TOTALLY FEEBLY CONTINUOUS FUNCTIONS

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Abstract: The aim of this paper is to present some new mappings like feebly clopen mapping, totally feebly continuous, strongly feebly continuous and slightly feebly continuous functions. The notion of feebly clopen separation space is introduced. Among several results we prove the composition of functions and characterizations of totally feebly continuous functions. Some fundamental properties are investigated by using the concepts of feebly clopen T_1, feebly clopen T_2, the separation axioms.

Keywords: Feebly open , feebly open mapping, feebly continuous, feebly-T_1,feebly-T_2, feebly clopen T_1, feebly clopen T_2.

Introduction:

Preliminaries: During the last few years the study of generalized closed and feebly closed mappings has found considerable interest among general topologists. Feebly closed and generalized mappings suggest some new concepts which have been to be very useful in study of a topology. In fact S.N. Maheswari and P.C. Jain [6] introduced the concepts of feebly open and feebly closed sets in a topological space (X,τ) . Dalal [2] proved that the map $f:X \to Y$ is feebly continuous if every singleton set in X is feebly open. In 1991, Mahide Kucuk and Idris Zorlutuna introduced feebly normality and feebly regularity in" S-separable spaces, feebly continuous functions and feebly separation axioms", Ganit11, (1-2), 19-24 MR95a:54036 ZbI 818.54022. We recall that in the subset A of X , the closure of A is the intersection of all closed sets containing A and the interior of A is the union of all open sets contained in A, denoted by cl (A) and int (A) respectively. A is said to be *semi-open* if $A \subseteq$ cl int (A) and semi-closed if int cl (A) $\subseteq$ A.

Definition 1.1[5]: A subset A of a topological space (X,τ) is said to be *feebly open* (resp. *feebly closed*) if $A \subset$ s cl int (A) (resp. s int cl(A) $\subset$ A). The feebly closure of A is the intersection of all feebly closed set containing A and is denoted by f cl A.

Remark 1.2 [7]: Let X and Y be the set of real numbers with usual topology, let the mapping f: $X \to Y$ be defined as follows $f(x) = x$ if $x \neq 0$ and $x \neq 1$,$f(0) = 1$ and $f(1) = 0$. Then f is one-one. Recall that a function f: $X \to Y$ is continuous if the inverse image of every open set in Y is open in X.

Definition 1.3 [8]: A map f : $X \to Y$ is said to be

(i)*Feebly closed* (resp. *feebly open*) if the image of each closed set in X (resp. open set) is feebly closed set in Y (resp. feebly open in Y).

(ii)*Feebly continuous* if f^{-1} (V) is feebly open in X for each open set V of Y.

Remark 1.4[1]: In a topological space,

(i) Every open set is feebly open

(ii) Every closed set is feebly closed

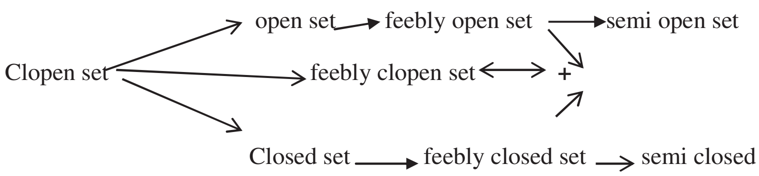
Definition 1.5[1]: A topological space (X,τ) is called

1. *Feebly-T_1* if there are feebly open sets U and V , x and y are the distinct points such that $x\in U$, $x \notin V$ and $y \in V$, $y\notin U$.
2. *Feebly-T_2* (*Feebly Hausdorff*) if for any pair of distinct points x,y of X, there exists disjoint feebly open sets U and V such that $x\in U$ and $y\in V$.
3. *Feebly-regular* if for all $x\in X$ and for all closed set A containing x there exists feebly open set H such that $x\in H \subset f\,cl\,H \subset A$.
4. *Feebly-normal* if for all disjoint closed sets H_1 , H_2 in X , there exists feebly open sets U_1 , U_2 in X such that $H_1 \subset U_1$,$H_2 \subset U_2$ and $U_1 \cap U_2 = \varphi$.

The Main Results:

Definition 2.1: Let (X,τ) be a topological spaces. Any subset A of X is called a *feebly clopen* subset of X if it is both feebly open and feebly closed.

Definition 2.2 : A function f : $X \to Y$ is said to be *feebly clopen* if the image of every open and closed set in Xis respectively feebly open and feebly closed in Y. We have the following diagram



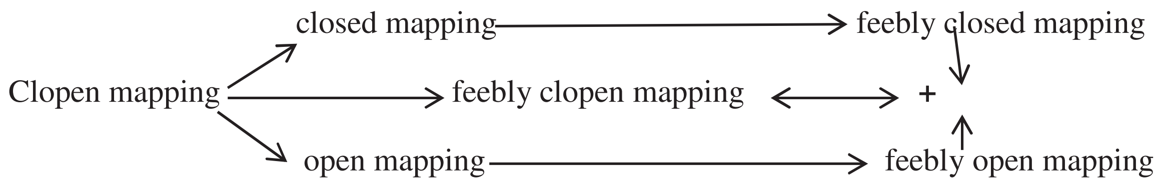
Theorem 2.3: Every feebly clopen mapping is feebly closed map and feebly open map.

proof: Let f: $X \to Y$ be a feebly clopen mapping. Let H be any clopen subset of X. Since f is feebly clopen mapping , f(H) is feebly closed and feebly open in Y. Hence f is feebly clopen mapping. The converse of this theorem is trivial.

