

IMPACT OF MONOTONIC PLANT GROWTH FUNCTIONS ON THE DYNAMICS OF PLANT-HERBIVORE MODELS USING MULTI-DIMENSIONAL DISCRETE TIME BEVERTON-HOLT EQUATION

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Abstract: In this paper, we formulate the general plant-herbivore interaction models with monotone plant growth functions. We study the impact of monotone plant growth functions on the dynamics of general plant-herbivore interaction models. The multi-dimensional discrete-time competitive Beverton-Holt equation is used to investigate how the plants with different population dynamics contribute to these interactions. The relation between the n plant populations is given by the following system of first order difference equation, $x_i(t+1) = \frac{a_i x_i(t)}{1 + b_i \sum_{j=1}^n x_j(t)}$

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Introduction: There is a strong ecological evidence indicating that the population dynamics of plants has an important effect on the plant-herbivore interactions. We use the multi-dimensional discrete-time competitive Beverton-Holt equation to investigate how the plants with different population dynamics, contribute to the interactions.

The following is the multi-dimensional discrete-time Beverton-Holt competition model:

For $1 \leq i \leq n$, let $x_i(t)$ be the population size of the i^{th} plant species at time $t = 0, 1, 2, \dots$. The interactions between these n populations is given by the following system of first order difference equation,

$$x_i(t+1) = \frac{a_i x_i(t)}{1 + b_i \sum_{j=1}^n x_j(t)}, \quad x_i(0) > 0$$

for $1 \leq i \leq n$ and $t \geq 0$. (1)

Assume that

$$a_i > 1, b_i > 0 \text{ for } 1 \leq i \leq n. \quad (2)$$

Furthermore, the first plant species is assumed to have the largest carrying capacity,

$$\frac{a_1 - 1}{b_1} > \frac{a_2 - 1}{b_2} > \dots > \frac{a_{n-1} - 1}{b_{n-1}} > \frac{a_n - 1}{b_n} \quad (3)$$

Now (1) can be written as

$$x_i(t+1) = x_i(t) f(x_i(t)) \quad (4)$$

Where $x_i(t)$ is the density of the i^{th} plant species in the season t and $f(\cdot)$ is the per capita growth rate. In the absence of intraspecific competition, the latter is given by $f(0)$.

In Section 2, we define two classes of monotone dynamics of the multiple plant species. We formulate the general plant-herbivore models for the plant dynamics in Section 3. In Section 4, we analyze the dynamical behavior of these two general models, i.e., the global stability of the boundary equilibrium and uniform persistence. Section 5 provides the conclusion [1], [9].

2. Monotone growth dynamics of multiple plant species:

Consider,

$$x_i(t+1) = x_i(t) f(a_i, x_i(t))$$

$$= F(a_i, x_i(t)) \quad (5)$$

where $x_i(t)$ is the density of biomass of the i^{th} plant species at generation t , $F(a_i, x_i(t))$ is the growth function of the biomass density and $f(a_i, x_i(t))$ is the per capita growth rate of the biomass density. Without intraspecific competition, we have $f(a_i, 0) = a_i$, i.e., a_i is the maximal per capita growth rate of the i^{th} plant species. Our model with per capita growth rate that declines has an increasing population density:

$$\frac{\partial f(a_i, x_i(t))}{\partial x_i(t)} < 0, \quad x_i(t) > 0 \quad (6)$$

In biological terms, this means that the per capita growth rate $f(a_i, x_i(t))$ decreases due to the negative density dependent mechanism such as intra-specific competition between the individuals within a population. For convenience, we use $F(x_i(t))$, $f(x_i(t))$ instead of $F(a_i, x_i(t))$, $f(a_i, x_i(t))$. Since a_i is a fixed parameter.

The prototype of model (5) under this biological assumption is the Beverton-Holt model. We focus on the Beverton-Holt prototype, i.e., the dynamics of the i^{th} plant species is monotonically increasing,

$$F'(x_i(t)) = \frac{dF(x_i(t))}{dx_i(t)} \geq 0 \quad (7)$$

We can characterize the growth models of the multiple plant species with the assumptions H_1 or H_2 or both H_1 and H_2 :

