COMMON FIXED POINT THEOREMS FOR SUB COMPATIBLE AND SUB SEQUENTIALLY CONTINUOUS MAPS IN INTUITIONISTIC FUZZY METRIC SPACES

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Abstract: The purpose of this paper is to prove common fixed point theorems for the new concepts of sub compatibility and sub sequential continuity in intuitionistic fuzzy metric space using implicit relation for six self mappings which are weaker than occasionally weak compatibility and reciprocal continuity. In general all known results on commuting, weakly commuting, compatible, weak compatible, semi compatible and occasionally weak compatible maps in intuitionistic fuzzy metric spaces are generalized in this note.

Keywords: Intuitionistic fuzzy metric spaces, Sub compatibility and Sub sequential continuity, Common fixed point, implicit relation.

Introduction: Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [12]. In 2004, Park [8] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al. [1] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm as a generalization of fuzzy metric space due to Kramosil and Michalek [4]. Further, Alaca et al. [1] proved intuitionistic fuzzy Banach and intuitionistic fuzzy Edelstein contraction theorems, with the different defnition of Cauchy sequences and completeness than the ones given in [8]. Popa [6, 7] introduced the idea of implicit function to prove a common fixed point theorem in metric spaces Singh and Jain [10] further extended the result of Popa [6, 7] in fuzzy metric spaces. In this paper, we use the concepts of sub compatibility and sub sequential continuity in intuitionistic fuzzy metric spaces using implicit relation which are respectively weaker than occasionally weak compatibility and reciprocal continuity. With them, we establish a common fixed point theorem for six maps.

Preliminaries:

Definition 2.1 : [12] A 5-tuple (X, M, N,∗,◊) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set,∗ is a continuous t-norm,◊ is a continuous t- conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions, for all x, y, z ∈ X and s, t > 0,

1. M(x, y, t) + N(x, y, t) ≤ 1.
2. M(x, y, 0) = 0 for all x, y in X.
3. M(x, y, t) = 1 for all x, y in X and t > 0 if and only if x = y.
4. M(x, y, t) = M(y, x, t), for all x,y in X, and t > 0.
5. M(x, y, t)∗M(y, z, s) ≤ M(x, z, t + s).
6. M(x, y, .) : [0,∞)⟶[0, 1] is left continuous.
7. $\lim_{t\to\infty}$M(x, y, t) = 1 for all x, y in X and t > 0.
8. N(x, y, 0) = 1 for all x, y in X.
9. N(x, y, t) = 0 for all x, y in X and t > 0 if and only if x = y.
10. N(x, y, t) = N(y, x, t), for all x, y in X, and t > 0.
11. N(x, y,t) ◊ N(y, z,s) ≥ N(x, z,t + s).
12. N(x, y, .) : [0,∞) ⟶ [0, 1] is right continuous.
13. $\lim_{t\to\infty}$N(x, y, t) = 0 for all x, y in X and t > 0.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Example 2.2: Let X ={$1/n$: n =1,2,...}∪ {0} and let ∗ be the continuous t-norm and ◊ be the continuous t - conorm defined by

a ∗ b = ab and a ◊ b = min {1, a + b }, respectively, for all a, b∈ [0, 1]. For each t ∈ (0, ∞) and x, y in X, define (M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & t > 0 \\ 0, & t = 0 \end{cases} \quad \text{and } N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & t > 0 \\ 1, & t = 0 \end{cases}$$

Clearly, (X, M, N, ∗,◊) is complete intuitionistic fuzzy metric space.

Definition 2.3: [11] Let A and S be maps from an intuitionistic fuzzy metric space (X, M, N, ∗,◊) into itself. The maps A and S are said to be weakly commuting if M(ASz, SAz, t) ≥ M(Az, Sz, t) and N(ASz, SAz, t) ≤ N(Az, Sz, t) for all z ∈ X and t > 0.

Definition 2.4: [13] Let A and S be maps from an IFM-space (X, M, N, ∗,◊) into itself. The maps A and S are said to be compatible if for all t > 0,

$\lim_{n\to\infty} M(AS x_n, SA x_n, t) = 1$ and

$\lim_{n\to\infty} N(AS x_n, SA x_n, t) = 0$ when ever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some z ∈ X.

