DECOMPOSITIONS OF FUZZY $\ddot{g}$ -CONTINUITY

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Abstract: The aim of this paper is to give decompositions of a weaker form of fuzzy continuity, namely fuzzy $\ddot{g}$ -continuity, by providing the concepts of fuzzy $\ddot{g}_t$ -sets, fuzzy $\ddot{g}_{\alpha^*}$ -sets, fuzzy $\ddot{g}_t$ -continuity and fuzzy $\ddot{g}_{\alpha^*}$ -continuity.

Keywords: Fuzzy $\ddot{g}$ -closed set, fuzzy $\ddot{g}_\alpha$ -closed set, fuzzy $\ddot{g}_p$ -closed set, fuzzy $\ddot{g}_t$ -set, fuzzy $\ddot{g}_{\alpha^*}$ -set, fuzzy $\ddot{g}_t$ -continuity and fuzzy $\ddot{g}_{\alpha^*}$ -continuity.

Introduction: Various types of generalizations of fuzzy continuous functions were introduced and studied by various authors in the recent development of fuzzy topology. The decomposition of fuzzy continuity is one of many problems in fuzzy topology. Tong [11] obtained a decomposition of fuzzy continuity by introducing two weak notions of fuzzy continuity namely, fuzzy strong semi-continuity and fuzzy pre-continuity. Rajamani [8] obtained a decomposition of fuzzy continuity.

In this paper, we obtain decompositions of fuzzy $\ddot{g}$ -continuity in fuzzy topological spaces using fuzzy $\ddot{g}_p$ -continuity, fuzzy $\ddot{g}_\alpha$ -continuity, fuzzy $\ddot{g}_t$ -continuity and fuzzy $\ddot{g}_{\alpha^*}$ -continuity.

Preliminaries:

Definition 2.1 [9] [12]: If X is a set, then any function A: X→[0, 1] (from X to the closed unit interval [0, 1]) is called a fuzzy set in X.

Definition 2.2 [9]: If X is a set, then A, B : X→[0, 1] are fuzzy sets in X.

(i)The complement of a fuzzy set A, denoted by A′, is defined by

$A'(x) = 1 - A(x), \forall x \in X$,

(ii)Union of two fuzzy sets A and B, denoted by A∪B, is defined by

$(A \vee B)(x) = \max\{A(x), B(x)\}, \forall x \in X$,

(iii)Intersection of two fuzzy sets A and B, denoted by A ∩ B, is defined by

$(A \wedge B)(x) = \min\{A(x), B(x)\} \forall x \in X$.

Definition 2.3 [9] [12]

Let f : X→Y be a function from a set X into a set Y. Let A be a fuzzy subset in X and B be a fuzzy subset in Y. Then the Zadeh's functions f(A) and $f^{-1}(B)$ are defined by

(i) f(A) is a fuzzy subset of Y where

$$f(A)=\begin{cases} supA(z), & if\ f^{-1}(y) \neq \emptyset \\ z \in f^{-1}(y) & \\ 0, & otherwise \end{cases}$$

for each y ∈ Y.

(ii) $f^{-1}(B)$ is a fuzzy subset of X where $f^{-1}(B)(x) = B(f(x))$, for each x ∈ X.

Definition 2.4 [5] [9]: Let X be a set and τ be a family of fuzzy sets in X. Then τ is called a fuzzy topology if τ satisfies the following conditions:

1. 0, 1 $\in \tau$,
2. If $A_i \in \tau$, $i \in I$ then $\cup_{i\in I} A_i \in \tau$ or $\vee_{i \in I} A_i \in \tau$,
3. If A, B $\in \tau$ then A ∩ B $\in \tau$ or A ∧ B $\in \tau$.

The pair (X, τ) is called a fuzzy topological space (briefly fts). The elements of τ are called fuzzy open sets. Complements of fuzzy open sets are called fuzzy closed sets.

Definition 2.5 [9]: Let A be a fuzzy set in a fts (X, τ). Then,

(i) the closure of A, denoted by cl(A), is defined by

cl(A)=∧{ F : A≤ F and F is a fuzzy closed},

(i) the interior of A, denoted by int(A), is defined by

int(A)=∨{G : G≤ A and G is a fuzzy open}.

Definition 2.6

A subset A of a fts (X, τ) is called:

(i) fuzzy semi-open set [1] if A≤ cl(int(A)),

(ii)fuzzy preopen set [4] if A≤ int(cl(A)),

(iii)fuzzy α-ope set[4] if A≤ int(cl(int(A))).