Proposition 2.4 [2] : (i) A mapping f : $X \to Y$ is feebly open if $f(H^0) \subset (f(H))^0$ for every $H \subset X$ **(ii)** A mapping f : $X \to Y$ is feebly closed if $f(H) \subset f\,(H)$ for every $H \subset X$

Theorem 2.5: A mapping $f : X \rightarrow Y$ is feebly clopen iff f satisfies the conditions $f(H^0) \subset (f(H))^0$ for every $H \subset X$ and $f(H) \subset f(H)$ for every $H \subset X$

Proof: This theorem follows from proposition 2.4 and the following implication diagrams.



Totally feebly continuity :

In this section, we introduce totally feebly continuous, slightly feebly continuous and strongly feebly continuous functions and derived some theorems, and also the characterizations of these functions with its related functions are discussed.

Definition 3.1: A function $f : (X,\tau) \rightarrow (Y,\sigma)$ is said to be *totally feebly continuous* if the inverse image of every open subset of (Y,σ) is a feebly clopen subset of (X,τ).

Definition 3.2 : A function $f : (X,\tau) \rightarrow (Y,\sigma)$ is said to be *strongly feebly continuous* if the inverse image of every subset of Y is feebly clopen subset of (X,τ).

Definition 3.3 : A function $f :(X,\tau) \rightarrow (Y,\sigma)$ is said to be *slightly feebly continuous* if the inverse image of every feebly clopen set in Y is feebly open in X.

Theroem 3.4 : Every totally feebly continuous mapping is feebly continuous.

Proof: Let $f : X \rightarrow Y$ be the totally feebly continuous. Now we have to show that f is feebly continuous . Let H be any open set in Y. Since f is totally feebly continuous then $f^{-1}(H)$ is feebly clopen in X. By definition 3.1 , $f^{-1}(H)$ is feebly open and feebly closed set in X. This implies that $f^{-1}(H)$ is feebly open in X. Hence f is feebly continuous.

Proposition 3.5 [6]: Every continuous map is feebly continuous map.

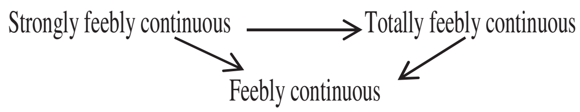
Theorem 3.6 : A function $f : X \rightarrow Y$ is totally feebly continuous if and only if the inverse image of every closed subset of Y is feebly clopen in X.

Proof: Let B be any closed set in Y. Then $Y - B$ is open set in Y. By definition 3.1, $f^{-1}(Y - B)$ is feebly clopen in X. That is $X - f^{-1}(B)$ is feebly clopen in X. This implies that the $f^{-1}(B)$ is feebly clopen in X. On the other hand if V is open in Y then Y-V is closed in Y. By hypothesis $f^{-1}(Y - V) = X - f^{-1}(V)$ is feebly clopen in X, which implies $f^{-1}(V)$ is feebly clopen in X. Thus the inverse image of every open set in Y is feebly clopen in X. Therefore, f is totally feebly continuous function.

Theorem 3.7 : Every strongly feebly continuous function is totally feebly continuous function.

Proof: Suppose $f : X \rightarrow Y$ is strongly feebly continuous function. Let A be any subset in Y. By definition, $f^{-1}(A)$ is feebly clopen in X. Thus the inverse image of each open set in Y is feebly clopen in X. Therefore f is totally feebly continuous.

Remark 3.8: The following implication gives the inter-relationship.



Theorem 3.9: Let $f : X \rightarrow Y$ be a function, where X and Y are topological spaces. The following are equivalent (i) f is totally feebly continuous (ii) for each $x \in X$ and each open set V in Y with $f(x) \in V$, there is a feebly clopen set U in X such that $x \in U$ and $f(U) \subset V$.

Proof : (i)⇒(ii) Suppose $f : X \rightarrow Y$ is totally feebly continuous and V be any open set in Y containing f(x), so that $x \in f^{-1}(V)$. Since f is totally feebly continuous, $f^{-1}(V)$ is feebly clopen in X. Let $U = f^{-1}(V)$ then U is feebly clopen set in X and $x \in U$. Also $f(U) = f(f^{-1}(V)) \subset V$. This implies that $f(U) \subset V$.

(ii) ⇒(i) Let V be open in Y. Let $x \in f^{-1}(V)$ be any arbitrary point. This implies that $f(x) \in V$. Therefore by (ii) there is a feebly clopen set $f(G_x) \subset X$ containing x such that $f(G_x) \subset V$, which implies $G_x \subset f^{-1}(V)$, we have $x \in G_x \subset f^{-1}(V)$. This implies that $f^{-1}(V)$ is feebly clopen neighbourhood of each of its points. Hence it is feebly clopen set in X. Therefore, f is totally continuous.

