ON WEAKLY g'''-CLOSED SETS IN FUZZY TOPOLOGY

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Abstract: In this paper, we offer a new class of sets called weakly fuzzy g'''-closed sets in fuzzy topological spaces. It turns out that this class lies between the class of fuzzy closed sets and the class of fuzzy generalized closed sets.

Keywords: Fuzzy Topological space, fuzzy g'''-closed set, fαg-closed set, fgsp-closed set and weakly fuzzy g'''-closed set.

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Introduction : Quite Recently, Jeyaraman et al. [6] have introduced the concept of fuzzy g'''-closed sets and studied its basic fundamental properties in fuzzy topological spaces. In this paper, we introduce a new class of fuzzy generalized closed sets called weakly fuzzy g'''-closed sets which contains the above mentioned class. Also, we investigate the relationships among related fuzzy generalized closed sets.

2.Preliminaries: Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

We recall the following definitions which are useful in the sequel.

2.1Definition: A fuzzy subset A of a space (X, τ) is called:

1. fuzzy semi-open set [1] if $A \leq cl(int(A))$;
2. fuzzy α-open set [4] if $A \leq int(cl(int(A)))$;
3. fuzzy semi preopen set[13] if $A \leq cl(int(cl(A)))$;
4. fuzzy regular open set [1] if $A = int(cl(A))$.

The complements of the above mentioned open sets are called their respective closed sets.

The fuzzy semi-closure [15] (resp. fuzzy α-closure [7], fuzzy semi-preclosure [13]) of a fuzzy subset A of X, denoted by scl(A) (resp. αcl(A), spcl(A)) is defined to be the intersection of all fuzzy semi-closed (resp. fuzzy α-closed, fuzzy semi-preclosed) sets of (X, τ) containing A. It is known that scl(A) (resp. αcl(A), spcl(A)) is a fuzzy semi-closed (resp. fuzzy α-closed, fuzzy semi-preclosed) set.

2.2Definition:

A fuzzy subset A of a space (X, τ) is called:

1. A fuzzy generalized closed (briefly fg-closed) set [2] if $cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in (X, τ). The complement of fuzzy g-closed set is called fuzzy g-open set;
2. A fuzzy semi-generalized closed (briefly fsg-closed) set [3] if $scl(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi-open in (X,τ). The complement of fsg-closed set is called fsg-open set;
3. A fuzzy generalized semi-closed (briefly fgs-closed) set [10] if $scl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in (X, τ). The complement of fgs-closed set is called fgs-open set;
4. A fuzzy α-generalized closed (briefly fαg-closed) set [11] if $\alpha cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in (X, τ). The complement of fαg-closed set is called fαg-open set;
5. A fuzzy generalized semi-preclosed (briefly fgsp-closed) set [9] if $spcl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in (X,τ). The complement of fgsp-closed set is called fgsp-open set;
6. A fuzzy g'''-closed set [6] if $cl(A) \leq U$ whenever $A \leq U$ and U is fgs-open in (X, τ). The complement of fuzzy g'''-closed set is called fuzzy g'''-open set.

Remark 2.4: Every fuzzy open set is fgs-open set but not conversely.

Example 2.5: Let X = {a, b} with $\tau = \{0_x, A, 1_x\}$ where A is fuzzy set in X defined by A(a)=1, A(b)=0. Then (X, τ) is a fuzzy topological space. Clearly B defined by B(a)=0.5, B(b)=0 is fgs-open set but not fuzzy open.

3.Weakly Fuzzy g'''-Closed Sets: We introduce the definition of weakly fuzzy g'''-closed sets in fuzzy topological spaces and study the relationships of such sets.

Definition 3.1: A fuzzy subset A of a fuzzy topological space (X, τ) is called a weakly fuzzy g'''-closed (briefly w fg'''-closed) set if $cl(int(A)) \leq U$ whenever $A \leq U$ and U is fgs-open in (X, τ).

The complement of weakly fuzzy g'''-closed set is called weakly fuzzy g'''-open set.

Proposition 3.2: Every fuzzy closed set is weakly fuzzy g'''-closed but not conversely.

Proof: If A is any fuzzy closed set in (X, τ) and G is any fgs-open set such that $A \leq G$, then cl(A) = A, $cl(int(A)) \leq cl(A) = A$. We have $cl(int(A)) \leq A \leq G$ whenever $A \leq G$ and G is fgs-open. Hence A is weakly fuzzy g'''-closed.

