SUB DIVIDED UNIFORM SHELL BOW GRAPHS ARE ONE MODULO THREE GRACEFUL

J. JEBA JESINTHA, K. EZHILARASI HILDA

Abstract: Shell graphs, also known as fans are the join of K_1 and the path P_m with 'm' vertices. When each edge in the path alone are sub divided, we say that it is a sub divided shell graph. Also a shell bow graph is a double shell having a common end point called the apex. When the paths are having the same order we call it as a uniform shell bow graph. This becomes a sub divided uniform shell bow graph when the edges in the paths are sub divided. In this paper we prove that all sub divided uniform shell bow graphs are one modulo three graceful .

Keywords: One modulo three graceful labeling, shell graph , sub divided shell graphs, sub divided uniform shell bow graph.

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Introduction: In 1967 Rosa[6] introduced the labeling method called *β - valuation* as a tool for decomposing the complete graph into isomorphic sub graphs. Later on, this β - valuation was renamed as *graceful labeling* by Golomb[3]. A *labeling* of a graph G with *q* and vertex set V is an injection $f : V(G) \rightarrow \{0, 1, 2, \ldots, q\}$ with the property that the resulting edge labels are also distinct, where an edge incident with vertices *u* and *v* is assigned the label $|f(u) - f(v)|$. A graph which admits a graceful labeling is called a *graceful graph*. Various kinds of graphs are shown to be graceful. In particular, cycle-related graphs have been a major focus of attention for nearly five decades.

Deb and Limaye[1] have defined a *shell graph* as a cycle C_n with *(n -3)* chords sharing a common end point called the *apex*. Shell graphs are denoted as *C(n, n-3).* In [4] we have defined a *bow graph* as a double shell in which each shell has any order. Shell graphs, also known as fans are the join of K_1 and P_m , the path with 'm' vertices. When each edge in the path alone are sub divided, we say that it is a sub divided shell graph. In [5] we have proved that all sub divided uniform shell bow graphs and sub divided shell graphs are odd graceful. In this paper we prove yet another labeling on this graph namely one modulo three graceful. A variation of graceful labeling is one modulo three graceful labelibg. Sekar [7] calls an injective function $\varphi : V(G) \rightarrow \{0, 1, 3, 4, 6, 7, \ldots, (3q-3), (3q-2)\}$ as *one modulo three graceful* if the edge labels induced by labeling each edge uv with $|\varphi(u) - \varphi(v)|$ is $\{1, 4, 7, \ldots, 3q - 2\}$. He proves that the following graphs are one modulo three graceful. The paths, Cycles C_n when $n \equiv 0 \bmod 4$, the complete bipartite graphs, caterpillars; stars, lobsters, banana trees; rooted trees of height 2, ladders are one modulo three graceful. He conjectures that every one modulo three graceful graph is graceful. For an exhaustive survey, refer to the dynamic survey by Gallian[2]. In this paper we prove that sub divided uniform shell bow graphs are one modulo three graceful.

Main Result:

Theorem: All sub divided uniform shell bow graphs are one modulo three graceful.

Proof: Let G be a Subdivided uniform shell bow graph with '*n'* vertices and '*q*' edges as shown in figure 1. We describe the Subdivided uniform shell bow graph G as follows: Denote the apex of G as v_0. Let 'm' be the number of vertices in each path. Denote the vertices in the path of the right shell of G from bottom to top as $v_1, v_2 \ldots v_m$. The vertices in the path of the left shell are denoted from top to bottom *as* $v_{m+1}, v_{m+2}, \ldots, v_{(2m-1)}, v_{2m}$. G has n = 2m+ 1 vertices and q = (3m- 1) edges.

