DETOUR DISTANCE ENERGY OF SOME GRAPHS

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Abstract: The detour distance energy of a graph G is defined as the sum of the absolute values of itseigen values of the detour distance matrix of G. In this paper the detourdistance energy of cycle graph, star graph, complete bipartite graph, crown graph,circulant graph are computed.

Keywords: Energy, detour distance matrix, detour distance energy.

Introduction: The concept of energy of a graph was introduced by I.Gutman [9] in the year 1978. Let G be a graph with n vertices and m edges. Let $A = (a_{ij})_{nxn}$ be the adjacency matrix of G. The eigen values $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ of A, assumed in non increasing order, are the eigen values of the graph G. As A is real symmetric, the eigen values of G are real with sum equal to zero. The Energy E(G) of G is defined to be the sum of the absolute values of the eigen values of G.

Let G be a graph. The distance matrix of a graph G is defined as a square matrix $D = D(G) = [d_{ij}]$, where d_{ij} is the distance between the vertices v_i and v_j in G.

The eigen values of the distance matrix D(G) are denoted by $\mu_1, \mu_2, \mu_3, \ldots, \mu_n$ and are said to be the D-eigen values of G. Since the distance matrix is symmetric, its eigen values are real and can be ordered as $\mu_1 \geq \mu_2 \geq \mu_3 \geq \ldots \geq \mu_n$. The distance energy $E_D = E_D(G)$ of a graph G is defined as $E_D = E_D(G) = \sum_{i=1}^{n} \left| \mu_i \right|$ [5, 6, 7,8,13,14,16]

For more results on Various types of Energyof graphs, see [1, 2,4,10,11,12,15].

Let G be a graph with vertex set $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and edge set E. The Detour Distance Matrix of G is the n × nmatrix defined by $A_{DD}(G) = (D_{ij})$, where D_{ij} = longest distance between v_i and v_j.The characteristic polynomial of $A_{DD}(G)$ is denoted by $f_n(G, \gamma) = \det(\gamma_I - A_{DD}(G))$. The detour distance eigen values of the graph G are the eigen values of $A_{DD}(G)$. Since $A_{DD}(G)$ is real and symmetric, its eigen values are real numbers and it can be ordered as $\gamma_1 \geq \gamma_2 \geq \gamma_3 \geq \ldots \geq \gamma_n$.

The detour distance energy of G is defined as

$$E_{DD}(G) = \sum_{i=1}^{n} \left| \gamma_i \right|.$$

The detour distance energy of complete bipartite graph is $8n^2$-10n+4when m=n[3]. The detour distance energy of complete graph is $2(n-1)^2$ [3]. Graphs for which the Detour Distance Energy is greater than $2(n-1)^2$ are called detour hyper energetic graphs. If $E_{DD}(G) \leq 2(n-1)^2$, G is called

non-detourhyperenergetic. In this paper the Detour energy of some standard graphs and some special graphs like cocktail party graph and crown graph is computed.

Detour Distance Energy Of Some Graphs:

Definition :2.1: The cocktail party graph is denoted by $K_{n\times2}$, is a graph having the vertex set $V = \bigcup_{i=1}^{n} \{u_i, v_i\}$ and the edge set$E = \{u_iu_j, v_iv_j, :i \neq j\} \cup \{u_iu_j, v_iv_j, : 1 \leq i < j \leq n\}$

Theorem: 2.2: The Detour Distance energy of cocktail party graph $K_{n\times2}$ is $2(2n - 1)^2$.

