BAYESIAN DOUBLE SAMPLING PLAN USING MINIMUM RISK

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Abstract: A Bayesian acceptance sampling plan, which is used for prior information on the process variation for taking decisions about the submitted lots, can be employed as an alternative to conventional plans. This paper focuses on Bayesian double sampling inspection plan (0, 1) under the condition of Gamma-Poisson distribution. The design parameters of the proposed plan is constructed with specified two points on operating characteristic such as acceptable and limiting quality level along with producer's and consumer's risk. The optimal parameters for Bayesian Double sampling plan are determined using minimum sum of risk.

Keywords: Bayesian Acceptance Sampling Plan, Double sampling plan, Gamma-Poisson distribution.

Introduction: Acceptance sampling is one the major area in statistical quality control. An acceptance sampling plan deals with decision either to accept or reject a submitted lot based on the results of inspection of samples. Acceptance sampling helps manufacture products according to the desired specification limits and hence yields quality of assurance. Acceptance sampling approach is mainly classified as sampling inspection by variables and sampling inspection by attributes. Many types of sampling plans are available, including single sampling plans, double sampling plans, multiple sampling plans, continuous sampling plans, and etc. The proposed result is developing a new attributes Bayesian double sampling plan

A classical study is directed towards the use of sample information. In addition to the sample information two other types of information are typically relevant. The first is knowledge of the possible consequences of the decision and the second source of non-sampling information is prior information. Thomas Bayes (1702) was first to use the prior information in inductive inference and the approach to statistics, which formally seeks to utilize prior information is called Bayesian analysis. Suppose a product in a series is produced, due to random fluctuations, variation can be separated in to within lot (sampling) variation of individual units and between lot (sampling and process) variations.

Bayesian Acceptance Sampling approach is associated with the utilization of prior process history for the selected distributions (viz., Gamma Poisson, Beta Binomial), to describe the random fluctuations involved in Acceptance Sampling. Bayesian sampling plans requires the user to specify explicitly the distribution of defectives from lot to lot. The prior distribution is the expected distribution of a lot quality on which the sampling plan is going to operate. The distribution is called prior because it is formulated prior to the taking of samples. The combination of prior knowledge, represented with the prior distribution, and the empirical knowledge based on the samples which leads to the decision on the lot.

Hald [2] has also provided an excellent comparison of Classical and Bayesian Theory and methodology for attributes acceptance sampling. Suresh and Latha [7] studies on average probability of acceptance function for single sampling plan with Gamma Prior distribution. They also derived the plan with inflection point and tangent at the inflection point. Saminathan [5] have developed a method for finding the parameters of average sample numbers and a combination of two quality levels for Bayesian Double Sampling plan under the condition of Gamma-Poisson.

Golub [3] has given a method and tables for finding the acceptance number c of a single sampling involving a minimum sum of producer's and consumer's risks for given AQL and LQL when the sample size is fixed. Soundararajan [6] has extended his approach to single sampling plan under the condition for application of Poisson model. Govindaraju and Subramanian [4] has developed double sampling plan involving minimum sum of risks for specified acceptable and limiting quality level.

Double Sampling Attributes Plan

The *DSP-(0, 1)* plan is valid under general conditions for application of attributes sampling inspection. However, this plan will specially be useful to product characteristics that involve costly or destructive testing. The plan is as follows-

- From a lot, select a random sample of size n_1 units and observe the number of nonconforming units d_1.
  If $d_1=0$, accept the lot. If $d_1 \geq 1$, reject the lot.
  If $d_1=1$, select a second random sample of n_2 units and observe the number of nonconforming units, d_2.
- If $d_2=0$, accept the lot; else reject it.