Definition 2.5:[5] Two mappings A and S of a IFM-space (X, M, N, ∗,◊) will be called reciprocally continuous if $ASu_n \longrightarrow Az$ and $SAu_n \longrightarrow Sz$ whenever$\{u_n\}$ is a sequence such that $Au_n, Su_n \longrightarrow$ z forsome z in X.

Definition 2.6: Let (X, M, N, ∗,◊) be a intuitionistic fuzzy metric space. A and S be self maps on X. A point x in X is called a coincidence point of A and S iff Ax = Sx. In this case, w = Ax = Sx is called a point of coincidence of A and S.

Definition 2.7: A pair of self mappings (A, S) of a IFM- space (X, M, N, $*,\lozenge$) is said to be weakly compatible if they commute at the coincidence points i.e., if Au = Su for some u in X, then ASu = SAu. It is easy to see that two compatible maps are weakly compatible but converse is not true.

Definition 2.8: [2] Two self mappings A and S of a IFM space (X, M, N, $*,\lozenge$) are said to be occasionally weakly compatible (OWC) iff there is a point x in X which is coincidence point of A and S at which A and S commute.

Definition 2.9: Let (X, M, N, $*,\lozenge$) be a intuitionistic fuzzy metric space. Self maps A and S on X are said to be sub compatible iff there exists a sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some z ∈ X and satisfy $\lim_{n\to\infty} M(ASx_n , SAx_n , t) = 1$ and $\lim_{n\to\infty} N(ASx_n , SAx_n , t) = 0$

Example 2.10: Let X ∈ [0, ∞). For each t ∈ (0, ∞) and x, y ∈ X, define (M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|} , & t > 0 \\ 0 , & t = 0 \end{cases}$$ and

$$N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|} , & t > 0 \\ 1 , & t = 0 \end{cases}$$

Define A and S as follows A(x) = x, S(x) = $\{x^2$, if x ∈ [2, 3]}.

Let $\{ x_n\}$ be a sequence in X defined by

$x_n = \left(1 + \frac{1}{n}\right)^n$ Then

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$ = e,(2 ≤ e ≤ 3)and

$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{2n}$ = e, $e \in X$ (2 ≤ e ≤ 3)

and $SA(x_n) \longrightarrow$ e, $AS(x_n) \longrightarrow$ e, when n⟶∞.

Thus, $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$

and $\lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$.

Hence A and S are sub compatible.

Definition 2.11: Let (X, M, N, $*,\lozenge$) be a intuitionistic fuzzy metric space. Self maps A and S on X are said to be sub sequentially continuous iff there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$, for some t∈ X and satisfy $\lim_{n\to\infty} ASx_n = At$, $\lim_{n\to\infty} SAx_n = St$.

Example 2.12: Let X ∈ [0, ∞) for each t ∈ (0, ∞) and x, y ∈ X, define (M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|} , & t > 0 \\ 0 , & t = 0 \end{cases}$$ and

$$N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|} , & t > 0 \\ 1 , & t = 0 \end{cases}$$

Define A and S as follows

$$A(x) = \begin{cases} \frac{1+x}{2} & if\ x \in [0, 2) \\ \frac{2}{3}x + 1 & if\ x \in [2, \infty) \end{cases}$$ and

$$S(x) = \begin{cases} x + \frac{1}{2} & if\ x \in [0, 2] \\ \frac{4}{3}x - \frac{1}{3} & if\ x \in (2, \infty) \end{cases}$$

Let $\{x_n\}$ be a sequence in X defined by

$x_n = \frac{1}{n^2}$ for n = 1, 2, 3,. . .

Then $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \frac{1}{2}$, $\frac{1}{2} \in$ X and $ASx_n \longrightarrow \frac{3}{4}$ = A($\frac{1}{2}$), $SAx_n \longrightarrow \frac{3}{4}$ = S($\frac{1}{2}$) when n ⟶∞, therefore A and S are sub sequentially continuous.

Now, Let $\{x_n\}$ be a sequence in X defined by

$x_n = 2 + \frac{1}{n}$ for n = 1, 2, 3, . . . Then

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \frac{7}{3}$, $\frac{7}{3} \in$ X and

$ASx_n \longrightarrow \frac{23}{9}$ = A ($\frac{7}{3}$), $SAx_n \longrightarrow \frac{25}{9}$ = S ($\frac{7}{3}$)

when n⟶∞. Therefore A and S are sub sequentially continuous and A and S are reciprocally continuous. Clearly, from this example finally we conclude that every reciprocally continuous is sub sequentially continuous but converse need not be true.