Proof: Let $G = K_{n\times2}$ be the cocktail party graph with vertex set $V(G) = \bigcup_{i=1}^{n} \{u_i, v_i\}$. Thenthe detour distance matrix of G is a 2nx2n matrix and is given by

$$A_{DD}(K_{n\times2}) = \begin{array}{c} \\ u_1 \\ v_1 \\ u_2 \\ v_2 \\ \vdots \\ \vdots \\ \vdots \\ u_{n-1} \\ v_{n-1} \\ u_n \\ v_n \end{array} \begin{array}{c} \begin{array}{ccccccccc} u_1 & v_1 & u_2 & v_2 & \cdots & u_{n-1} & v_{n-1} & u_n & v_n \end{array} \\ \left[\begin{array}{ccccccccc} 0 & 2n-1 & 2n-1 & 2n-1 & \cdots & 2n-1 & 2n-1 & 2n-1 & 2n-1 \\ 2n-1 & 0 & 2n-1 & 2n-1 & \cdots & 2n-1 & 2n-1 & 2n-1 & 2n-1 \\ 2n-1 & 2n-1 & 0 & 2n-1 & \cdots & 2n-1 & 2n-1 & 2n-1 & 2n-1 \\ 2n-1 & 2n-1 & 2n-1 & 0 & \cdots & 2n-1 & 2n-1 & 2n-1 & 2n-1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 2n-1 & 2n-1 & 2n-1 & 2n-1 & \cdots & 0 & 2n-1 & 2n-1 & 2n-1 \\ 2n-1 & 2n-1 & 2n-1 & 2n-1 & \cdots & 2n-1 & 0 & 2n-1 & 2n-1 \\ 2n-1 & 2n-1 & 2n-1 & 2n-1 & \cdots & 2n-1 & 2n-1 & 0 & 2n-1 \\ 2n-1 & 2n-1 & 2n-1 & 2n-1 & \cdots & 2n-1 & 2n-1 & 2n-1 & 0 \end{array}\right] \end{array}$$

The characteristic polynomial of $A_{DD}(K_{n\times2})$ is

$$\begin{vmatrix} \gamma & -(2n-1) & -(2n-1) & . \; . \; . & -(2n-1) \\ -(2n-1) & \gamma & -(2n-1) & . \; . \; . & -(2n-1) \\ -(2n-1) & -(2n-1) & \gamma & . \; . \; . & -(2n-1) \\ . & . & . & . \; . \; . & . \\ . & . & . & . \; . \; . & . \\ . & . & . & . \; . \; . & . \\ -(2n-1) & -(2n-1) & -(2n-1) & . \; . \; . & \gamma \end{vmatrix}$$

Characteristic equation$K_{n\times2}$is

$(\gamma + (2n - 1))^{2(n-1)}(\gamma + (2n - 1))(\gamma - (2n - 1)^2) = 0$

Hence the Detour Distance eigen values are $\gamma = -(2n - 1)$ [2(n – 1) times],

$\gamma = -(2n - 1)$ & $\gamma = (2n - 1)^2$

The Detour Distance energyof $K_{n\times2}$ is $E_{DD}(K_{n\times2})$ = | -(2n

$- 1) \mid 2(n-1) +$
$\mid -(2n-1) \mid + \mid (2n-1)^2 \mid$
$= (2n-1)2(n-1) + (2n-1) + (2n-1)^2$
$= 2n - 1(2n - 2 + 1) + (2n-1)^2$
$= (2n-1)^2 + (2n-1)^2$
$= 2(2n-1)^2$

Theorem :2.3: For $n \geq 3$, the detour distance energy of star graph $K_{1,n-1}$ is $2(n-2) + 2\sqrt{n^2 - 3n + 3}$.

Proof: Let $K_{1,n-1}$ be the star graph with vertex set $V = \{v_0, v_1, v_2, v_3, \ldots, v_{n-1}\}$ and $E(G) = \{v_0v_i ;\ 1 \leq i \leq n-1\}$. The Detour Distance matrix of $K_{1,n-1}$ is a nxn matrix and is given by

$$A_{DD}(K_{1,n-1}) = \begin{array}{c} \\ v_0 \\ v_1 \\ v_2 \\ \vdots \\ v_{n-2} \\ v_{n-1} \end{array} \begin{array}{c} \begin{array}{ccccccc} v_0 & v_1 & v_2 & \ldots & & & v_{n-1} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & \ldots & & & 1 \\ 1 & 0 & 2 & \ldots & & & 2 \\ 1 & 2 & 0 & \ldots & & & 2 \\ \vdots & \vdots & \vdots & & & & \vdots \\ 1 & 2 & 2 & \ldots & & 0 & 2 \\ 1 & 2 & 2 & \ldots & & 2 & 0 \end{bmatrix} \end{array}$$