Thus, the *DSP-(0, 1)* plan is specified by the constants n_1 and n_2. It is here assumed that n_1, n_2=n. Under the condition for application of Poisson model with acceptance number $c_1=0$ and $c_2=1$ is given by Dodge and Romig [1] as

$$P_a(p) = e^{-n_1 p} + n_1 p\, e^{-(n_1+n_2)p} \quad (1)$$

From the past history of inspection, suppose the product quality p has a Gamma prior distribution with density function is given by

$$w(p) = \frac{e^{-pt} p^{s-1} t^s}{\Gamma s}, \text{ s, t} > 0, \text{ p} > 0 \quad (2)$$

Where s and t are parameters with mean $\overline{P} = \frac{s}{t} = \mu$ (say).the Average Probability of Acceptance for Gamma

Prior distribution is, $\overline{P} = \int_0^\infty P(p/n_1, n_2) w(p) dp$

From equation (1) and (2), APA is obtained as

$$\overline{P} = \left(\frac{s}{s+n\mu}\right)^s + n\mu\left(\frac{s}{s+n\mu}\right)^{s+1}\left(\frac{s}{s+n\mu}\right)^s$$

Selection of Minimum Risk Bayesian Double Sampling Plan:

The following procedure is used for selecting plan for given μ_1, μ_2, α and β.

1. Compute the operating ratio μ_2 / μ_1
2. With the computed value of μ_2 / μ_1 enter Table 1 in the row headed by μ_2 / μ_1 , which is equal or just smaller than the computed ratio.
3. For determining the parameter c_1, c_2 and s, one proceeds from left to right in the row identified in step 2 such that the tabulated producer's and the consumer's risks are equal to or just less than the desired values.
4. The sample size n is obtained as $n = n\mu_1 / \mu_1$ values are given in the column heading to the acceptance numbers obtained in step 3.

For example, for given μ_1 = 0.02 and μ_2 = 0.40 with α = 0.05 and β = 0.10, from table one finds the *BDSP-(0,1)* involving minimum sum of risks as follows

1. Tabulated $\frac{\mu_2}{\mu_1} = \frac{0.40}{0.02} = 20$
2. Corresponding to c_1=0, c_2=1 and s = 3 given in the Table (1) one obtain α=0.01 and β = 0.2 against the desired α = 0.05 and β = 0.10
3. $n = \frac{n\mu_1}{\mu_1}\frac{0.50}{0.02} = 25$

Construction of Tables: The probability of acceptance function for Bayesian *DSP-(0, 1)*

$$\overline{P} = \left(\frac{s}{s+n\mu}\right)^s + n\mu\left(\frac{s}{s+n\mu}\right)^{s+1}\left(\frac{s}{s+n\mu}\right)^s$$

The expression for the sum of producer's and consumer's risk is given by

$$\alpha + \beta = 1 - P_a(\mu_1) + P_a(\mu_2) \qquad (3)$$

If the operating ratio μ_2 / μ_1 and $n\mu_1$ are known, then $n\mu_2$ can be written as

$$n\mu_2 = n\mu_1(\mu_2 / \mu_1) \qquad (4)$$

For fixed $n\mu_1$, the value of $n\mu_2$ is calculated from equation (4) and is used in equation (3). Hence the Table 1 provided in this paper can be used to select the Bayesian double sampling plan discussed the above which involve the minimum sum of risk for given μ_1, μ_2, α and β.

Conclusion: The result presented in this paper are mainly related with new procedure and necessary tables for selection of sampling system through minimum sum of risks involving producer's and consumer's quality levels. The emphasis in the present work is that the selection of sampling system with this procedure is more advantages to the producer and consumer. Tables are provided here which are tailor made, handy and ready- made uses to the industrial shop-floor conditions.