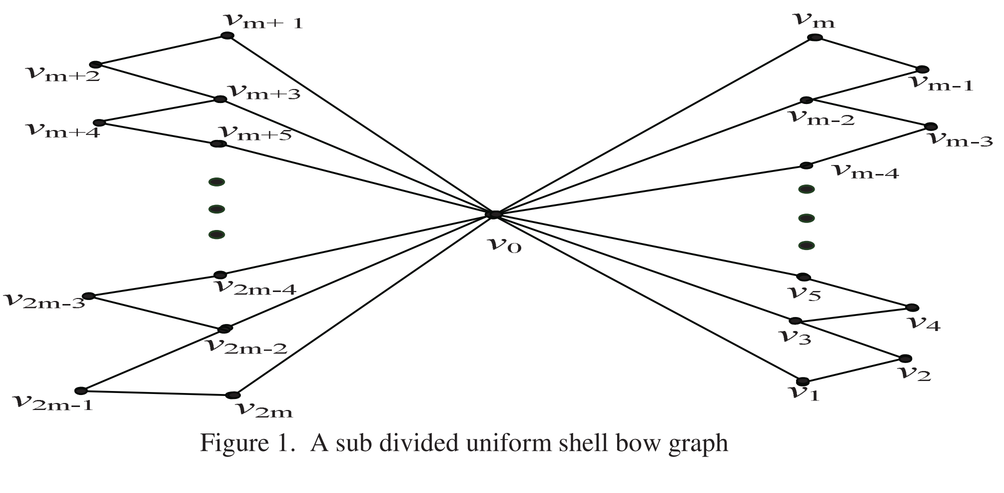

Figure 1. A sub divided uniform shell bow graph

The vertex labelings are defined as follows:-

$$f(v_0) = 0 \quad (1)$$

$$f(v_{2i-1}) = \begin{cases} 6m+3i-8, & \text{for } 1 \le i \le (m+1)/2 \\ 9m-3i, & \text{for } (m+3)/2 \le i \le m \end{cases} \quad (2)$$

$$f(v_{2i}) = \begin{cases} 3m-3i, & \text{for } 1 \le i \le (m-1)/2 \\ 6m+3i-5, & \text{for } (m+1)/2 \le i \le m \end{cases} \quad (3)$$

Equations (1), (2) and (3) show that the vertices have distinct labeling. No vertex label is of the form 3t – 1, t = 1, 2, 3, . . .(q-1). If so then we would get a contradiction to the fact that m is an integer. For instance, if 6m+3i -8 = 3t – 1 when i = (m+1)/2 then m = (11+6t)/15. But 11+6t is not a multiple of 15 for t = 1, 2, 3, ..(q-1). We get 'm' as a fraction which is a contradiction as m is the number of vertices in the path.

The edge labelings are calculated as follows:-

$$|f(v_0) - f(v_{2i-1})| = |6m+3i-8|, \quad \text{for } 1 \le i \le (m+1)/2 \quad (4)$$

$$|f(v_0) - f(v_{2i})| = |6m+3i-5|, \quad \text{for } (m+1)/2 \le i \le m \quad (5)$$

$$|f(v_{2i-1}) - f(v_{2i})| = \begin{cases} |3m+6i-8|, & \text{for } 1 \le i \le (m-1)/2 \\ |3m-6i+5|, & \text{for } (m+3)/2 \le i \le m \end{cases} \quad (6)$$

$$|f(v_{2i}) - f(v_{2i+1})| = \begin{cases} |3m+6i-5|, & \text{for } 1 \le i \le (m-1)/2 \\ |3m-6i+2|, & \text{for } (m+1)/2 \le i \le m-1 \end{cases} \quad (7)$$

From (4), (5), (6) and (7) we can see that the edge labels are distinct. Also all edge labels are of the form 3t+1, t= 0, 1, 2, . . . If any edge label is of the form 3t-1, or 3t then we would get a contradiction to the fact 'm' is an integer. An illustration is given in the figure 2.