The Characteristic polynomial of $K_{1,n-1}$ is

$$\begin{vmatrix} \gamma & -1 & -1 & \ldots & & -1 \\ -1 & \gamma & -2 & \ldots & & -2 \\ -1 & -2 & \gamma & \ldots & & -2 \\ \vdots & \vdots & \vdots & & & \vdots \\ -1 & -2 & -2 & \ldots & \gamma & -2 \\ -1 & -2 & -2 & \ldots & -2 & \gamma \end{vmatrix}$$

Characteristic equation is

$$(\gamma + 2)^{n-2}[\gamma^2 - (2n-4)\gamma - (n-1)] = 0$$

The Detour Distance eigen values are given by

$\gamma = -2[(n-2)$ times$]$,

$$\gamma = \frac{2(n-2) \pm 2\sqrt{n^2 - 3n + 3}}{2}$$

Hence the Detour Distance Energy of $K_{1,n-1}$ is $E_{DD}(K_{1,n-1}) = \mid -2 \mid (n-2) + \left|(n-2) + \sqrt{n^2 - 3n + 3}\right| + \left|(n-2) - \sqrt{n^2 - 3n + 3}\right|$

$= 2(n-2) + 2\sqrt{n^2 - 3n + 3}$

Definition:2.4: The crown graph S_n^o for an integer $n \geq 2$ is the graph with vertex set $\{u_1, u_2, u_3, \ldots, u_n, v_1, v_2, v_3, \ldots, v_n\}$ and edge set $\{u_iv_j : 1 \leq i, j \leq n, i \neq j\}$.

Theorem 2.5: The Detour Distance Energy of crown graph S_n^o for an integer $n > 3$ is $8n^2 - 10n + 4$.

Proof: Let S_n^o be the crown graph .The Detour Distance matrix of order 2nx2n is given by

$$= \begin{array}{c} \\ u_1 \\ u_2 \\ \vdots \\ \vdots \\ \vdots \\ u_n \\ v_1 \\ v_2 \\ \vdots \\ \vdots \\ \vdots \\ v_n \end{array} \begin{array}{c} \begin{array}{ccccccccc} u_1 & u_2 & u_3 & \cdots & u_n & v_1 & v_2 & \cdots & v_n \end{array} \\ \begin{bmatrix} 0 & 2(n-1) & 2(n-1) & \cdots & 2(n-1) & 2n-1 & 2n-1 & \cdots & 2n-1 \\ 2(n-1) & 0 & 2(n-1) & \cdots & 2n-1 & 2n-1 & 2n-1 & \cdots & 2n-1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 2(n-1) & 2(n-1) & 2(n-1) & \cdots & 0 & 2n-1 & 2n-1 & \cdots & 2n-1 \\ 2n-1 & 2n-1 & 2n-1 & \cdots & 2n-1 & 0 & 2(n-1) & \cdots & 2(n-1) \\ 2n-1 & 2n-1 & 2n-1 & \cdots & 2n-1 & 2(n-1) & 0 & \cdots & 2(n-1) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 2n-1 & 2n-1 & 2n-1 & \cdots & 2n-1 & 2(n-1) & 2(n-1) & \cdots & 0 \end{bmatrix} \end{array}$$