$$H_1: F(0) = 0; F(x_i(t))|_{x_i(t) > 0} > 0; F'(x_i(t)) > 0;$$

$$\text{and } \lim_{x_i(t) \rightarrow \infty} F(x_i(t)) = C > 0.$$

$$H_2: f(x_i(t))|_{x_i(t) \geq 0} \geq 0; f'(x_i(t)) < 0;$$

$$\text{and } \lim_{x_i(t) \rightarrow \infty} f(x_i(t)) = 0.$$

In biological sense, H_1 implies that the population density $F(x_i(t))$ is an increasing function and its per capita growth function $f(x_i(t))$ may be increasing or decreasing or both, which implies that the i^{th} plant species suffers from extremes of contest intraspecific competitive interactions. H_2 implies that the per capita growth function of the i^{th} plant species is a decreasing

function due to intraspecific competition and the population density of the i^{th} plant species can be an increasing or decreasing or both, which implies that the i^{th} plant species suffers from extremes of scramble interspecific competitive interactions [2],[7].

Plant-Herbivore Models: Herbivore and plant survival rates often appear to be non-linear functions of plant and herbivore density. In discrete-time models, we therefore assume that the plant population growth is a non-linear function of the herbivore and the plant density and that the plant population growth decreases gradually with increasing herbivore density. Similarly, we assume that the density of herbivore population depends on both the plant and herbivore density rather than only the herbivore density. A key feature of the plant-herbivore interactions is that, in the absence of the herbivore, we have monotone growth dynamics.

As per these general considerations, we construct our models as follows. Let $x_i(t)$ represent the density of the i^{th} plant species biomass in generation t and $Y(t)$ represent the population density of the herbivore. The effect of herbivore on the plant's population growth rate is described by the function $g(a_i, Y(t))$. We assume that the herbivore population growth is proportional to a nonlinear, saturating function of plant density, $h(x_i(t))$. The structure of our model is:

$$x_i(t+1) = x_i(t) f(x_i(t)) g(a_i, Y(t)) \tag{8}$$

$$Y(t+1) = Y(t) h(x_i(t)) \tag{9}$$

Our model has the following features: Without the herbivore, we assume a monotone growth rate, i.e., H_1 holds. The growth function $F(x_i(t))$ determines the amount of new leaves available for consumption for the herbivore in the i^{th} plant species at generation t . We assume that the herbivores search for plants randomly. The area consumed is measured by the parameter a_i , for $1 \leq i \leq n$. i.e., a_i is a constant that correlates to the total amount of biomass that a herbivore consumes. After attack by the herbivores, the biomass of the i^{th} plant population is reduced to

$$x_i(t+1) = x_i(t) f(x_i(t)) e^{-a_i Y(t)} \tag{10}$$

where $g(a_i, Y(t))$ in (8) is defined as

$$g(a_i, Y(t)) = e^{-a_i Y(t)}. \tag{11}$$

If the herbivore attacks the i^{th} plant species before it grows, then we have $h(x_i(t)) = x_i(t)$, otherwise, $h(x_i(t)) = x_i(t) f(x_i(t))$. Since the biomass of the herbivore comes from whatever they eat, $h(x_i(t))$ is the available biomass of a plant that can be converted into the herbivore's biomass. The term $Y(t)$ describes the fraction of $h(x_i(t))$ that can be used by the herbivore, i.e., $Y(t) = 1 - e^{-a_i Y(t)}$. Therefore, the evolution of the plant-herbivore system is described by Model I :

$$x_i(t+1) = F(x_i(t)) e^{-a_i Y(t)} \tag{12}$$

$$Y(t+1) = x_i(t) [1 - e^{-a_i Y(t)}] \tag{13}$$

describing the dynamics of a system where the i^{th} plant species is attacked before it has a chance to grow. While Model II :

$$x_i(t+1) = F(x_i(t)) e^{-a_i Y(t)} \tag{14}$$

$$Y(t+1) = F(x_i(t)) [1 - e^{-a_i Y(t)}] \tag{15}$$

describes the dynamics of a system where the i^{th} plant species grows first, before being attacked [1],[3],[6].

Mathematical Analysis:

4.1. Equilibrium and Stability

The solutions $(x_1(t), x_2(t), \dots, x_n(t))$ of system (1) are positive, bounded and there exists $n + 1$ equilibria,

$$E_0 = (0, 0, 0, \dots, 0)$$

$$E_1 = (\frac{a_1-1}{b_1}, 0, 0, \dots, 0)$$

$$E_2 = (0, \frac{a_2-1}{b_2}, 0, \dots, 0)$$

⋮

⋮

⋮

$$E_n = (0, 0, 0, \dots, \frac{a_n-1}{b_n})$$

Theorem 4.1.1

Assume (2) and (3) holds. The solutions $(x_1(t), x_2(t), \dots, x_n(t))$ of the system (1) converges to the steady state $E_1 = (\frac{a_1-1}{b_1}, 0, 0, \dots, 0)$ exponentially.