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nμ μ_1/μ_1	0.50	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.50	0.55	0.60	0.70
50	0,1,3 0.00,0.2	0,1,3 0.05,0.02	0,1,3 0.00,0.05	0,1,3 0.04,0.02	0,1,3 0.02,0.05	0,1,3 0.03,0.06	0,1,3 0.01,0.11	0,1,3 0.03,0.11	0,1,3 0.00,0.2	0,1,3 0.02,0.2	0,1,3 0.03,0.22	0,1,3 0.00,0.3
49	0,1,3 0.00,0.2	0,1,3 0.02,0.05	0,1,3 0.03,0.02	0,1,3 0.04,0.02	0,1,3 0.02,0.05	0,1,3 0.00,0.1	0,1,3 0.01,0.11	0,1,3 0.03,0.11	0,1,3 0.00,0.2	0,1,3 0.02,0.2	0,1,3 0.03,0.22	0,1,3 0.00,0.3
44	0,1,3 0.00,0.2	0,1,3 0.03,0.05	0,1,3 0.04,0.02	0,1,3 0.04,0.02	0,1,3 0.01,0.07	0,1,3 0.00,0.1	0,1,3 0.01,0.11	0,1,3 0.04,0.11	0,1,3 0.00,0.2	0,1,3 0.02,0.2	0,1,3 0.03,0.22	0,1,3 0.00,0.3
41	0,1,3 0.00,0.2	0,1,3 0.08,0.01	0,1,3 0.03,0.03	0,1,3 0.04,0.02	0,1,3 0.01,0.07	0,1,3 0.00,0.1	0,1,3 0.01,0.11	0,1,3 0.04,0.11	0,1,3 0.00,0.2	0,1,3 0.02,0.2	0,1,3 0.03,0.22	0,1,3 0.00,0.3
40	0,1,3 0.00,0.2	0,1,3 0.0,0.10	0,1,3 0.05,0.01	0,1,3 0.02,0.05	0,1,3 0.01,0.07	0,1,3 0.00,0.1	0,1,3 0.01,0.11	0,1,3 0.04,0.11	0,1,3 0.00,0.2	0,1,3 0.02,0.2	0,1,3 0.03,0.22	0,1,3 0.00,0.3
38	0,1,3 0.00,0.2	0,1,3 0.01,0.1	0,1,3 0.04,0.03	0,1,3 0.02,0.05	0,1,3 0.01,0.07	0,1,3 0.00,0.1	0,1,3 0.01,0.11	0,1,3 0.04,0.11	0,1,3 0.00,0.2	0,1,3 0.02,0.2	0,1,3 0.03,0.22	0,1,3 0.00,0.3
35	0,1,3 0.00,0.2	0,1,3 0.02.0.1	0,1,3 0.04,0.04	0,1,3 0.02,0.05	0,1,3 0.01,0.07	0,1,3 0.00,0.1	0,1,3 0.01,0.11	0,1,3 0.04,0.11	0,1,3 0.00,0.2	0,1,3 0.02,0.21	0,1,3 0.03,0.22	0,1,3 0.00,0.3
31	0,1,3 0.00,0.2	0,1,3 0.05,0.1	0,1,3 0.04,0.05	0,1,3 0.01,0.07	0,1,3 0.04,0.05	0,1,3 0.01,0.1	0,1,3 0.02,0.11	0,1,3 0.04,0.11	0,1,3 0.00,0.2	0,1,3 0.02,0.21	0,1,3 0.03,0.22	0,1,3 0.00,0.3
28	0,1,3 0.00,0.2	0,1,3 0.03,0.15	0,1,3 0.01,0.1	0,1,3 0.03,0.06	0,1,3 0.04,0.05	0,1,3 0.01,0.1	0,1,3 0.02,0.11	0,1,3 0.04,0.11	0,1,3 0.00,0.2	0,1,3 0.02,0.21	0,1,3 0.03,0.22	0,1,3 0.00,0.3
25	0,1,3 0.00,0.2	0,1,3 0.01,0.21	0,1,3 0.03,0.1	0,1,3 0.00,0.1	0,1,3 0.00,0.1	0,1,3 0.01,0.1	0,1,3 0.02,0.11	0,1,3 0.04,0.11	0,1,3 0.00,0.2	0,1,3 0.02,0.21	0,1,3 0.03,0.22	0,1,3 0.00,0.3