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3: Let X = {a, b} with $\tau = \{0_x, A, 1_x\}$ where A is fuzzy set in X defined by A(a)=1, A(b)=0. Then (X,τ) is a fuzzy topological space. Clearly B defined by

B(a)=0.5, B(b)=1 is weakly fuzzy g'''-closed set but not fuzzy closed.

Proposition 3.4: Every fuzzy g'''-closed set is weakly fuzzy g'''-closed but not conversely.

Proof: If A is a fuzzy g'''-closed set in (X, τ) and G is any fgs-open set such that A ≤ G, then cl(int(A)) ≤ cl(A) ≤ G. Hence A is weakly fuzzy g'''-closed.

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5: Let X = {a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by (a)=0.6, λ(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly μ defined by (a)=0.4, μ(b)=0.4 is weakly fuzzy g'''-closed set but not fuzzy g'''-closed set in (X, τ) .

Proposition 3.6: Every fuzzy regular closed set is weakly fuzzy g'''-closed but not conversely.

Proof: If A is any regular closed set and G is any fgs-open set such that A ≤ G, then A = cl(int(A)) ≤ G. Hence A is weakly fuzzy g'''-closed.

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7: Let X = {a, b} with τ = {0_x, A, 1_x} where A is fuzzy set in X defined by A(a)=1, A(b)=0. Then (X, τ) is a fuzzy topological space. Clearly B defined by B(a)=0.5, B(b)=1 is weakly fuzzy g'''-closed set but not fuzzy regular closed set.

Proposition 3.8: Every weakly fuzzy g'''-closed set is fgsp-closed but not conversely.

Proof: Suppose that A ≤ G and G is fuzzy open in (X, τ) . Since every fuzzy open set is fgs-open set and A is weakly fuzzy g'''-closed set, therefore spcl(A) ≤ G. Hence A is fgsp-closed set.

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9: Let X = {a, b} with τ = {0_x, α, 1_x} where α is fuzzy set in X defined by α(a)=0.4, α(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly β defined by β(a)=0.5, β(b)= 0.5 is fgsp-closed set but not weakly fuzzy g'''-closed set in (X, τ).

Proposition 3.10: Every fuzzy g-closed set is weakly fuzzy g'''-closed but not conversely.

Proof: Suppose that A ≤ G and G is fuzzy open in (X, τ) . Since every fuzzy open set is fgs-open set and A is fuzzy g-closed set, cl(int(A)) ≤ cl(A) ≤ G. Hence A is weakly fuzzy g'''-closed set.

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11: Let X = {a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly μ defined by (a)=0.5, μ(b)=0.5 is weakly fuzzy g'''-closed set but not fuzzy g -closed set in (X, τ).

Theorem 3.12: If a subset A of a fuzzy topological space (X, τ) is both fuzzy closed and fαg-closed, then it is weakly fuzzy g'''-closed set in (X, τ).

Proof : If A is fαg-closed set in (X, τ) and G is fuzzy open set such that A ≤ G, since A is fuzzy closed,we have cl(int(A)) = cl(int(cl(A))) ≤ A ∪ cl(int(cl(A))) = αcl(A) ≤ G. Hence A is weakly fuzzy g'''-closed set in (X, τ).

Theorem 3.13: If a subset A of a fuzzy topological space (X, τ) is both fuzzy open and weakly fuzzy g'''-closed set, then it is fuzzy closed.

Proof: If A is both fuzzy open and weakly fuzzy g'''-closed set in (X, τ), A ≥ cl(int(A)) = cl(A). Hence A is fuzzy closed in (X, τ).

Corollary 3.14: If a subset A of a fuzzy topological space (X, τ) is both fuzzy open and weakly fuzzy g'''-closed set, then it is both fuzzy regular open and fuzzy regular closed in (X, τ).

Proof: If A is both fuzzy open and weakly fuzzy g'''-closed set in (X, τ), and by Theorem 3.13, we have cl(int(A)) = A and int(cl(A)) = A. Hence A is both fuzzy regular open and fuzzy regular closed in (X, τ).

Remark 3.15: The following examples show that weakly fuzzy g'''-closed set and fuzzy semi-closed set are independent.