In particular, E_1 is globally asymptotically stable.

Theorem 4.1.2

Assume (2) and there exists $1 \leq l \leq n$ such that

$$\frac{a_1-1}{b_1} = \frac{a_2-1}{b_2} = \dots = \frac{a_l-1}{b_l} > \frac{a_{l+1}-1}{b_{l+1}} \geq \frac{a_{l+2}-1}{b_{l+2}} \geq \dots \geq \frac{a_n-1}{b_n} \tag{16}$$

Then the solutions of system (1) satisfies,

$$\lim_{t \rightarrow \infty} x_j(t) = \begin{cases} s_j > 0 & \text{for } 1 \leq j \leq l \\ 0 & \text{for } l + 1 \leq j \leq n \end{cases}$$

exponentially with $\sum_{j=1}^l s_j = \frac{a_1-1}{b_1}$ and $\{s_j\}$, $j=1$ to l depends on

$$(x_1(0), x_2(0), \dots, x_n(0)).$$

In contrast, for the case $l = n$, the convergence becomes monotone as well, and it is given in the corollary 4.1.3.

Corollary 4.1.3

Assume $l = n$ in (16). Let

$$S = \left\{ \begin{array}{l} s \in R^n; s_j > 0 \\ \text{for } 1 \leq j \leq n, \sum_{j=1}^n s_j = \frac{a_1-1}{b_1} \end{array} \right\}.$$

If $\sum_{j=1}^n x_j(0) = \frac{a_1-1}{b_1}$, then $x_j(t) = x_j(0)$, for $1 \leq j \leq n$ & $t \geq 1$. Otherwise, there exists $s = (s_1, s_2, \dots, s_n) \in S$ such that $\lim_{t \rightarrow \infty} (x_1(t), x_2(t), \dots, x_n(t)) = s$ exponentially and monotonically.

Introduce $\alpha_i = a_i/b_i$ and $\beta_i = 1/b_i$. Then (2) and (3) are equivalent to

$$\alpha_1 - \beta_1 > \alpha_2 - \beta_2 > \dots > \alpha_{n-1} - \beta_{n-1} > \alpha_n - \beta_n > 0 \tag{17}$$

and (16) is equivalent to $\alpha_1 - \beta_1 = \alpha_2 - \beta_2 = \dots = \alpha_l - \beta_l > \alpha_{l+1} - \beta_{l+1} \geq \dots \geq \alpha_n - \beta_n > 0$ (18)

In the following theorems, we will analyze the system which is equivalent to (1) i.e.,

$$x_i(t+1) = \frac{\alpha_i x_i(t)}{\beta_i + \sum_{j=1}^n x_j(t)}, x_i(0) > 0$$

for $1 \leq i \leq n$ and $t \geq 0$. (19)

Theorem 4.1.4

Assume (18) and $u(0) > 0; v(0) > 0$: Define

$$u(t+1) = \min_{1 \leq i \leq n} \frac{\alpha_i u(t)}{\beta_i + u(t)} \text{ and}$$

$$v(t+1) = \max_{1 \leq i \leq n} \frac{\alpha_i v(t)}{\beta_i + v(t)} \text{ for } t \geq 0.$$

Then $\lim_{t \rightarrow \infty} u(t) = \alpha_n - \beta_n$

and $\lim_{t \rightarrow \infty} v(t) = \alpha_1 - \beta_1$ both at exponential rates.

Theorem 4.1.5

Assume (17) and $z(0) > 0; w(0) > 0$: Define

$$\left\{ \begin{array}{l} z(t+1) = \min_{1 \leq i \leq n-1} \frac{\alpha_i z(t)}{\beta_i + z(t) + w(t)} \\ w(t+1) = \frac{\alpha_n w(t)}{\beta_n + z(t) + w(t)} \end{array} \right. \quad (20)$$

Then $\lim_{t \rightarrow \infty} z(t) = \alpha_{n-1} - \beta_{n-1}$ and $\lim_{t \rightarrow \infty} w(t) = 0$ both at exponential rates.