The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets.

For a subset A of a fuzzy topological space X, the fuzzy α-closure (resp. fuzzy semi-closure, fuzzy pre-closure) of A, denoted by αcl(A) (resp. scl(A), pcl(A)), is the intersection of all fuzzy α-closed (resp. fuzzy semi-closed, fuzzy preclosed) subsets of X containing A. The union of all fuzzy α-open (resp. fuzzy semi-open, fuzzy preopen) subsets of X contained in A is called the fuzzy α-interior (resp. fuzzy semi-interior, fuzzy pre-interior) of A and denoted by αint(A) (resp. sint(A), pint(A)).

Definition 2.7 [5] [9]

A function f : X→Y is said to be fuzzy continuous if

$f^{-1}(\lambda)$ is fuzzy open in X for each fuzzy open set λ in Y.

Definition 2.8
A subset A of a fuzzy topological space (X, τ) is called:
(i) a fuzzy semi-generalized closed (briefly fsg-closed) set [3] if scl(A) ≤ U whenever A ≤ U and U is fuzzy semi-open in (X, τ). The complement of fsg-closed set is called fsg-open set.
(ii) a fuzzy $\ddot{g}$-closed set [7] if cl(A) ≤ U whenever A ≤ U and U is fsg-open in (X, τ). The complement of fuzzy $\ddot{g}$-closed set is called fuzzy $\ddot{g}$-open set.
The collection of all fuzzy $\ddot{g}$-closed (resp. fuzzy $\ddot{g}$-open) sets is denoted by F$\ddot{G}$ C(X) (resp. F$\ddot{G}$ O(X)).
(iii) a fuzzy $\ddot{g}_{\alpha}$-closed set [7] if αcl(A) ≤ U whenever A ≤ U and U is fsg-open in (X,τ). The complement of fuzzy $\ddot{g}_{\alpha}$-closed set is called fuzzy $\ddot{g}_{\alpha}$-open set.

Definition 2.9: A fuzzy subset A of a space (X, τ) is called:
(i) fuzzy t-set [11] if int(A) = int(cl(A)),
(ii) fuzzy α^*-set [6] if int(A) = int(cl(int(A))).

Proposition 2.10[7]: Every fuzzy $\ddot{g}$-closed set is fuzzy $\ddot{g}_{\alpha}$-closed but not conversely.

Example 2.11: Let X={a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly μ defined by μ(a)=0.4, μ(b)=0.4 is fuzzy $\ddot{g}_{\alpha}$-closed set but not fuzzy $\ddot{g}$-closed set in (X, τ) .

Proposition 2.12: Every fuzzy $\ddot{g}$-continuous function is fuzzy $\ddot{g}_{\alpha}$-continuous but not conversely.

Example 2.13: Let X=Y={a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5 and σ = {0_y, β, 1_y} where β is fuzzy set in Y defined by β(a)=0.6, β(b)=0.6. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let
f : (X, τ)→(Y, σ) be the identity fuzzy function. Clearly f is fuzzy $\ddot{g}_{\alpha}$-continuous but not fuzzy $\ddot{g}$-continuous.

Decompositions Of Fuzzy $\ddot{g}$-Continuity: We introduce the following definition.

Definition 3.1: A subset λ in a fuzzy topological space X is called fuzzy $\ddot{g}_{\alpha^*}$-set if

$\lambda = \alpha \wedge \beta$ where α is fuzzy $\ddot{g}$-open in X and β is fuzzy α^*-set in X.

The family of all fuzzy $\ddot{g}_{\alpha^*}$-sets in a space (X, τ) is denoted by F$\ddot{g}_{\alpha^*}$ (X, τ).

Example 3.2: Let X={a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly μ defined by μ(a)=0.4, μ(b)=0.5 is fuzzy $\ddot{g}_{\alpha^*}$-set.

Proposition 3.3: Every fuzzy $\ddot{g}$-closed set is fuzzy $\ddot{g}_{\alpha^*}$-set but not conversely.

Example 3.4: Let X={a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5.
Then (X, τ) is a fuzzy topological space. Clearly μ defined by μ(a)=0.4, μ(b)=0.4 is fuzzy $\ddot{g}_{\alpha^*}$-set but not fuzzy $\ddot{g}$-closed set in (X, τ) .