New Implicit Relation: Let M_6 be the set of all real continuous functions $\Phi, \Psi : [0,1]^6 \longrightarrow \mathbb{R}$, non decreasing in the first argument and satisfying the following conditions:

A. Φ (1, 1, u, u, 1, u) ≥ 0 ⟹ u ≥ 1

B. Ψ (0, 0, u, u, 0, u) ≤ 0 ⟹ u ≤ 0 for all u ≥ 0

Example 3.1: Define as $\Phi, \Psi : [0,1]^6 \longrightarrow \mathbb{R}$ as $\Phi(t_1, t_2, t_3, t_4, t_5, t_6) = - 8 t_1 + 5t_2 - 4t_3 + 2t_4 + t_5 + 4t_6$ and

$\Psi (t_1, t_2, t_3, t_4, t_5, t_6) = 10t_1 - 9t_2 - 2t_3 + 3t_4 - 3t_5 + t_6$

Clearly Φ and Ψ satisfies the conditions A and B. Therefore $\Phi , \Psi \in M_6$

4. Main Result :

Theorem 4.1 : Let A, B, S, T, P and Q be six self maps of a complete intuitionistic fuzzy metric space (X, M, N, $*$, $\lozenge$) with continuous t - norm * defined by t * t ≥ t and continuous t - conorm $\lozenge$ defined by (1 – t) $\lozenge$ (1 – t) ≤ (1 – t) for all t ∈ [0, 1]. If the pairs (A, PQ) and (B, ST) are sub compatible and sub sequentially continuous, then

- A and PQ have a coincidence point,
- B and ST have a coincidence point,
- The pairs (A, PQ), (B, ST), (B, Q),
- (ST, Q) are commutes,
- for some $\Phi , \Psi \in M_6$ and for all

x, y ∈ X and every t > 0,

$$\varphi\{M(Ax, PQx, t), M(STy, By, t), \frac{M(Ax, By, t) + M(PQx, By, t)}{2}, \frac{\alpha M(PQx,STy,t)+\beta M(STy,Ax,t)}{\alpha + \beta}, \frac{\gamma M(STy,By,t)+\delta M(Ax,PQx,t)}{\gamma + \delta}, M(Ax, STy, t)\} \geq 0$$

$\Psi\{N(Ax, PQx, t), N(STy, By, t),$

$\frac{N(Ax,By,t)+N(PQx,By,t)}{2}$,
$\frac{\alpha N(PQx,STy,t)+\beta N(STy,Ax,t)}{\alpha+\beta}$,
$\frac{\gamma N(STy,By,t)+\delta N(Ax,PQx,t)}{\gamma+\delta}$,
$N(Ax,STy,t)\} \leq 0$

where $\alpha,\beta,\gamma,\delta \in \mathbb{R}$, Then A, B, S, T, P and Q have a unique common fixed point.

Proof: Since the pairs (A, PQ) and (B, ST) are sub compatible and sub sequentially continuous, then there exist two sequence $\{x_n\}$, $\{y_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} PQx_n = z$,

$z \in X$ and satisfy $\lim_{n\to\infty} M(A(PQ)x_n,(PQ)Ax_n,t) = M(Az,PQz,t) = 1$ and

$\lim_{n\to\infty} N(A(PQ)x_n,(PQ)Ax_n,t) = N(Az,PQz,t) = 0$

$\lim_{n\to\infty} By_n = \lim_{n\to\infty} STy_n = z'$, $z' \in X$ and satisfy

$\lim_{n\to\infty} M(B(ST)y_n,(ST)By_n,t) = M(Bz',STz',t) = 1$ and

$\lim_{n\to\infty} N(B(ST)y_n,(ST)By_n,t) = N(Bz',STz',t) = 0$

Therefore $Az = PQz$ and $Bz' = STz'$. z is a coincidence point of A, PQ and z' is a coincidence point of B, ST. Now prove $z = z'$. Put $x = x_n$ and $y = y_n$ in (d) we get

$\Phi\{M(Ax_n,PQx_n,t), M(STy_n,By_n,t)$
$\frac{M(Ax_n,By_n,t)+M(PQx_n,By_n,t)}{2}$,
$\frac{\alpha M(PQx_n,STy_n,t)+\beta M(STy_n,Ax_n,t)}{\alpha+\beta}$
$\frac{\gamma M(STy_n,By_n,t)+\delta M(Ax_n,PQx_n,t)}{\gamma+\delta}$,
$M(Ax_n,STy_n,t)\} \geq 0$.