Lemma 4.1.6

If H_1 holds, then

$\limsup_{t \rightarrow \infty} \max\{x_i(t), Y(t)\} \leq \frac{\alpha_1 - 1}{b_1}$ for both Model I and II.

Proof : For model I,

$$Y(t+1) = x_i(t) [1 - e^{-a_i Y(t)}] \leq x_i(t) \quad (21)$$

For model II,

$$Y(t+1) = x_i(t) f(x_i(t)) [1 - e^{-a_i Y(t)}] \leq F(x_i(t)) \quad (22)$$

Since condition H_1 holds for $F(x_i(t))$, we can conclude that for any $\epsilon > 0$, there exists N large enough, such that for all $t > N$, the following holds.

$$x_i(t+1) = F(x_i(t)) e^{-a_i Y(t)} \leq F(x_i(t)) \leq \frac{\alpha_1 - 1}{b_1} + \epsilon \quad (23)$$

Therefore, $\limsup_{t \rightarrow \infty} \max\{x_i(t), Y(t)\} \leq \frac{\alpha_1 - 1}{b_1}$ for both Model I and II.

Theorem 4.1.7: Assume that H_1 holds for both Model I and II. If $F'(0) < 1$ and $(\frac{\alpha_1 - 1}{b_1}, 0, 0, \dots, 0)$ is the only boundary equilibrium, then Model I and II are globally stable at $E_1 = (\frac{\alpha_1 - 1}{b_1}, 0, 0, 0, \dots, 0)$. More generally, if $a_i \frac{\alpha_1 - 1}{b_1} < 1$, then $\lim_{t \rightarrow \infty} Y(t) = 0$ for both model I and II.

Proof

From Lemma (4.1.6), we know that for any $\epsilon > 0$, there exists N large enough, such that for all $t > N$, we have

$$x_i(t+1) = F(x_i(t)) e^{-a_i Y(t)} \leq F(x_i(t)) \leq \frac{\alpha_1 - 1}{b_1} + \epsilon \quad (24)$$

Since $a_i \frac{\alpha_1 - 1}{b_1} < 1$, for ϵ small enough, we have

$$a_i x_i(t) \leq a_i (\frac{\alpha_1 - 1}{b_1} + \epsilon) < 1 \text{ and}$$

$$a_i F(x_i(t)) \leq a_i (\frac{\alpha_1 - 1}{b_1} + \epsilon) < 1 \text{ for all } t \geq N$$

Thus, for Model I,

$$Y(t+1) = x_i(t) [1 - e^{-a_i Y(t)}] = x_i(t) Y(t) \frac{[1 - e^{-a_i Y(t)}]}{Y(t)} \leq a_i x_i(t) Y(t) \leq a_i (\frac{\alpha_1 - 1}{b_1} + \epsilon) Y(t) \quad (25)$$

And for Model II,

$$Y(t+1) = F(x_i(t)) [1 - e^{-a_i Y(t)}] = F(x_i(t)) Y(t) \frac{[1 - e^{-a_i Y(t)}]}{Y(t)} \leq a_i F(x_i(t)) Y(t) \leq a_i (\frac{\alpha_1 - 1}{b_1} + \epsilon) Y(t) \quad (26)$$

Hence, the solutions of Model I and II are globally attracted to boundary dynamics.

Theorem 4.1.7 indicates that the herbivore cannot maintain its population if its attacking rate is too small or there is no enough food [8].

4.2. Unique Interior Equilibrium:

The number of interior equilibrium is determined by the intercepts of nullclines. $y = F(x_i(t))$ is a differentiable and monotone function of $x_i(t)$. Its inverse exists and can be written as

$x_i(t) = F^{-1}(y)$. If $(x_i(t), Y(t))$ is an interior equilibrium, then

(i). The nullclines of model I can be written as,

$$x_i(t) = x_i(t) f(x_i(t)) e^{-a_i Y(t)} \Rightarrow f(x_i(t)) = e^{a_i Y(t)} \quad (t) = f^{-1}(e^{a_i Y(t)}) \quad (27)$$

$$Y(t) = x_i(t) [1 - e^{-a_i Y(t)}] \Rightarrow x_i(t) = \frac{Y(t)}{1 - e^{-a_i Y(t)}} \quad (28)$$