Remark 3.5: Fuzzy $\ddot{g}_{\alpha}$-closed sets and fuzzy $\ddot{g}_{\alpha^*}$-sets are independent of each other.

Example 3.6: Let X={a, b} with τ ={0_x, A, 1_x} where A is fuzzy set in X defined by A(a)=1, A(b)=0. Then (X, τ) is a fuzzy topological space. Clearly B defined by B(a)=0, B(b)=0.5 is a fuzzy $\ddot{g}_{\alpha}$-closed set but not fuzzy $\ddot{g}_{\alpha^*}$-set in (X, τ).

Example 3.7: Let X={a, b} with τ ={0_x, β_1, 1_x} and β_1 and β_2 are fuzzy sets in X defined by β_1(a)= 0.4, β_1(b)= 0.5 and β_2(a)= 0.5, β_2(b)= 0.5. Then (X, τ) is a fuzzy topological space. Clearly β_2 is fuzzy $\ddot{g}_{\alpha^*}$-set but not fuzzy $\ddot{g}_{\alpha}$-closed set in (X, τ).

Lemma 3.8: (i) A fuzzy subset A of (X, τ) is fuzzy $\ddot{g}$-open if and only if F≤ int(A) whenever 1− F is fsg-open and F ≤ A.
(ii) A fuzzy subset A of (X, τ) is fuzzy $\ddot{g}_{\alpha}$-open if and only if F≤αint(A) whenever 1−F is fsg-open and F ≤ A.

Proof:
(i) Necessity. Assume that A is fuzzy $\ddot{g}$-open in (X, τ). Let 1−F be fsg-open such that F≤A. Then 1−A≤1−F where 1−A is fuzzy $\ddot{g}$-closed. Hence cl(1−A)≤1−F and F≤1−cl(1−A)=int(A).
Sufficiency. To prove that A is fuzzy $\ddot{g}$-open under the given conditions, we prove 1-A is fuzzy $\ddot{g}$-closed in (X, τ). Let U be any fsg-open set such that 1−A≤U. Then 1−U≤A. Taking F=1−U, we have F≤A where 1−F is fsg-open. By assumption F≤int(A) which implies 1−U≤int(A) and hence 1−int(A) ≤U. Thus cl(1−A) ≤U which proves that 1−A is fuzzy $\ddot{g}$-closed and A is fuzzy $\ddot{g}$-open.
(ii) The proof follows immediately from Lemma 3.8(i).

Theorem 3.9: A fuzzy subset S is fuzzy $\ddot{g}$-open in (X, τ) if and only if it is both fuzzy $\ddot{g}_{\alpha}$-open and an fuzzy $\ddot{g}_{\alpha^*}$-set in (X, τ).

Proof: Necessity. The proof is obvious.
Sufficiency. Let S be a fuzzy $\ddot{g}_{\alpha}$-open set and a

fuzzy $\ddot{g}_{\alpha^*}$ -set. Since S is a fuzzy $\ddot{g}_{\alpha^*}$ -set, S = A ∧ B, where A is fuzzy $\ddot{g}$ -open and B is a fuzzy α^* -set. Assume that
F ≤ S, where F is fsg-closed in X. Since A is fuzzy $\ddot{g}$ -open, by Lemma 3.8 (i),
F ≤ int(A). Since S is fuzzy $\ddot{g}_{\alpha}$ -open in X, by Lemma 3.8 (ii),
F ≤ αint(S) = S ∧ int(cl(int(S))) = (A ∧ B) ∧ int(cl(int(A ∧ B))) ≤ A ∧ B ∧ int(cl(int(A))) ∧ int(cl(int(B))) = A ∧ B ∧ int(cl(int(A))) ∧ int(B) ≤ int(B). Therefore, we obtain F ≤ int(B) and hence F ≤ int(A) ∧ int(B) = int(S). Hence S is fuzzy $\ddot{g}$ -open, by Lemma 3.8 (i).

Definition 3.10: A function f : (X, τ)→(Y, σ) is said to be fuzzy $\ddot{g}_{\alpha^*}$ -continuous if for each fuzzy closed set λ of Y, $f^{-1}(\lambda)$ is fuzzy $\ddot{g}_{\alpha^*}$ -set in X.