$\Psi\{N(Ax_n,PQx_n,t), N(STy_n,By_n,t)$,
$\frac{N(Ax_n,By_n,t)+N(PQx_n,By_n,t)}{2}$,
$\frac{\alpha N(PQx_n,STy_n,t)+\beta N(STy_n,Ax_n,t)}{\alpha+\beta}$,
$\frac{\gamma N(STy_n,By_n,t)+\delta N(Ax_n,PQx_n,t)}{\gamma+\delta}$,
$N(Ax_n,STy_n,t)\} \leq 0$

Taking limit as $n\to\infty$, we get

$\Phi\{M(z,z,t), M(z',z',t)$,
$\frac{M(z,z',t)+M(z,z',t)}{2}$, $\frac{\alpha M(z,z',t)+\beta M(z',z,t)}{\alpha+\beta}$,
$\frac{\gamma M(z',z',t)+\delta M(z,z,t)}{\gamma+\delta}$, $M(z,z',t)\} \geq 0$

$\Psi\{N(z,z,t), N(z',z',t)$,
$\frac{N(z,z',t)+N(z,z',t)}{2}$, $\frac{\alpha N(z,z',t)+\beta N(z',z,t)}{\alpha+\beta}$,
$\frac{\gamma N(z',z',t)+\delta N(z,z,t)}{\gamma+\delta}$, $N(z,z',t)\} \leq 0$

$\Phi\{1,1,M(z,z',t),M(z,z',t),1,M(z,z',t)\} \geq 0$

$\Psi\{0,0,N(z,z',t),N(z,z',t),0,N(z,z',t)\} \leq 0$

By new implicit relation we have $M(z,z',t) \geq 1$, and $N(z,z',t) \leq 0$, for all $t > 0$.

This gives $z = z'$. Again we claim that $Az = z$. Put $x = z$ and $y = y_n$ in (d) we get

$\Phi\{M(Az,PQz,t), M(STy_n,By_n,t)$,
$\frac{M(Az,By_n,t)+M(PQz,By_n,t)}{2}$,
$\frac{\alpha M(PQz,STy_n,t)+\beta M(STy_n,Az,t)}{\alpha+\beta}$,
$\frac{\gamma M(STy_n,By_n,t)+\delta M(Az,PQz,t)}{\gamma+\delta}$,
$M(Az,STy_n,t)\} \geq 0$

$\Psi\{N(Az,PQz,t), N(STy_n,By_n,t)$,
$\frac{N(Az,By_n,t)+N(PQz,By_n,t)}{2}$,
$\frac{\alpha N(PQz,STy_n,t)+\beta N(STy_n,Az,t)}{\alpha+\beta}$,
$\frac{\gamma N(STy_n,By_n,t)+\delta N(Az,PQz,t)}{\gamma+\delta}$,
$N(Az,STy_n,t)\} \leq 0$

Taking limit as $n\to\infty$, we get

$\Phi\{M(Az,Az,t), M(z',z',t)$,
$\frac{M(Az,z',t)+M(Az,z',t)}{2}$, $\frac{\alpha M(Az,z',t)+\beta M(z',Az,t)}{\alpha+\beta}$,
$\frac{\gamma M(z',z',t)+\delta M(Az,Az,t)}{\gamma+\delta}$, $M(Az,z',t) \geq 0$

$\Psi\{N(Az,Az,t), N(z',z',t)$,
$\frac{N(Az,z',t)+N(Az,z',t)}{2}$,
$\frac{\alpha N(Az,z',t)+\beta N(z',Az,t)}{\alpha+\beta}$,
$\frac{\gamma N(z',z',t)+\delta N(Az,Az,t)}{\gamma+\delta}$, $N(Az,z',t)\} \leq 0$

$\Phi\{1,1,M(Az,z',t),M(Az,z',t),1,M(Az,z',t)\} \geq 0$,

$\Psi\{0,0,N(Az,z',t),N(Az,z',t),0,N(Az,z',t)\} \leq 0$

By new implicit relation, we have

$M(Az,z',t) \geq 1$ *and* $N(Az,z',t) \leq 0$.