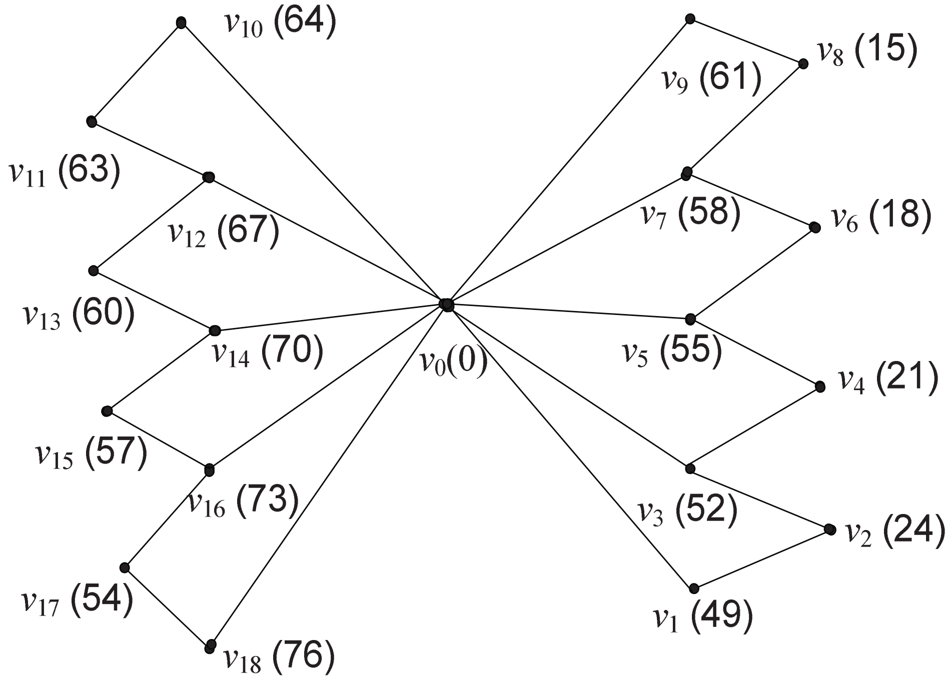

Figure 2. A one modulo 3 graceful sub divided uniform shell bow graph when m =9, q = 26, n = 19

Hence the vertices take labels from the set { 0, 1, 3, 4, 6, 7, . . .,(3q-3), (3q-2) }and the edge labels constitute the set {1, 4, 7, . . . , (3q-2)}. This shows that all subdivided uniform shell bow graphs are one modulo three graceful.

Conclusion: In this paper we have proved that sub divided uniform shell bow graphs satisfy the conditions of one modulo three graceful labeling. We are working out the possible conditions for changing a one modulo three graceful graph to a graceful graph.

References:

1. P. Deb and N. B. Limaye, 2002, "On Harmonious Labelling of some Cycle related graphs", Ars Combina. 65, pp 177-197.
2. A. Gallian Joseph,2010, " A Dynamic survey of Graph labelling" , Elec .J . of. Combina. Nov 13.
3. S. W. Golomb, 1972," How to Number a graph in Graph Theory and computing",R.C. Read,ed., Academic Press, New York 23- 37.
4. J. Jeba Jesintha , Ezhilarasi Hilda Stanley, 2012, " Butterfly graphs with Shell orders m and 2m+1 are graceful", Bonfring International Journal of Research in communication Engineering, Vol. 2, Number 2, pp 1-5
5. J.Jeba Jesintha , K. Ezhilarasi Hilda , 2014, Sub divided uniform shell bow Graphs and subdivided shell graphs are odd graceful, Proceedings of

International Conference on Applications of Mathematics in Modern technology, 27th March 2014, QMC, Chennai.
6. A .Rosa,1966, " On certain valuations of the vertices of a graph, Theory of graphs" ,Proceedings of the symposium, Rome,, Gordon and Breach, New York, (1967), pp 349-355.
7. C. Sekar, Studies in Graph Theory, Ph.D. Thesis, Madurai Kamaraj University, 2002.

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