Definition 3.11: A function f : (X, τ)→(Y, σ) is said to be
(i) fuzzy $\ddot{g}$ -continuous if for each fuzzy closed set λ of Y, $f^{-1}(\lambda)$ is fuzzy $\ddot{g}$ -closed in X,
(ii) fuzzy $\ddot{g}_{\alpha}$ -continuous if for each fuzzyclosed set λ of Y, $f^{-1}(\lambda)$ is fuzzy $\ddot{g}_{\alpha}$ -closed in X.
(iii) fuzzy $\ddot{g}_{p}$ -continuous if for each fuzzy closed set λ of Y, $f^{-1}(\lambda)$ is fuzzy $\ddot{g}_{p}$ -closed in X.

Example 3.12: Let X=Y={a, b} with τ = {0_x, α_1, 1_x} where α_1 is fuzzy set in X defined by α_1(a)=0.6, α_1(b)=0.5 and σ ={0_y, α_2, 1_y} where α_2 is fuzzy set in Y defined by α_2(a)=0.6, α_2(b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let
f : (X, τ)→(Y, σ) be the identity fuzzy function. Clearly f is fuzzy $\ddot{g}_{\alpha^*}$ -continuous.

Remark 3.13: Every fuzzy $\ddot{g}$ -continuous function is fuzzy $\ddot{g}_{\alpha^*}$ -continuous but not conversely.

Example 3.14: Let X=Y={a, b} with τ = {0_x, α_1, 1_x} where α_1 is fuzzy set in X defined by α_1 (a)=0.6, α_1 (b)=0.5 and σ = {0_y, α_2, 1_y} where α_2 is fuzzy set in Y defined by α_2(a)=0.6, α_2 (b)=0.6. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ)→(Y, σ) be the identity fuzzy function. Clearly f is fuzzy $\ddot{g}_{\alpha^*}$ -continuous but not fuzzy $\ddot{g}$ -continuous.

Remark 3.15: Fuzzy $\ddot{g}_{\alpha}$-continuity and fuzzy $\ddot{g}_{\alpha^*}$ -continuity are independent of each other.

Example 3.16: Let X=Y={a, b} with τ = {0_x, A, 1_x} where A is fuzzy set in X defined by A(a)=1, A(b)=0 and σ = {0_y, B, 1_y} where B is fuzzy set in Y defined by B(a)=1, B(b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let
f : (X, τ)→(Y, σ) be the identity fuzzy function. Clearly f is fuzzy $\ddot{g}_{\alpha}$ -continuous but not fuzzy $\ddot{g}_{\alpha^*}$ -continuous.

Example 3.17: Let X=Y={a, b} with τ ={0_x, α_1, 1_x} where α_1 is fuzzy set in X defined by α_1(a)=0.4, α_1(b)=0.5 and σ={0_y, α_2, 1_y} where α_2 is fuzzy set in Y defined by α_2(a)=0.5, α_2(b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let
f : (X, τ)→(Y, σ) be the identity fuzzy function. Clearly f is fuzzy $\ddot{g}_{\alpha^*}$ -continuous but not fuzzy $\ddot{g}_{\alpha}$ -continuous.

Theorem 3.18: A function f : (X, τ)→(Y, σ) is fuzzy $\ddot{g}$ -continuous if and only if it is both fuzzy $\ddot{g}_{\alpha}$ -continuous and fuzzy $\ddot{g}_{\alpha^*}$ -continuous.

Proof: The proof follows immediately from Theorem 3.9.

Definition 3.19: A subset A of X is called a fuzzy $\ddot{g}_{p}$ -closed set if pcl(A)≦U whenever A≦U and U is fsg-open in (X, τ).

Definition 3.20: A subset λ in a fuzzy topological space X is called fuzzy $\ddot{g}_{t}$ -set
if $\lambda = \alpha \wedge \beta$ where α is fuzzy $\ddot{g}$ -open in X and β is fuzzy t-set in X.
The family of all fuzzy $\ddot{g}_{t}$ -sets in a space (X, τ) is denoted by F $\ddot{g}_{t}$ (X, τ).

Example 3.21: Let X={a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly μ defined by μ(a)=0.4, μ(b)=0.5 is fuzzy $\ddot{g}_{t}$ -set.

Proposition 3.22: Every fuzzy $\ddot{g}$ -closed set is fuzzy $\ddot{g}_{t}$ -set but not conversely.

Example 3.23: Let X={a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly μ defined by μ(a)=0.4, μ(b)=0.4 is fuzzy $\ddot{g}_{t}$ -set but not fuzzy $\ddot{g}$ -closed set in (X, τ) .