This gives in view of Φ,Ψ . we get $Az = z'$ implies $Az = z = z' = PQz$. Again we claim that $Bz = z$. Put x= z and y=z in inequality(d), we get

$\Phi\{M(Az,PQz,t), M(STz,Bz,t)$,
$\frac{M(Az,Bz,t)+M(PQz,Bz,t)}{2}$, $\frac{\alpha M(PQz,STz,t)+\beta M(STz,Az,t)}{\alpha+\beta}$,
$\frac{\gamma M(STz,Bz,t)+\delta M(Az,PQz,t)}{\gamma+\delta}$, $M(Az,STz,t)\} \geq 0$

$\Psi\{N(Az,PQz,t), N(STz,Bz,t)$,
$\frac{N(Az,Bz,t)+N(PQz,Bz,t)}{2}$,
$\frac{\alpha N(PQz,STz,t)+\beta N(STz,Az,t)}{\alpha+\beta}$,
$\frac{\gamma N(STz,Bz,t)+\delta N(Az,PQz,t)}{\gamma+\delta}$, $N(Az,STz,t)\} \leq 0$

$\Phi\{M(Az,Az,t), M(Bz,Bz,t)$,
$\frac{M(Az,Bz,t)+M(Az,Bz,t)}{2}$,
$\frac{\alpha M(Az,Bz,t)+\beta M(Bz,Az,t)}{\alpha+\beta}$,

$\frac{\gamma M(Bz,Bz,t)+\delta M(Az,Az,t)}{\gamma+\delta}, M(Az,Bz,t)\} \geq 0$

$\Psi\{N(Az,Az,t), N(Bz,Bz,t),$
$\frac{N(Az,Bz,t)+N(Az,Bz,t)}{2}, \frac{\alpha N(Az,Bz,t)+\beta N(Bz,Az,t)}{\alpha+\beta},$
$\frac{\gamma N(Bz,Bz,t)+\delta N(Az,Az,t)}{\gamma+\delta}, N(Az,Bz,t)\} \leq 0$

$\Phi\{M(z,z,t), M(Bz,Bz,t),$
$\frac{M(z,Bz,t)+M(z,Bz,t)}{2}, \frac{\alpha M(z,Bz,t)+\beta M(Bz,z,t)}{\alpha+\beta},$
$\frac{\gamma M(Bz,Bz,t)+\delta M(z,z,t)}{\gamma+\delta}, M(z,Bz,t)\} \geq 0$

$\Psi\{N(z,z,t), N(Bz,Bz,t),$
$\frac{N(z,Bz,t)+N(z,Bz,t)}{2}, \frac{\alpha N(z,Bz,t)+\beta N(Bz,z,t)}{\alpha+\beta},$
$\frac{\gamma N(Bz,Bz,t)+\delta N(z,z,t)}{\gamma+\delta}, N(z,Bz,t)\} \leq 0$

$\Phi\{1, 1, M(z,Bz,t), M(z,Bz,t), 1, M(z,Bz,t)\} \geq 0$ and $\Psi\{0, 0, N(z,Bz,t), N(z,Bz,t), 0, N(z,Bz,t)\} \leq 0$.

By new implicit relation, we have M(z , Bz, t) ≥ 1 and N(z , Bz, t) ≤ 0, for all t > 0. In view of Φ and Ψ, we get Bz = z.

This gives z = Bz = STz,

Therefore z = Az = Bz = PQ = STz.. . . (1)

Which shows that z is a common fixed point of A, PQ, B and ST. Now we claim that Pz = Qz = z. Put x = z and y = Qz in inequality (d) we get

$\Phi\{M(Az,PQz,t), M(STQz,BQz,t),$
$\frac{M(Az,BQz,t)+M(PQz,BQz,t)}{2},$
$\frac{\alpha M(PQz,STQz,t)+\beta M(STQz,Az,t)}{\alpha+\beta},$
$\frac{\gamma M(STQz,BQz,t)+\delta M(Az,PQz,t)}{\gamma+\delta},$
$M(Az,STQz,t\} \geq 0$