Theorem 3.10 : If $f : X \rightarrow Y$ is totally feebly continuous and A is feebly clopen subset of X, then the restriction $f/A : A \rightarrow Y$ is totally feebly continuous .

Proof: Consider the function $f/A : A \rightarrow Y$ and let V be any open set in Y. Since f is totally feebly continuous, $f^{-1}(V)$ is feebly clopen subset of X. Since A is feebly clopen subset of X and $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is feebly clopen in A, it follows $(f/A)^{-1}(V)$ is feebly clopen in A. Hence f/A is totally feebly continuous.

Composition of functions: Here we proved only the simple, similar and basic part of composition of functions.

Definition 4.1: A map $f : (X,\tau) \rightarrow (Y,\sigma)$ is said to be *feebly irresolute* if $f^{-1}(V)$ is feebly open in (X,τ) for each feebly open set V of (Y,σ). In [9], it is named as f^{**}eebly continuous function.

Definition 4.2: A map $f : (X,\tau) \to (Y,\sigma)$ is said to be *feebly clopen irresolute* if $f^{-1}(V)$ is feebly clopen in (X,τ) for each feebly clopen set V of (Y,σ).

Theorem 4.3: If $f : X \to Y$ is slightly feebly continuous and $g : Y \to Z$ is totally feebly continuous, then $g \circ f: X \to Z$ is feebly continuous.

Proof : Let V be a open set in Z. Since g is totally feebly continuous, $g^{-1}(V)$ is feebly clopen in Y. Now since f is slightly feebly continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is feebly open in X. Hence $g \circ f : X \to Z$ is feebly continuous.

Theorem 4.4 : If $f : X \to Y$ is feebly irresolute function and $g : Y \to Z$ is slightly feebly continuous, then $g \circ f : X \to Z$ is slightly feebly continuous.

Proof: Let V be feebly clopen in Z. Since g is slightly feebly continuous, $g^{-1}(V)$ is feebly open in Y. Now since f is feebly irresolute function, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1} (V)$ is feebly open in X. Hence $g \circ f : X \to Z$ is slightly feebly continuous.

Theorem 4.5: If $f : X \to Y$ is feebly clopen irresolute function and $g : Y \to Z$ is totally feebly continuous, then $g \circ f : X \to Z$ is totally feebly continuous.

Proof: Let V be open in Z. Since g is totally feebly continuous, $g^{-1}(V)$ is feebly clopen in Y. Now since f is feebly clopen irresolute function, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1} (V)$ is feebly clopen in X. Hence $g \circ f : X \to Z$ is totally feebly continuous.

Theorem 4.6: If $f : X \to Y$ is slightly feebly continuous and $g : Y \to Z$ is feebly clopen irresolute function, then $g \circ f : X \to Z$ is slightly feebly continuous.

Proof: Let V be feebly clopen in Z. Since g is feebly clopen irresolute function, $g^{-1}(V)$ is feebly clopen in Y. Now since f is feebly slightly feebly continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1} (V)$ is feebly open in X. Hence $g \circ f : X \to Z$ is slightly feebly continuous.

Theorem 4.7: If $f : X \to Y$ is feebly irresolute function and $g : Y \to Z$ is feebly continuous, then $g \circ f : X \to Z$ is feebly continuous.

Proof: Let V be open in Z. Since g is feebly continuous, $g^{-1}(V)$ is feebly open in Y. Now since f is feebly irresolute function, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1} (V)$ is feebly open in X. Hence $g \circ f : X \to Z$ is feebly continuous.

Separation Axioms:

Definition 5.1: A space X is said to be *feebly clopen- T_1* if for each pair of distinct points x and y of X, there exist feebly clopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

Definition 5.2: A space X is said to be *feebly clopen -T_2* (feebly clopen Hausdorff) if for each pair of distinct points x and y in X, there exist disjoint feebly clopen sets U and V in X such that $x \in U$ and $y \in V$.

Theorem 5.3: If $f: X \to Y$ is a slightly feebly continuous injection and Y is feebly clopen- T_1 then X is feebly- T_1.

Proof : Suppose that Y is feebly clopen- T_1. For any distinct points x and y in X., there exist feebly clopen set V and W such that $f(x) \in V$, $f(y) \notin V$ and $f(x) \notin W$, $f(y) \in W$. Since f is slightly feebly continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are feebly open sets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$ and $x \notin f^{-1}(W)$, $y \in f^{-1}(W)$. This shows that X is feebly- T_1.

Theorem 5.4: If $f : X \to Y$ is a slightly feebly continuous injection and Y is feebly clopen- T_2 then X is feebly -T_2.
Proof: For any pair of distinct points x and y in X, there exist disjoint of feebly clopen sets Uand V.

Proof: For any pair of distinct points x and y in X, there exist disjoint feebly clopen sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is slightly feebly continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are feebly open in X containing x and y respectively. Therefore $f^{-1}(U) \cap f^{-1}(V) = \varphi$, because $U \cap V = \varphi$. This shows that X is feebly -T_2.

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