COVERING APPROXIMATION IN NANO TOPOLOGY

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Abstract: The purpose of this article is that the concept of the nano topology can be used as the covering approximation space in an any binary relation. This paper describes about the application of data mining in an trading analysis.

Keywords: After Set , Boundary region , Covering, approximation space , Fore Set , Lower approximation Nano Topology, Reducible cover, Upper approximation.

Introduction: Lellis Thivagar et al[8] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space. In this paper , we introduce new definition for the approximations based on general binary relation and their properties are studied.

Preliminaries:

Definition: 2.1 [8] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{ U, \phi, L_R(X), U_R(X), B_R(X) \}$ where $X \subseteq U$, then $\tau_R(X)$ satisfies following axioms:

1. U and $\phi \in \tau_R(X)$.
2. The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U called as the nano topology on U with respect to X. We call $\{U, \tau_R(X)\}$ as the nano topological space.

Definition: 2.2 [5, 9] Let U be the domain of the discourse, C a family of subsets of U. If none subsets in C is empty, and $\cup C=U$, then C is called a covering of U.

3.Covering via Nano Topology:

In this section we are going to review the concepts of decision table and decision rules and also we introduce some definitions and theorem related to decision table.

Definition: 3.1 Let U be any universal set, and R be any binary relation on U. Then, after set (resp. fore set) of the element $x \in U$ is the class $(x)R=\{y\in U|xRy\}$(resp.$R(x)=\{y\in U| Rx\}$).

Definition: 3.2 Let U be the non empty finite set of object is called the universe and R be an any binary relation on U. Then the pair (U, R) is called the approximation space.

Definition:3.3 Let (U,R) be a approximation space and R be any binary relation on U. Then we can define a covering for U by using the concept of after set and the fore set as the following:

(i) Right Covering: $C_r=\{(x)R|\forall x\in U\}$, such that $U = \cup (x)R, \forall x\in U$

(ii) Left Covering: $C_l = \{(x)R|\forall x\in U\}$, such that $U=\cup Rx, \forall x\in U$.

Definition:3.4 Let (U,R) be an approximation space ,for each element $x\in U$, we can define two neighbourhoods of it as following:

(i) Right Neighborhood : $N_r(x) = \cap yR, \forall x\in yR$

(ii) Left Neighborhood : $N_l(x) = \cap Ry, \forall x\in Ry$

Definition: 3.5 Let C_r be a right covering (resp. left covering) of a universe U and $K\in C_r$. If K is a union of some sets in C_r-{K}.We say K is reducible element of C_r, otherwise K is an irreducible element of C_r.

Example : 3.6 Let U={a, b, c, d} with R={(a, a), (b, b), (c, c) ,(d, d), (a, b), (b, a), (b, c)} and aR={a, b}; bR={a, b, c}; cR={c}; dR = {a,b,c,d} and A={aR, bR, cR,dR}.Then bR is a reducible element in the covering A. Since bR= {aR $\cup$ cR} and dR is not a reducible element in the covering A.

Proposition:3.7 Let (U,R) be a approximation space, then for each i = r,l:

(i) $x\in N_i(x), \forall x\in U$

(ii) $N_i(x)\neq \emptyset , \forall x\in U$

Proof: (i) Straight forward from definition 3.4 (ii) Since ,from definition 3.4 $xR\in C_r$ and $Rx\in C_l$. Then $\forall x\in U$,there is atleast $y\in U$ suchthat $x\in Ry$.Thus,$N_i\neq \emptyset, \forall x \in U$. Hence the proof.

Definition: 3.8 Let U be a non-empty finite set of objects called the universe R be an any binary relation on U. Elements belonging to the after set (resp. fore set)of an element $x\in U$. The pair (U, R) where U is said to be the approximation space (U,R), and $\tau_R(X)$={U,ϕ, $L_R(X)$,$U_R(X)$, $B_R(X)$} where $X \subseteq U$ and if

(i) $L_R(X) = \bigcup_{x\in U}\{(x)R : N_r(x) \subseteq X\}$

(ii) $U_R(X) = \bigcup_{x\in U}\{(x)R : N_r(x) \cap X \neq \phi\}$

(iii) $B_R(X) = U_R(X) - L_R(X)$.

Where $\tau_R(X)$ forms a topology on U called the Covering with Nano topology on U with respect to X.We call (U,$\tau_R(X)$) as the Covering Nano topological space.topology $\tau_r(X)=\{U,\phi,\{a,b,c\},\{d\}\}$(resp.fore set).

Proposition: 3.10 Let (U, R) be an approximation space

.If x∈ $N_i(y)$. Then, $N_i(x) \subseteq N_i(y)$, ∀ i = r.l.