$\Psi\{N(Az,PQz,t), N(STQz,BQz,t)$
$\frac{N(Az,BQz,t)+N(PQz,BQz,t)}{2},$
$\frac{\alpha N(PQz,STQz,t)+\beta N(STQz,Az,t)}{\alpha+\beta},$
$\frac{\gamma N(STQz,BQz,t)+\delta N(Az,PQz,t)}{\gamma+\delta},$
$N(Az,STQz,t)\}\} \leq 0$

$\Phi\{M(Az,PQz,t), M(QSTz,QBz,t),$
$\frac{M(Az, QBz, t)+M(PQz, QBz, t)}{2},$
$\frac{\alpha M(PQz,QSTz,t)+\beta M(QSTz,Az,t)}{\alpha+\beta},$
$\frac{\gamma M(QSTz,QBz,t)+\delta M(Az,PQz,t)}{\gamma+\delta},$
$M(Az,QSTz,t)\} \geq 0$

$\Psi\{N(Az,PQz,t), N(QSTz,QBz,t),$
$\frac{N(Az,QBz,t)+N(PQz,QBz,t)}{2},$
$\frac{\alpha N(PQz,QSTz,t)+\beta N(QSTz,Az,t)}{\alpha+\beta},$
$\frac{\gamma N(QSTz,QBz,t)+\delta N(Az,PQz,t)}{\gamma+\delta},$
$N(Az,QSTz,t)\} \leq 0$

By equation (1) we have,

$\Phi\{M(z,z,t), M(Qz,Qz,t), \frac{M(z,Qz,t)+M(z,Qz,t)}{2},$
$\frac{\alpha M(z,Qz,t)+\beta M(Qz,z,t)}{\alpha+\beta},$
$\frac{\gamma M(Qz,Qz,t)+\delta M(z,z,t)}{\gamma+\delta}, M(z,Qz,t)\} \geq 0$

$\Psi\{N(z,z,t), N(Qz,Qz,t),$
$\frac{N(z,Qz,t)+N(z,Qz,t)}{2}, \frac{\alpha N(z,Qz,t)+\beta N(Qz,z,t)}{\alpha+\beta},$
$\frac{\gamma N(Qz,Qz,t)+\delta N(z,z,t)}{\gamma+\delta}, N(z,Qz,t)\} \leq 0$

$\Phi\{1, 1, M(z,Qz,t), M(z,Qz,t), 1, M(z,Qz,t)\} \geq 0$

$\Psi\{0, 0, N(z,Qz,t), N(z,Qz,t), 0, N(z,Qz,t)\} \leq 0$

By new implicit relation, we have M(z, Qz, t) ≥ 1 and N(z , Qz, t) ≤ 0,for all t > 0. In view of Φ and Ψ , we get Qz = z .

Therefore Qz = z and PQz = z ⟹ Pz = z. Hence Pz = Qz = z.

Now we claim that Sz = Tz = z.

Put x = z and y = Tz

$\Phi\{M(Az,PQz,t), M(STTz,BTz,t),$
$\frac{M(Az,BTz,t)+M(PQz,BTz,t)}{2},$
$\frac{\alpha M(PQz,STTz,t)+\beta M(STTz,Az,t)}{\alpha+\beta},$
$\frac{\gamma M(STTz,BTz,t)+\delta M(Az,PQz,t)}{\gamma+\delta},$
$M(Az,STTz,t)\} \geq 0$

$\Psi\{N(Az,PQz,t), N(STTz,BTz,t),$
$\frac{N(Az,BTz,t)+N(PQz,BTz,t)}{2},$
$\frac{\alpha N(PQz,STTz,t)+\beta N(STTz,Az,t)}{\alpha+\beta},$
$\frac{\gamma N(STTz,BTz,t)+\delta N(Az,PQz,t)}{\gamma+\delta},$
$N(Az,STTz,t)\}\} \leq 0$

$\Phi\{M(Az,PQz,t), M(TSTz,TBz,t),$
$\frac{M(Az,TBz,t)+M(PQz,TBz,t)}{2},$
$\frac{\alpha M(PQz,TSTz,t)+\beta M(TSTz,Az,t)}{\alpha+\beta},$
$\frac{\gamma M(TSTz,TBz,t)+\delta M(Az,PQz,t)}{\gamma+\delta},$
$M(Az,TSTzt)\} \geq 0$