Example 3.16: Let X = {a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly μ defined by (a)=0.5, μ(b)=0.5 is weakly fuzzy g'''-closed set but not fuzzy semi-closed set in (X, τ) .

Example 3.17: Let X = {a, b} with τ = {0_x, α, 1_x} where α is fuzzy set in X defined by α(a)=0.4, α(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly β defined by β(a)=0.5, β(b)=0.5 is fuzzy semi-closed set but not weakly fuzzy g'''-closed set in (X, τ).

Theorem 3.18: Every fuzzy open set is weakly fuzzy g'''-open set in (X, τ).

Proof: If A is a fuzzy open set in a fuzzy topological space (X, τ). Then A^c is fuzzy closed in (X, τ). By Proposition 3.2, it follows that A^c is weakly fuzzy g'''-closed in (X, τ). Hence A is weakly fuzzy g'''-open in (X, τ).

The converse of Theorem 3.18 need not be true as seen from the following example.

Example 3.19: Let X = {a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by (a)=0.6, λ(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly μ defined by (a)=0.5, μ(b)=0.5 is weakly fuzzy g'''-open set but not fuzzy open set in (X, τ).

Theorem 3.20:

1. Every fuzzy g'''-open set is weakly fuzzy g'''-open set but not conversely.
2. Every fuzzy regular open set is weakly fuzzy g'''-open set but not conversely.

3. Every fuzzy g-open set is weakly fuzzy g'''-open set but not conversely.
4. Every weakly fuzzy g'''-open set is fgsp-open set but not conversely.

Proof: (i) If A is a fuzzy g'''-open set in a fuzzy topological space (X, τ), then A^c is fuzzy g'''-closed set in (X, τ). By Proposition 3.4, it follows that A^c is weakly fuzzy g'''-closed set in (X, τ). Hence A is weakly fuzzy g'''-open set in (X, τ).

(ii) If A is a fuzzy regular open set in a fuzzy topological space (X, τ), then A^c is fuzzy regular closed set in (X, τ). By Propositon 3.6, it follows that A^c is weakly fuzzy g'''-closed set in (X, τ). Hence A is weakly fuzzy g'''-open set in (X, τ).

(iii) If A is a fuzzy g-open set in a fuzzy topological space (X, τ), then A^c is fuzzy g-closed set in (X, τ). By Proposition 3.10, it follows that A^c is weakly fuzzy g'''-closed set in (X, τ). Hence A is weakly fuzzy g'''-open set in (X, τ).

(iv) If A is a weakly fuzzy g'''-open set in a fuzzy topological space (X, τ), then A^c is weakly fuzzy g'''-closed set in (X, τ). By Proposition 3.8, it follows that A^c is fgsp-closed set in (X, τ). Hence A is fgsp-open set in (X, τ).

The separate converses of Theorem 3.20 need not be true as seen from the following examples.

Example 3.21: Let X = {a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly μ defined by μ(a)=0.5, μ(b)=0.5 is weakly fuzzy g'''-open set but not fuzzy g'''-open set. Also it is neither fuzzy regular open set nor fuzzy g-open set in (X, τ) .

Example 3.22: Let X = {a, b} with τ = {0_x, α, 1_x} where α is fuzzy set in X defined by (a)=0.4, α(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly β defined by β(a)=0.5, β(b)= 0.5 is fgsp-open set but not weakly fuzzy g'''-open set in (X, τ).

Theorem 3.23: A subset A of a fuzzy topological space X is weakly fuzzy g'''-open if G ≤ int(cl(A)) whenever G ≤ A and G is fgs-closed.

Proof: Let A be any weakly fuzzy g'''-open set. Then A^c is weakly fuzzy g''' -closed set. Let G be a fgs-closed set such that G ≤ A. Then G^c is a fgs-open set such that $A^c \leq G^c$. Since A^c is weakly fuzzy g'''-closed, we have $cl(int(A^c)) \leq G^c$. Therefore G ≤ int(cl(A)).

Conversely, we suppose that G ≤ int(cl(A)) whenever G ≤ A and G is fgs-closed set. Then G^c is a fgs-open set such that $A^c \leq G^c$ and $G^c \geq (int(cl(A)))^c$. It follows that $G^c \geq cl(int(A^c))$. Hence A^c is weakly fuzzy g'''-closed set and so A is weakly fuzzy g'''-open set .

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