23	0,1,3 0.00,0.2	0,1,3 0.01,0.22	0,1,3 0.04,0.1	0,1,3 0.01,0.1	0,1,3 0.01,0.1	0,1,3 0.01,0.11	0,1,3 0.02,0.12	0,1,3 0.01,0.15	0,1,3 0.00,0.2	0,1,3 0.02,0.21	0,1,3 0.03,0.23	0,1,3 0.00,0.3
20	0,1,3 0.01,0.2	0,1,3 0.05,0.23	0,1,3 0.03,0.14	0,1,3 0.02,0.11	0,1,3 0.01,0.11	0,1,3 0.01,0.12	0,1,3 0.02,0.12	0,1,3 0.01,0.15	0,1,3 0.01,0.2	0,1,3 0.02,0.21	0,1,3 0.03,0.23	0,1,3 0.01,0.3
17	0,1,3 0.01,0.2	0,1,3 0.04,0.01	0,1,3 0.02,0.2	0,1,3 0.03,0.13	0,1,3 0.05,0.1	0,1,3 0.01,0.14	0,1,3 0.02,0.14	0,1,3 0.02,0.15	0,1,3 0.01,0.2	0,1,3 0.02,0.22	0,1,3 0.03,0.23	0,1,3 0.01,0.3
15	0,1,3 0.02,0.2	0,1,3 0.03.0.36	0,1,3 0.01,0.15	0,1,3 0.05,0.14	0,1,3 0.03,0.14	0,1,3 0.03,0.13	0,1,3 0.01,0.16	0,1,3 0.04,0.14	0,1,3 0.02,0.2	0,1,3 0.01,0.23	0,1,3 0.03,0.24	0,1,3 0.01,0.3
13	0,1,3 0.03,0.2	0,1,3 0.02,0.43	0,1,3 0.01,0.3	0,1,3 0.03,0.2	0,1,3 0.00,0.2	0,1,3 0.04,0.14	0,1,3 0.04,0.15	0,1,3 0.00,0.2	0,1,3 0.02,0.21	0,1,3 0.04,0.21	0,1,3 0.03,0.24	0,1,3 0.02,0.3
12	0,1,3 0.04,0.2	0,1,3 0.05,0.44	0,1,3 0.02,0.32	0,1,3 0.01,0.24	0,1,3 0.01,0.2	0,1,3 0.00,0.2	0,1,3 0.00,0.2	0,1,3 0.00.0.2	0,1,3 0.02,0.22	0,1,3 0.03,0.23	0,1,3 0.03,0.25	0,1,3 0.02,0.3
11	0,1,3 0.04,0.2	0,1,3 0.05,0.54	0,1,3 0.05,0.22	0,1,3 0.03,0.25	0,1,3 0.02,0.22	0,1,3 0.02,0.2	0,1,3 0.001,0.2	0,1,3 0.02,0.2	0,1,3 0.02,0.22	0,1,3 0.03,0.23	0,1,3 0.03,0.25	0,1,3 0.02,0.3
10	0,1,3 0.05,0.2	0,1,3 0.05,0.52	0,1,3 0.01,0.4	0,1,3 0.02,0.3	0,1,3 0.02,0.24	0,1,3 0.02,0.22	0,1,3 0.03,0.2	0,1,3 0.01,0.22	0,1,3 0.02,0.23	0,1,3 0.04,0.23	0,1,3 0.03,0.26	0,1,3 0.03,0.3
7	0,1,3 0.01,0.3	0,1,3 0.00,0.7	0,1,3 0.03,0.53	0,1,3 0.01,0.45	0,1,3 0.05,0.34	0,1,3 0.03,0.31	0,1,3 0.02,0.3	0,1,3 0.01,0.3	0,1,3 0.01,0.3	0,1,3 0.02,0.3	0,1,3 0.03,0.3	0,1,3 0.03,0.33
5	0,1,3 0.04,0.35	0,1,3 0.01,0.8	0,1,3 0.03,0.63	0,1,3 0.00,0.6	0,1,3 0.03,0.5	0,1,3 0.05,0.42	0,1,3 0.02,0.42	0,1,3 0.01,0.4	0,1,3 0.04,0.35	0,1,3 0.05,0.34	0,1,3 0.00.0.4	0,1,3 0.02,0.4

Table I: Parametric values for Bayesian Double sampling plan using minimum sum of risks

Key c_1, c_2, m, α and β