$\Psi\{N(Az,PQz,t), N(TSTz,TBz,t),$
$\frac{N(Az,TBz,t)+N(PQz,TBz,t)}{2},$
$\frac{\alpha N(PQz,TSTz,t)+\beta N(TSTz,Az,t)}{\alpha+\beta},$
$\frac{\gamma N(TSTz,TBz,t)+\delta N(Az,PQz,t)}{\gamma+\delta},$
$N(Az,TSTz,t)\} \leq 0$

$\Phi\{M(z,z,t), M(Tz,Tz,t),$
$\frac{M(z,Tz,t)+M(z,Tz,t)}{2}, \frac{\alpha M(z,Tz,t)+\beta M(Tz,z,t)}{\alpha+\beta},$
$\frac{\gamma M(Tz,Tz,t)+\delta M(z,z,t)}{\gamma+\delta}, M(z,Tz,t)\} \geq 0$

$\Psi\{N(z,z,t), N(Tz,Tz,t),$

$\frac{N(z,Tz,t) + N(z,Tz,t)}{2}, \frac{\alpha N(z,Tz,t)+\beta N(Tz,z,t)}{\alpha + \beta},$
$\frac{\gamma N(Tz,Tz,t)+\delta N(z,z,t)}{\gamma + \delta}, \quad N(z,Tz,t)\} \leq 0$

$$\Phi\left\{\begin{matrix} 1, 1, M(z,Tz,t), M(z,Tz,t), 1, \\ M(z,Tz,t) \end{matrix}\right\} \geq 0$$

$$\Psi\left\{\begin{matrix} 0, 0, N(z,Tz,t), N(z,Tz,t), 0, \\ N(z,Tz,t) \end{matrix}\right\} \leq 0.$$

By new implicit relation, we have M(z , Tz, t) ≥ 1 and N(z , Tz, t) ≤ 0, for all t > 0 In view of Φ and Ψ, we get Tz = z

Therefore Tz = z and STz = z ⟹ Sz = z. Hence Sz = Tz = z.

i.e., Az = Bz = Sz = Tz = Pz = Qz = z . Hence z is a common fixed point of A, B, S, T, and Q.

Uniqueness : Let w be another common fixed point of A, B, S, T, P and Q (w $\neq z$).

Put x = z and y = w in (d) we get

$$\Phi\{M(Az,PQz,t), M(STw,Bw,t), \frac{M(Az,Bw,t) + M(PQz,Bw,t)}{2}, \frac{\alpha M(PQz,STw,t)+\beta M(STw,Az,t)}{\alpha + \beta}, \frac{\gamma M(STw,Bw,t)+\delta M(Az,PQz,t)}{\gamma + \delta}, M(Az,STw,t)\} \geq 0$$

$$\Psi\{N(Az,PQz,t), N(STw,Bw,t), \frac{N(Az,Bw,t)+N(PQz,Bw,t)}{2}, \frac{\alpha N(PQz,STw,t)+\beta N(STw,Az,t)}{\alpha + \beta}, \frac{\gamma N(STw,Bw,t)+\delta N(Az,PQz,t)}{\gamma + \delta}, N(Az,STw,t)\} \leq 0$$

$$\Phi\{M(z,z,t), M(w,w,t), \frac{M(z,w,t) + M(z,w,t)}{2} \frac{M(z,w,t) + M(z,w,t)}{2} \frac{\alpha M(z,w,t)+\beta M(w,z,t)}{\alpha + \beta}, \frac{\gamma M(w,w,t)+\delta M(z,z,t)}{\gamma + \delta}, M(z,w,t)\} \geq 0$$

$$\Psi\{N(z,z,t), N(w,w,t), \frac{N(z,w,t) + N(z,w,t)}{2}, \frac{\alpha N(z,w,t)+\beta N(w,z,t)}{\alpha + \beta}, \frac{\gamma N(w,w,t)+\delta N(z,z,t)}{\gamma + \delta}, N(z,w,t)\} \leq 0$$

$$\Phi\{1, 1, M(z,w,t), M(z,w,t), 1, M(z,w,t)\} \geq 0$$
$$\Psi\{0, 0, N(z,w,t), N(z,w,t), 0, N(z,w,t)\} \leq 0$$

By new implicit relation, we have M(z , w, t) ≥ 1 and N(z , w, t) ≤ 0, for all t > 0. In view of Φ and Ψ , we get w = z . Hence z is a unique common fixed point of A, B, S,T, P and Q.