The Characteristic polynomial of S_n^o is

$$\begin{array}{c} \\ u_1 \\ u_2 \\ \vdots \\ \vdots \\ \vdots \\ u_n \\ v_1 \\ v_2 \\ \vdots \\ \vdots \\ \vdots \\ v_n \end{array} \begin{array}{c} \begin{array}{ccccccccc} u_1 & u_2 & u_3 & \cdots & u_n & v_1 & v_2 & \cdots & v_n \end{array} \\ \begin{vmatrix} \gamma & -2(n-1) & -2(n-1) & \cdots & -2(n-1) & -(2n-1) & -(2n-1) & -(2n-1) & \cdots & -(2n-1) \\ -2(n-1) & \gamma & -2(n-1) & \cdots & -(2n-1) & -(2n-1) & -(2n-1) & \cdots & -(2n-1) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -2(n-1) & -2(n-1) & -2(n-1) & \cdots & \gamma & -(2n-1) & -(2n-1) & \cdots & -(2n-1) \\ -(2n-1) & -(2n-1) & -(2n-1) & \cdots & -(2n-1) & \gamma & -2(n-1) & \cdots & -2(n-1) \\ -(2n-1) & -(2n-1) & -(2n-1) & \cdots & -(2n-1) & -2(n-1) & \gamma & \cdots & -2(n-1) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -(2n-1) & -(2n-1) & -(2n-1) & \cdots & -(2n-1) & -2(n-1) & -2(n-1) & \cdots & \gamma \end{vmatrix} \end{array}$$

Characteristic equation is

$$(\gamma + 2(n-1))^{2(n-1)}(\gamma + (3n-2))(\gamma - (4n^2 - 5n + 2)] = 0$$

The Detour Distance eigen values are

$\gamma = -2(n-1)$ $[2(n-1)$ times$]$, $\gamma = -(3n-2)$ and $\gamma = 4n^2 - 5n + 2$

Hence the Detour Distance Energy of $\left(S_n^o\right)$ is

$E_{DD}(S_n^o) = \mid -2(n-1) \mid [2(n-1)] + \mid -(3n-2) \mid + \mid 4n^2 - 5n + 2 \mid$

$= 4(n-1)^2 + 3n - 2 + 4n^2 - 5n + 2$
$= 8n^2 - 10n + 4.$

Theorem :2.6: The Detour Distance Energy of complete bipartite graph $K_{m,n}$ is

$$2m(n-1)+2m-2(m-1)+2\sqrt{m^4-2m^3+m^2n^2+2m^3n+3m^2-2m^2n-mn-2m+1}\ \text{when } m<n.$$

Proof:
Let $K_{m,n}$ be the complete bipartite graph. The Detour Distance matrix of order
m+n x m+n when m < n is given by

$$A_{DD}(K_{m,n}) = \begin{bmatrix} 0 & 2m-2 & 2m-2 & \cdots & 2m-2 & 2m-1 & 2m-1 & \cdots & 2m-1 \\ 2m-2 & 0 & 2m-2 & \cdots & 2m-2 & 2m-1 & 2m-1 & \cdots & 2m-1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 2m-2 & 2m-2 & 2m-2 & \cdots & 0 & 2m-1 & 2m-1 & \cdots & 2m-1 \\ 2m-1 & 2m-1 & 2m-1 & \cdots & 2m-1 & 0 & 2m & \cdots & 2m \\ 2m-1 & 2m-1 & 2m-1 & \cdots & 2m-1 & 2m & 0 & \cdots & 2m \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 2m-1 & 2m-1 & 2m-1 & \cdots & 2m-1 & 2m & 2m & \cdots & 0 \end{bmatrix}$$

Characteristic polynomial of $K_{m,n}$ is

$$\begin{vmatrix} \gamma & -(2m-2) & -(2m-2) & \cdots & -(2m-2) & -(2m-1) & -(2m-1) & \cdots & -(2m-1) \\ -(2m-2) & \gamma & -(2m-2) & \cdots & -(2m-2) & -(2m-1) & -(2m-1) & \cdots & -(2m-1) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -(2m-2) & -(2m-2) & -(2m-2) & \cdots & \gamma & -(2m-1) & -(2m-1) & \cdots & -(2m-1) \\ -(2m-1) & -(2m-1) & -(2m-1) & \cdots & -(2m-1) & \gamma & -2m & \cdots & -2m \\ -(2m-1) & -(2m-1) & -(2m-1) & \cdots & -(2m-1) & -2m & \gamma & \cdots & -2m \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -(2m-1) & -(2m-1) & -(2m-1) & \cdots & -(2m-1) & -2m & -2m & \cdots & \gamma \end{vmatrix}$$