Remark 3.24: Fuzzy $\ddot{g}_{p}$ -closed sets and fuzzy $\ddot{g}_{t}$ -sets are independent of each other.

Example 3.25: Let X={a, b} with τ = {0_x, A, 1_x} where A is fuzzy set in X defined by A(a)=1, A(b)=0. Then (X, τ) is a fuzzy topological space. Clearly B defined by B(a)=0, B(b)=0.5 is a fuzzy $\ddot{g}_{p}$ -closed set but not fuzzy $\ddot{g}_{t}$ -set in (X, τ).

Example 3.26: Let X={a, b} with $\tau = \{0_x, \beta_1, 1_x\}$ and β_1 and β_2 are fuzzy sets in X defined by β_1(a)= 0.4, β_1(b)= 0.5 and β_2(a)= 0.5, β_2(b)= 0.5. Then (X, τ) is a fuzzy topological space. Clearly β_2 is fuzzy $\ddot{g}_t$-set but not fuzzy $\ddot{g}_p$-closed set in (X, τ).

Lemma 3.27: (i) A fuzzy subset A of (X, τ) is fuzzy $\ddot{g}$-open if and only if F≤ int(A) whenever 1– F is fsg-open and F ≤ A.

(ii) A fuzzy subset A of (X, τ) is fuzzy $\ddot{g}_p$-open if and only if F≤pint(A) whenever 1–F is fsg-open and F ≤ A.

Theorem 3.28: A fuzzy subset S is fuzzy $\ddot{g}$-open in (X, τ) if and only if it is both fuzzy $\ddot{g}_p$-open and an fuzzy $\ddot{g}_t$-set in (X, τ).

Proof: Similar to Theorem 3.9.

Definition 3.29: A function f : (X, τ)→(Y, σ) is said to be fuzzy $\ddot{g}_t$-continuous if for each fuzzy closed set λ of Y, $f^{-1}(\lambda)$ is fuzzy $\ddot{g}_t$-set in X.

Example 3.30: Let X=Y={a, b} with $\tau = \{0_x, \alpha_1, 1_x\}$ where α_1 is fuzzy set in X defined by α_1(a)=0.6, α_1(b)=0.5 and σ ={0_y, α_2, 1_y} where α_2 is fuzzy set in Y defined by α_2(a)=0.6, α_2(b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ)→(Y, σ) be the identity fuzzy function. Clearly f is fuzzy $\ddot{g}_t$-continuous.

Remark 3.31: Every fuzzy $\ddot{g}$-continuous function is fuzzy $\ddot{g}_t$-continuous but not conversely.

Example 3.32: Let X=Y={a, b} with τ ={0_x, α_1, 1_x} where α_1 is fuzzy set in X defined by α_1 (a)=0.6, α_1 (b)=0.5 and σ = {0_y, α_2, 1_y} where α_2 is fuzzy set in Y defined by α_2(a)=0.6, α_2(b)=0.6. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ)→(Y, σ) be the identity fuzzy function. Clearly f is fuzzy $\ddot{g}_t$-continuous but not fuzzy $\ddot{g}$-continuous

Remark 3.33: Fuzzy $\ddot{g}_p$-continuity and fuzzy $\ddot{g}_t$-continuity are independent of each other.

Example 3.34: Let X=Y={a, b} with τ = {0_x, A, 1_x} where A is fuzzy set in X defined by A(a)=1, A(b)=0 and σ = {0_y, B, 1_y} where B is fuzzy set in Y defined by B(a)=1, B(b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ)→(Y, σ) be the identity fuzzy function. Clearly f is fuzzy $\ddot{g}_p$-continuous but not fuzzy $\ddot{g}_t$-continuous.

Example 3.35: Let X=Y={a, b} with τ ={0_x, α_1, 1_x} where α_1 is fuzzy set in X defined by α_1 (a)=0.4, α_1(b)=0.5 and σ = {0_y, α_2, 1_y} where α_2 is fuzzy set in Y defined by α_2 (a)=0.5, α_2 (b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ)→(Y, σ) be the identity fuzzy function. Clearly f is fuzzy $\ddot{g}_t$-continuous but not fuzzy $\ddot{g}_p$-continuous.

Theorem 3.36: A function f : (X, τ)→(Y, σ) is fuzzy $\ddot{g}$-continuous if and only if it is both fuzzy $\ddot{g}_p$-continuous and fuzzy $\ddot{g}_t$-continuous.

Proof: The proof follows immediately from Theorem 3.28.

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