Remark 4.2 : If we put A = B, $T= Q = I_X$ in the theorem 4.1 we have the following.

Corollary 4.3: Let A, S and P be three self maps of an intuitionistic fuzzy metric space (X, M, N, * , ◊) with continuous t - norm * and continuous t - conorm ◊ defined by t * t ≥ t and (1 – t) ◊ (1 – t)≤ (1 – t) for all t ∈ [0, 1]. If the pairs (A, P) and (A, S) are sub compatible and sub sequentially continuous, then

a) A and P have a coincidence point ,
b) A and S have a coincidence point ,
c) for some Φ , Ψ ∈ M_6 and for all x, y∈ X and every t > 0

$$\Phi\{M(Ax,Px,t), M(Sy,Ay,t), \frac{M(Ax,Ay,t) + M(Px,Ay,t)}{2}, \frac{\alpha M(Px,Sy,t)+\beta N(Sy,Ax,t)}{\alpha + \beta}, \frac{\gamma M(Sy,Ay,t)+\delta M(Ax,Px,t)}{\gamma + \delta}, M(Ax,Sy,t)\} \geq 0$$

$$\Psi\{N(Ax,Px,t), N(Sy,Ay,t), \frac{N(Ax,Ay,t) + N(Px,Ay,t)}{2}, \frac{\alpha N(Px,Sy,t)+\beta N(Sy,Ax,t)}{\alpha + \beta}, \frac{\gamma N(Sy,Ay,t)+\delta N(Ax,Px,t)}{\gamma + \delta}, N(Ax,Sy,t)\} \leq 0$$

where $\alpha, \beta, \gamma, \delta \in \mathbb{R}$, Then A, S and P have a unique common fixed point.

Conclusion:

This article investigates common fixed point theorems for six self mappings. The concept of sub compatible maps and sub sequentially continuous maps in intuitionistic fuzzy metric spaces using implicit relations has also been used. Several fixed point theorems in intuitionistic fuzzy metric spaces such as fixed point theorems for four, three and two self mappings have been derived in the present study as particular cases.

References:

1. Alaca.C., Turkoglu.D, and Yildiz.C, Fixed points in intuitionistic fuzzy metric spaces,Chaos, Soliton & Fractals,**29**(2006), 1073-1078..
2. Al-Thagafi.M.A and N. Shahzad. Generalised I-non expansive self maps and invariants approximations .Acta Math.Sin.,24(5):867:876, 2008.
3. Atanassov.K, Intuitionistic Fuzzy sets, Fuzzy sets and system, 20(1986) 87-96.
4. Kramosil.I and Michalek.J. Fuzzy metric and statistical metric spaces. Kybernetica, 11:336:344, 1975.
5. Muralisankar.S and Kalpana.G Commonfixed point theorems in intuitionistic fuzzy metric spaces using reneral contractive condition of integral type. Int. J. Contemp. math.Sci., 4(11):505:518, 2009.
6. Popa.V. Some fixed point theorems for compatible mappings satisfying on implicit relation. Demonsratio Math., 32:157:163,1999.
7. Popa.V. A general coincidence theorem for compatible multivalued mappings satisfying an

implicit relation.Demonsratio Math., 33:159:164, 2000.
8. Park. J.H. Intuitionistic fuzzy metric spaces. Chaos, Solitons& Fractals, 22:1039:1046, 2004.
9. Schweizer. B and Sklar. A. Statistical metric spaces. Pacific J. Math., 0:313:334, 1960.
10. Singh. B, Jain. A, and Masoodi. A. A Semi-compatibility, weak compatibility and fixed point theorem in fuzzy metric space. Int. Math. Forum, 5(61):3041:3051, 2010.
11. Sessa.S. On a weak commutativity condition of mappings in fixed point considerations. Publ. Inst. Math.(Beogard) N.S.,32(46):149:153,1982.
12. Zadeh.L.A. Fuzzy sets. Inform. and control., 89:338:353, 1965.

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