebly continuous, if every singleton set in X is soft feebly open.

proof: Let $\{x\}$ be soft feebly open set in X for every $x \in X$. Let p be any arbitrary point in X and $(G,E) \widetilde{\subseteq}$ Y be an arbitrary soft open set such that f (p)$\in (G,E) \Rightarrow$ p$\in f^{-1}(G,E) \Rightarrow \{p\} \widetilde{\subseteq} f^{-1}(G,E) \Rightarrow f(\{p\}) \widetilde{\subseteq} (G,E)$. Also, p$\in X$ and by hypothesis $\{p\}$ is soft feebly open in X. Thus we have shown that, given any soft open set $(G,E) \widetilde{\subseteq}$ Y such that f (p) $\in (G,E)$, there exists soft feebly open set $\{p\}$ in X with p$\in \{p\}$ such that $f(\{p\}) \widetilde{\subseteq} (G,E)$. Hence f is soft feebly continuous at p. □

Theorem 4.7: Let X,Y be two soft topological spaces, and let $X = (A,E) \widetilde{\cup} (B,E)$. Also $f|_A$ and $f|_B$ be soft feebly continuous mappings provided (A,E) and (B,E) are soft open in X, then $f : X \rightarrow Y$ is soft feebly continuous.

Proof: Let $(H,E) \widetilde{\subseteq} Y$ be soft open. To prove $f^{-1}(H,E)$ is soft feebly open in X. By hypothesis, $f|_A$: $A \rightarrow Y$ and $f|_B$: $B \rightarrow Y$ are both soft feebly continuous mappings, then $f^{-1}|_A(H,E)$ and $f^{-1}|_B(H,E)$ are soft feebly open subsets of (A,E) and (B,E) respectively. Since (A,E) and (B,E) are soft open subsets of X, then by Lemma 3.4, $f^{-1}|_A(H,E)$ and $f^{-1}|_B(H,E)$ are soft feebly open subsets of X. Since $X = (A,E) \widetilde{\cup} (B,E)$, we have $f^{-1}(H,E) = f^{-1}|_A(H,E) \widetilde{\cup} f^{-1}|_B(H,E)$. Hence $f^{-1}(H,E)$ is soft feebly open in X. □

Theorem 4.8: The function $f : X \rightarrow Y$ is soft feebly closed if and only if for each soft open set (H,K) of Y and for each soft open set (U,E) containing $f^{-1}(H,K)$,there exists a soft feebly open set (G,E) of Y containing (H,K) and $f^{-1}(G,K) \widetilde{\subseteq} (U,E)$.

Proof: Let f be soft feebly closed. Let $(H,K) \widetilde{\subseteq} Y$ and

(U,E) is soft open in X so that $f^{-1}(H,K) \tilde{\subseteq} (U,E)$.Let $(G,K) = Y - f(X-(U,E))$. Since (U,E) is soft open in X, $X-(U,E)$ is soft closed in X .Since f is soft feebly closed, $f(X-(U,E))$ is soft feebly closed in Y. Thus $(G,K) = Y-(X-(U,E))$ is soft feebly open in Y .Now to prove: $(H,K) \tilde{\subseteq} (G,K)$, let $y \notin (G,K)$, we will prove $y \notin (H,K)$,then $y \in f(X-(U,E)$.Thus there exists $x \in (X-(U,E))$ such that $f(x) = y$, then $x \notin (U,E)$ since $f^{-1}(H,K) \tilde{\subseteq} (U,E)$,then $x \notin f^{-1}(H,K)$.Therefore $y \notin (H,K)$.Thus $(H,K) \tilde{\subseteq} G,K)$. Now we prove $f^{-1}(G,K) \tilde{\subseteq} (U,E)$. Since $(G,K) = Y - f(X-(U,E))$, $f^{-1}(G,K) = f^{-1}[Y - f(X-(U,E))] = f^{-1}(Y) - f^{-1}[X-(U,E)] \tilde{\subseteq} X-(X-(U,E)) = (U,E)$.Thus $f^{-1}(G,K) \tilde{\subseteq} (U,E)$.Conversely, let (A,E)be a soft closed in X. Then $f^{-1}(H,K) = f^{-1}[Y-f(A,E)] = f^{-1}(Y) - f^{-1}[f^{-1}(A,E)] \tilde{\subseteq} X-(A,E) = (U,E)$. By hypothesis, there exists soft feebly open set (G,K) in Y such that $f^{-1}(G,K) \tilde{\subseteq} (U,E)$ and $(H,K) \tilde{\subseteq} (G,K)$.That is $Y-f(A,E) \tilde{\subseteq} (G,K)$ and $f^{-1}(G,K) \tilde{\subseteq} f(X-(A,E))$.we get $Y-f(A,E) \tilde{\subseteq} (G,K)$ and $G \tilde{\subseteq} f(X-(A,E))$.Thus $f(A,E) \tilde{\subseteq} (G,K)$ and $(G,K) \tilde{\subseteq} Y-f(A,E)$.Then $G = Y-f(A,E))$. Since G is soft feebly open in Y, we have $Y-f(A,E)$ is soft feebly open in Y. Thus $f(A,E)$ is soft feebly closed in Y. Hence f is soft feebly closed. □

Theorem 4.9: If $f : X \rightarrow Y$ is soft continuous, soft feebly closed surjection from a soft normal space X to Y, then Y is soft feebly normal.