Characteristic equation of $K_{m,n}$ is

$$\left[(\gamma+2m)^{n-1}\right]\left[(\gamma+(2m-2))^{m-1}\right]$$
$$\left[\gamma^2-[2m(m+n)-(6m-2)]\gamma-[(4m^2-3m)(m+n)-(5m^2-4m)]\right]=0$$

The Detour Distance eigen values are
$\gamma = -2m$ [(n-1)times],
$\gamma = -(2m-2)$[(m-1) times]

$$\gamma = m(m+n)-(3m-1)\pm\sqrt{m^4-2m^3+m^2n^2+2m^3n+3m^2-2m^2n-mn-2m+1}$$

Hence the Detour Distance energy of $K_{m,n}$ is
$E_{DD}(K_{m,n})=|-2m|(n-1)+$
$|-(2m-2)|(m-1)+$

$$\left|m(m+n)-(3m-1)\pm\sqrt{m^4-2m^3+m^2n^2+2m^3n+3m^2-2m^2n-mn-2m+1}\right|$$

$$2m(n-1)+2m-2(m-1)+2\sqrt{m^4-2m^3+m^2n^2+2m^3n+3m^2-2m^2n-mn-2m+1}\ \text{when } m<n.$$

Definition: 2.7: An nxncirculantmatrix C is defined as

$$C=\begin{bmatrix} C_o & C_{n-1} & . & . & . & C_2 & C_1 \\ C_1 & C_o & C_{n-1} & . & . & . & C_2 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ C_{n-2} & . & . & . & . & . & C_{n-1} \\ C_{n-1} & C_{n-2} & . & . & . & C_1 & C_0 \end{bmatrix}.$$

The eigen values of circulant matrix are

$$\lambda_j = C_o + C_{n-1}\omega^j + C_{n-2}\omega^{2j} + \ldots\ldots + C_1\omega^{(n-1)j}$$
$$, j = 0,1,2,\ldots..n-1.\ where\ \omega^j = \exp(\frac{2\pi ij}{n})$$ are

the n^{th} roots of unity.

Theorem:2.8: The Detour Distance Energy of C_n is $\frac{3n^2 - 4n + 1}{2}$ if n is odd and $\frac{3n^2 - 4n}{2}$ if n is even.

Proof: Let C_n be the cycle graph with vertex set $\{v_1, v_2, v_3, \ldots, v_n\}$ and edge set
$E = \{v_iv_{i+1}, 1 \le i \le n-1\} \cup \{v_1v_n\}$ respectively.

Case (i): when n is odd
The Detour Distance matrix of C_3 is a circulant matrix with first row (0,2,2) and hence
theeigen values of the Detour distance circulant matrix $A_{DD}(C_3)$ are given by

$\gamma_j = 2\omega_j + 2\omega_j^2$, j = 0, 1, 2.

$\gamma_j = 4\cos\left(\frac{2\pi j}{3}\right)$, j = 0, 1, 2.

$\gamma_0 = 4$, $\gamma_1 = -2$, $\gamma_2 = -2$.
$E_{DD}(C_3) = 8$.

The Detour Distance matrix of C_5 is a circulant matrix with first row (0,4,3,3,4) and hence
theeigen values of the Detour distance circulant matrix $A_{DD}(C_5)$ are given by

$\gamma_j = 4\omega_j + 3\omega_j^2 + 3\omega_j^3 + 4\omega_j^4$, j = 0, 1, 2,3, 4.

$= 8\cos\left(\frac{2\pi j}{5}\right) + 6\cos\left(\frac{4\pi j}{5}\right)$, j = 0, 1, 2, 3, 4.

$\gamma_0 = 14$, $\gamma_1 = -2.382$, $\gamma_2 = -4.618$,
$\gamma_3 = -4.618$, $\gamma_4 = -2.382$.

$E_{DD}(C_5) = 28$.