(ii) The nullclines of model II can be written as,

$$Y(t) = x_i(t) f(x_i(t)) [1 - e^{-a_i Y(t)}] \Rightarrow x_i(t) f(x_i(t)) e^{-a_i Y(t)} [e^{a_i Y(t)} - 1] Y(t) = x_i(t) [e^{a_i Y(t)} - 1] \Rightarrow x_i(t) = \frac{Y(t)}{e^{a_i Y(t)} - 1} \quad (29)$$

In addition,

$$Y(t) = F(x_i(t)) [1 - e^{-a_i Y(t)}] \Rightarrow F(x_i(t)) = \frac{Y(t)}{[1 - e^{-a_i Y(t)}]} \Rightarrow x_i(t) = F^{-1}(\frac{Y(t)}{[1 - e^{-a_i Y(t)}]}) \quad (30)$$

Proposition 4.2.1

If $F(x_i(t)) \in H_1$ and $f(x_i(t)) \in H_2$, then Model I has at most one interior equilibrium. The interior equilibrium emerges at $a_i \frac{\alpha_1 - 1}{b_1} = 1$.

Proof: The interior equilibria of Model I are determined by the interceptions of the following two functions (27) and (28):

$$x_i(t) = f^{-1}(e^{a_i Y(t)})$$

$$x_i(t) = \frac{Y(t)}{1 - e^{-a_i Y(t)}}$$

Since (27) is a decreasing function and (28) is an increasing function, they have only one interior intercept if

$$f^{-1}(1) > \frac{1}{a_i}$$

Proposition: Let $f(x_i(t)) \in H_2$, then $x_i(t) = x_i(t) f(x_i(t))$ has at most two roots, i.e., $x_i(t) = 0$ and $1 = f(x_i(t))$.

Proof : Since $f(x_i(t)) \in H_2$, $f(x_i(t))$ is a differentiable and decreasing function. This implies that $1 = f(x_i(t))$ has at most one solution.

By this proposition, we know that $\frac{a_1-1}{b_1} = f^{-1}(1)$

Hence, Model I has at most one interior equilibrium. The interior equilibrium emerges at $a_i \frac{a_1-1}{b_1} = 1$.

Proposition 4.2.2: If $F(x_i(t)) \in H_1$, then Model II has at most one interior equilibrium. The interior equilibrium emerges at $a_i \frac{a_1-1}{b_1} = 1$.

Proof: The interior equilibria of Model II are determined by the interceptions of the following two functions (30) and (29):

$$x_i(t) = F^{-1}\left(\frac{Y(t)}{1-e^{-a_i Y(t)}}\right)$$

$$x_i(t) = \frac{Y(t)}{e^{a_i Y(t)} - 1}$$

Since (30) is an increasing function and (29) is a decreasing function, they have only one interior intercept if $F^{-1}\left(\frac{1}{a_i}\right) > \frac{1}{a_i}$ has the same $n + 1$ solutions as $x_i(t) =$

$F(x_i(t))$ with $\frac{a_n-1}{b_n} < \dots < \frac{a_i-1}{b_i} < \dots < \frac{a_1-1}{b_1}$. Therefore, the only interior equilibrium emerges at $a_i \frac{a_1-1}{b_1} = 1$ [8].

4.3. Uniform persistence of Model I and II:

We define the sets,

$$X = \{(x_i(t), Y(t)); x_i(t) \geq 0, Y(t) \geq 0\} \quad (31)$$

$$X_{11} = \{(x_i(t), Y(t)) \in X; x_i(t) Y(t) > 0\} \quad (32)$$

$$\partial X_{11} = X \setminus X_{11} \quad (33)$$

and consider the additional hypothesis

H_3 : The smallest positive root $\frac{a_n-1}{b_n}$ of

$$x_i(t) = F(x_i(t)) = x_i(t) f(x_i(t))$$

satisfies $a_i \frac{a_n-1}{b_n} > 1$, and in addition,

$$f(0) > 1.$$

In the following, we show that Model I and II are uniformly persistent with respect to $(X_{11}, \partial X_{11})$, if both H_1 and H_3 holds. i.e., for any initial condition $(x_i(0), Y(0)) \in X_{11}$, there exists some $\epsilon > 0$ such that

$$\liminf_{t \rightarrow \infty} \min\{x_i(t), Y(t)\} \geq \epsilon \quad (34)$$

Lemma 4.3.1: X_{11} and ∂X_{11} are positively invariant for (12)-(13) and (14)-(15). The following theorem is the main result of this lemma.