Proof: Let (A,K) and (B,K) be disjoint soft closed sets of Y. Since f is soft continuous, then $f^{-1}(A,K)$ and $f^{-1}(B,K)$ are disjoint soft closed sets in X. Since X is soft normal there exist disjoint soft open sets (U_1,E) and (U_2,E) of X such that $f^{-1}(A,K) \tilde{\subseteq} (U_1,E)$ and $f^{-1}(B,K) \tilde{\subseteq} (U_2,E)$. By Theorem 4.8, there exist soft feebly open set (G,K) and (H,K) such that $(A,K) \tilde{\subseteq} (G,K)$ and $(B,K) \tilde{\subseteq} (G,K)$ and $f^{-1}(G,K) \tilde{\subseteq} (U_1,E)$ and $f^{-1}(H,K) \tilde{\subseteq} (U_2,E)$.Then, $f^{-1}(G,K) \tilde{\cap} f^{-1}(H,K) = \phi$ and hence $(G,K) \tilde{\cap} (H,K) = \phi$. Since (A,K) and (B,K) are soft closed then (A,K) and (B,K) are soft feebly closed sets, since (G,K) is soft feebly open and (A,K) is soft feebly closed,$(A,K) \tilde{\subseteq} (G,K) \Rightarrow (A,K) \tilde{\subseteq} \widetilde{fcl}(G,K)$. Similarly, $(B,K) \tilde{\subseteq} \widetilde{fcl}(H,K)$. Hence $\widetilde{fcl}(G,K) \tilde{\cap} \widetilde{fcl}(H,K) = (G,K) \tilde{\cap} (H,K) = \phi$ and hence Y is feebly normal. □

Theorem 4.10: Let $f : X \rightarrow Y$ be soft continuous, soft feebly open and soft feebly closed surjection from a soft regular space X to a space Y then Y is soft feebly regular

Proof: Let (U,K) be a soft open set containing a point y in Y. Let x be any point of X such that $y = f(x)$. Since X is a soft regular space, there exist an soft open set (H,K) in X such that $x \in (H,K) \tilde{\subseteq} \widetilde{cl}(H,K) \tilde{\subseteq} f^{-1}(U,K)$,where $f^{-1}(U,K)$ is soft open, since f is soft continuous. Then $y \in f(H,K) \tilde{\subseteq} f(\widetilde{cl}(H,K)) \tilde{\subseteq} (U,K)$ where $f(H,K)$ is soft feebly open, since f is soft feebly open. Since (U,K) is soft open, (U,K) is soft feebly open,$\widetilde{cl}(H,K)$ is soft closed in X. By hypothesis f is soft feebly closed. Then $f(\widetilde{cl}(H,K))$ is soft feebly closed in Y. This implies $\widetilde{fcl}(f(\widetilde{cl}(H,K))) \tilde{\subseteq} (U,K)$ and since $(H,K) \tilde{\subseteq} \widetilde{cl}(H,K)$,we have $f(H,K) \tilde{\subseteq} f(\widetilde{cl}(H,K))$.Then by Result 3.5, $\widetilde{fcl}(f(H,K)) \tilde{\subseteq} \widetilde{fcl}(f(\widetilde{cl}(H,K))) \tilde{\subseteq} (U,K) \Rightarrow \widetilde{fcl}(f(H,K)) \tilde{\subseteq} (U,K)$.Thus $y \in f(H,K) \tilde{\subseteq} \widetilde{fcl}(f(H,K)) \tilde{\subseteq} (U,K)$, $f(H,K)$ is soft feebly open in Y. Hence Y is soft feebly regular.□

References:

1. A.Aygunoglu, H.Aygun."Some note on soft topological spaces",Neural Computting and Appli., 21,113-119, 2012.
2. Bin.chen,"Soft semiopen sets andrelated properties in soft topological spaces", Appl.Math.Inf.Sci.,7(1),287-294, 2013.
3. M.Irfan Ali,F.Feng,X.Liu,W.K.Min and M.shabir,"On some new operations in soft set theory, Comp.Math.Appl.,57, 1547-1553, 2009.
4. Janaki .C, Jeyanthi.V,"On soft πgr-closed sets in soft topological spaces", Jou.of Advan.in.Math. 4,(3),478-485,2013.
5. P.K.Maji,R.Biswas,R.roy,"Soft set theory",Compu.Math.Appl.37,19-31,1999.
6. D.Molodtsov,"Soft set theory-first results",Comp and Math. with Appl,37, (4-5),19-31,1999.
7. Z.Pawlak,"Rough sets",International jou.of info.and Comp.sci,11,341-356,1982.
8. D.Pei and D.Miao,"From soft sets to information systems",Proc.of the IEEE confe on Granular Comp,2,617-621,2005.
9. M.Shabir and M.Naz,"On soft topological spaces",Comp. and Math. with Appl,61,(7),1786-1799,2011. [10]
10. L.A.Zadeh,"Fuzzy sets",Information and control, 8, 338-353, 1965.
11. I.Zorlutuna,M.Akdag,W.K.Min and S.Atmaca, "Remarks on soft topological spaces",to appear in Annals of Fuzzy Math and Info.,3(2),171-185, 2011.
12. Metin Akdag and Alkan Ozkan,"Soft α-Open Sets and Soft α-Cont Functions", Abst.and Appl.Analy.,1-7, 2014.
13. A.P.Dhana Balan, RM.Sivagama Sundari, C.Santhi, "Soft p-sets and soft optimally continuous functions", Math. Sci. Int. Res. Jou.,(accepted).

***

A.P.Dhana Balan/Assistant Professor of Mathematics/
C.Santhi/ Rm.Sivagama Sundari/ Research Scholars/Department of Mathematics;
Alagappa Govt. Arts College, Karaikudi-630003; Tamil Nadu; India. e.mail: danabalanap@yahoo.com/
kumarsanthi11@yahoo.com/kanish9621@hotmail.com/ Mob: 94882 90720