The Detour Distance matrix of C_7 is a circulant matrix with first row (0,6,5,4,4,5,6,) and hence the eigen values of the Detour distance circulant matrix $A_{DD}(C_7)$ are given by

$\gamma_j = 6\omega_j + 5\omega_j^2 + 4\omega_j^3 + 4\omega_j^4 + 5\omega_j^5 + 6\omega_j^6$, j = 0, 1, 2, . . . , 6.

$= 12\cos\left(\frac{2\pi j}{7}\right) + 10\cos\left(\frac{4\pi j}{7}\right) + 8\cos\left(\frac{6\pi j}{7}\right)$

$\gamma_0 = 30$, $\gamma_1 = -1.951$, $\gamma_2 = -6.692$, $\gamma_3 = -6.357$, $\gamma_4 = -6.357$, $\gamma_5 = -6.692$, $\gamma_6 = -1.951$.

$E_{DD}(C_7) = 60$.

In general, the eigen values of the Detour Distance circulant matrix $A_{DD}(C_n)$ are given by

$$\gamma_j = (n-1)\omega_j + (n-2)\omega_j^2 + (n-3)\omega_j^3 + \ldots + \frac{n-1}{2}\omega_j^{\frac{n-1}{2}} + \frac{n-1}{2}\omega_j^{\frac{n+1}{2}} + \ldots + (n-2)\omega_j^{n-2} + (n-1)\omega_j^{n-1}$$

$$\gamma_j = \sum_{k=1}^{\frac{n-1}{2}} 2(n-k)\cos\left(\frac{2\pi jk}{n}\right), j = 0,1,2,3,\ldots n-1.$$

$$E_{DD}(C_n) = \left|\gamma_j\right| = \sum_{j=0}^{n-1}\left|\sum_{k=1}^{\frac{n-1}{2}} 2(n-k)\cos\left(\frac{2\pi jk}{n}\right)\right|$$

$$E_{DD}(C_n) = \frac{3n^2 - 4n + 1}{2}.$$

Case (ii): When n is even

The Detour Distance matrix of C_4 is a circulant matrix with first row (0,3,2,3) and hence
theeigen values of the Detour distance circulant matrix $A_{DD}(C_4)$ are given by

$\gamma_j = 3\omega_j + 2\omega_j^2 + 3\omega_j^3$, j = 0, 1, 2, 3.

$= 6\cos\left(\frac{2\pi j}{4}\right) + 2\cos\left(\frac{4\pi j}{4}\right)$, j = 0, 1, 2, 3.

$\gamma_0 = 8$, $\gamma_1 = -2$, $\gamma_2 = -4$, $\gamma_3 = -2$.

$E_{DD}(C_4) = 16$.

The Detour Distance matrix of C_6 is a circulant matrix with first row (0,5,4,3,4,5) and hencetheeigen values of the Detour distance circulant matrix $A_{DD}(C_6)$ are given by

$\gamma_j = 5\omega_j + 4\omega_j^2 + 3\omega_j^3 + 4\omega_j^4 + 5\omega_j^5$,
j = 0, 1, . . . , 5.

$= 10\cos\left(\frac{2\pi j}{6}\right) + 8\cos\left(\frac{4\pi j}{6}\right) + 3\cos\left(\frac{6\pi j}{6}\right)$,
j = 0, 1, . . . , 5.

$\gamma_0 = 21$, $\gamma_1 = -2$, $\gamma_2 = -6$, $\gamma_3 = -5$, $\gamma_4 = -6$, $\gamma_5 = -2$.

$E_{DD}(C_6) = 42$.

The Detour Distance matrix of C_8 is a circulant matrix with first row (0,7,6,5,4,5,6,7) and hence the eigen values of the Detour distance circulant matrix $A_{DD}(C_8)$ are given by

$\gamma_j = 7\omega_j + 6\omega_j^2 + 5\omega_j^3 + 4\omega_j^4 + 5\omega_j^5 + 6\omega_j^6 + 7\omega_j^7$,
j = 0, 1, . . . , 7.

$= 14\cos\left(\frac{2\pi j}{8}\right) + 12\cos\left(\frac{4\pi j}{8}\right) + 10\cos\left(\frac{6\pi j}{8}\right) + 4\cos\left(\frac{8\pi j}{8}\right)$,
j = 0, 1, . . . ,7.