Theorem 4.3.2: If $a_i \frac{a_n-1}{b_n} > 1$, then (12)-(13) and (14)-(15) are uniformly persistent with respect to $(X_{11}, \partial X_{11})$

provided that they satisfy both H_1 and H_3 .

Proof: From Lemma 4.3.1, we obtain that the systems (12)-(13) and (14)-(15) are point dissipative. Notice that the omega limit set of $S_1 = \{(x_i(t), Y(t)) \in R^2_+; x_i(t) = 0\}$ is the trivial boundary equilibrium E_0 . Let $L(x_i(t), Y(t)) = x_i(t)$ be an average Lyapunov function, then we have $L(x_i(t), Y(t))|_{S_1} = 0$. Since the systems satisfy H_3 , the following inequality holds

$$\sup_{t \geq 0} \liminf_{(x_i(0), Y(0)) \rightarrow (0,0)} \frac{x_i(t)}{x_i(0)} = \sup_{t \geq 0} \liminf_{(x_i(0), Y(0)) \rightarrow (0,0)} \prod_{j=0}^{t-1} f(x_i(j)) e^{-a_i Y(j)} = \sup_{t \geq 0} (f(0))^t > 1 \quad (35)$$

where $(x_i(0), Y(0)) \in X \setminus S_1$. Therefore, by using Theorem 2.2 in [5] and its corollary to the systems (12)-(13) and (14)-(15), we obtain the persistence of the i^{th} plant population, i.e., for any initial condition $x_i(0) > 0$, we have $\liminf_{t \rightarrow \infty} x_i(t) \geq \epsilon$. The fact that the i^{th} plant population is uniformly persistent implies that the systems (12)-(13) and (14)-(15) can be restricted in $X \cap \{(x_i(t), Y(t)) \in R^2_+; x_i(t) \geq \epsilon\}$. We can conclude that the omega limit sets of $S_2 = \{(x_i(t), Y(t)) \in \partial X_{11}; x_i(t) > 0\}$ are $\{E_i; 1 \leq i \leq n\}$. Since $a_i \frac{a_n-1}{b_n} > 1$,

condition H_3 indicates that $a_i \frac{a_1-1}{b_i} > 1; 1 \leq i \leq n$.

Now define $L(x_i(t), Y(t)) = Y(t)$ as an average Lyapunov function.

Then we have $L(x_i(t), Y(t))|_{S_2} = 0$.

Moreover, for the model (12)-(13), we have

$$\sup_{t \geq 0} \liminf_{(x_i(0), Y(0)) \rightarrow (\frac{a_i-1}{b_i}, 0)} \frac{Y(t)}{Y(0)} = \sup_{t \geq 0} \liminf_{(x_i(0), Y(0)) \rightarrow (\frac{a_i-1}{b_i}, 0)} \left[\prod_{j=0}^{t-1} x_i(j) \frac{[1 - e^{-a_i Y(j)}]}{Y(j)} \right] = \sup_{t \geq 0} \left[a_i \frac{a_i-1}{b_i} \right]^t > 1 \quad (36)$$

and for the model (14)-(15) we have

$$\sup_{t \geq 0} \liminf_{(x_i(0), Y(0)) \rightarrow (\frac{a_i-1}{b_i}, 0)} \frac{Y(t)}{Y(0)} = \sup_{t \geq 0} \liminf_{(x_i(0), Y(0)) \rightarrow (\frac{a_i-1}{b_i}, 0)} \left[\prod_{j=0}^{t-1} F(x_i(j)) \frac{[1 - e^{-a_i Y(j)}]}{Y(j)} \right] = \sup_{t \geq 0} \left[a_i \frac{a_i-1}{b_i} \right]^t > 1 \quad (37)$$

where $(x_i(0), Y(0)) \in X_{11}$. Therefore, according to Theorem 2.2 and its corollary 2.3 in [5], we can show that the systems (12)-(13) and (14)-(15) are uniformly persistent.

Hence, the statement holds [4],[5].

Conclusion: We have investigated the impact of monotone plant growth functions on the dynamics of

plant-herbivore models using multi-dimensional Beverton-Holt equation. From this paper, we have come to the conclusion that

1. The equilibrium E_1 , is globally asymptotically stable.

2. The solutions of Model I and II are globally attracted to the boundary dynamics.

3. The monotone growth models generate a unique interior equilibrium.

4. Model I and II are uniformly persistent.

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