$\gamma_0 = 40$, $\gamma_1 = -1.1716$, $\gamma_2 = -8$, $\gamma_3 = -6.8284$, $\gamma_4 = 8$, $\gamma_5 = -6.8284$, $\gamma_6 = -8$, $\gamma_7 = -1.1716$,.

$E_{DD}(C_8) = 80$.

In general, the eigen values of the Detour Distance circulant matrix $A_{DD}(C_n)$ are given by

$$\gamma_j = (n-1)\omega_j + (n-2)\omega_j^2 + \ldots + \frac{n}{2}\omega_j^{\frac{n}{2}} + \frac{(n+2)}{2}\omega_j^{\frac{n+2}{2}} + \ldots + (n-2)\omega_j^2 + (n-1)\omega_j,\ j = 0, 1, \ldots, n-1.$$

$$\gamma_j = \sum_{k=1}^{\frac{n-2}{2}} 2(n-k)\cos\left(\frac{2\pi jk}{n}\right) + \frac{n}{2}\cos\left(\frac{n\pi j}{n}\right), j = 0,1,2,3,\ldots n-1.$$

$$E_{DD}(C_n) = \left|\gamma_j\right| = \sum_{j=0}^{n-1}\left|\sum_{k=1}^{\frac{n-2}{2}} 2(n-k)\cos\left(\frac{2\pi jk}{n}\right) + \frac{n}{2}\cos\left(\frac{n\pi j}{n}\right)\right|$$

$$E_{DD}(C_n) = \frac{3n^2 - 4n}{2}.$$

Definition:2.9: A circulant undirected graph,denoted by $C_n(\pm\{S\})$ where $S \subseteq \left\{1,2,3,\ldots,\left\lfloor\frac{n}{2}\right\rfloor\right\} n \geq 3$ is defined as an undirected graph consisting of the vertex set $V = \{1,2,3,\ldots,n\}$ and the edge set $E = \{(i,j) : there\ is\ s \in S\ such\ that\ |j-i| \equiv s(\mathrm{mod}\ n)\}$

Theorem :2.10: The Detour Distance Energy of circulant graph $C_n\left(\pm\left\{1, 2, \ldots \left\lfloor\frac{n}{2}\right\rfloor\right\}\right)$ is $2(n-1)^2$.

Proof: Let $C_n\left(\pm\left\{1, 2, \ldots \left\lfloor\frac{n}{2}\right\rfloor\right\}\right)$ be the circulant graph then

$$A_{DD}\left(C_n\left(\pm\left\{1, 2, \ldots \left\lfloor \frac{n}{2} \right\rfloor\right\}\right)\right) = \begin{bmatrix} 0 & (n-1) & (n-1) & \cdots & (n-1) \\ (n-1) & 0 & (n-1) & \cdots & (n-1) \\ (n-1) & (n-1) & 0 & \cdots & (n-1) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ (n-1) & (n-1) & (n-1) & \cdots & 0 \end{bmatrix}$$

Characteristic polynomial of circulant graph is

$$\begin{vmatrix} \gamma & -(n-1) & -(n-1) & . & . & . & -(n-1) \\ -(n-1) & \gamma & -(n-1) & . & . & . & -(n-1) \\ -(n-1) & -(n-1) & \gamma & . & . & . & -(n-1) \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ & . & . & . & . & . & . \\ -(n-1) & -(n-1) & -(n-1) & . & . & . & \gamma \end{vmatrix}$$

The Detour Distance energy is $2(n - 1)^2$

Result:2.1: The largest distance between any two vertices v_i and v_j in a wheel graph $w_n$is (n-1). Hence the Detour Distance Energy of w_n is $2(n - 1)^2$

Conclusion: Wheel Graph and circulant graph aredetourequienergitic. Sincethe Detour Distance Energy of Wheel graph and circulant graph is equal.Also the cocktail party graph ,crown graph are detour hyper energetic whereas the star graph,cycle graph are non detour hyper energetic.

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