Proof: Let x∈ $N_i(y)$, then x is contained in all after sets (resp. fore sets) that contains also the element y. Now, if z ∈ $N_i(x)$.Then z is contained in all after set (resp. fore set) that contains the element x and thus z is contained in all after sets(resp. fore sets) that contains also the element y. Then z ∈ $N_i(y)$ which implie $N_i(x) \subseteq N_i(y)$,∀ i = r.l. Hence the proof.

Theorem: 3.11 Let (U,R) be an approximation space and X, Y ⊆ U. Then, for each ∀ i = r.l.

(i) $L_{Ri}(U)=U_{Ri}(U)=U$

(ii) $L_{Ri}(\phi)=U_{Ri}(\phi)=\phi$

(iii) If $X \subseteq Y$, then $L_{Ri}(X) \subseteq L_{Ri}(Y)$

(iv) If $X \subseteq Y$, then $U_{Ri}(X) \subseteq U_{Ri}(Y)$

(v) $L_{Ri}(X \cup Y) \supseteq L_{Ri}(X) \cup L_{Ri}(Y)$

(vi) $L_{Ri}(X \cap Y) = L_{Ri}(X) \cap L_{Ri}(Y)$

(vii) $U_{Ri}(X \cap Y) \subseteq U_{Ri}(X) \cap U_{Ri}(Y)$

(viii) $U_{Ri}(X \cup Y) = U_{Ri}(X) \cup U_{Ri}(Y)$

Proof: (i) and (ii) are straight forward ,from the definition of approximation space in an covering nano topological space.

(iii) and (iv) Suppose that $X \subseteq Y$,and let Ni(x) ∈ $L_{Ri}(X)$. Then x ∈ X and Ni(X)) ⊆ X, which means that x ∈ Y and Ni(x)⊆ Y.Thus $L_{Ri}(X)$) ⊆ $L_{Ri}(Y)$.By the same way $U_{Ri}(X)$) ⊆ $U_{Ri}(Y)$.

(v) and (vi) Since $X \subseteq X \cup Y$ and $Y \subseteq X \cup Y$ .Then $L_{Ri}(X) \subseteq L_{Ri}(X \cup Y)$ and $L_{Ri}(Y) \subseteq L_{Ri}(X \cup Y)$ .Thus we have $L_{Ri}(X) \cup L_{Ri}(Y) \subseteq L_{Ri}(X \cup Y)$. By the same way $L_{Ri}(X \cap Y) \subseteq L_{Ri}(X) \cap L_{Ri}(Y)$.

(vii) and (viii) Let $N_i(x) \in L_{Ri}(X) \cap L_{Ri}(Y)$, then $N_i(x) \in L_{Ri}(X)$ and $N_i(x) \in L_{Ri}(Y)$.Thus x ∈ X, $N_i(x) \subseteq X$ and X∈ Y ,$N_i(x) \subseteq Y$ which means that x ∈ $X \cap Y$, $N_i(x) \subseteq X \cap Y$ .Then $N_i(x) \in L_{Ri}(X \cap Y)$ and this implies $L_{Ri}(X) \cap L_{Ri}(Y) \subseteq L_{Ri}(X \cap Y)$. Now, Let $N_i(x) \in L_{Ri}(X \cap Y)$ then x∈ $X \cap Y$ and $N_i(x) \subseteq X \cap Y$ .Thus x ∈ X, $N_i(x) \subseteq X$ and x∈ Y, $N_i(x) \subseteq Y$ which implies $N_i(x) \in L_{Ri}(X)$ and $N_i(x) \in L_{Ri}(Y)$. Then $N_i(x) \in L_{Ri}(X) \cap L_{Ri}(Y)$ and thus $L_{Ri}(X \cap Y) \subseteq L_{Ri}(X) \cap L_{Ri}(Y)$. Hence, $L_{Ri}(X \cap Y) = L_{Ri}(X) \cap L_{Ri}(Y)$. By the same way $U_{Ri}(X \cup Y) = U_{Ri}(X) \cup U_{Ri}(Y)$.

Example: 3.12 Data are usually given in a form of a data table, called also an arrtibute - value table ,an information table(or) a data base. Data table is a matrix rows of which are labelled by objects, whereas columns are labelled by attributes. Entries of the table are attribute values. In a data base table show below:

Store	E	Q	L	p
1	High	Good	NO	PROFIT
2	Medium	Good	NO	LOSS
3	Medium	Good	NO	PROFIT
4	NO	Average	NO	LOSS
5	Medium	Average	YES	LOSS
6	High	Average	YES	PROFIT

Trading Data Analysis Table: In the data base six stores are characterized by four attributes:

E- **E**mpowerment of sales personal

Q- Perceived **Q**uality of merchandise

L- High Traffic **L**ocation

P- Store Profit

Support we are interested which features are associated with profit (or) loss of stores. This problem can not be solved uniquely since the data are inconsistent ie., stores two and three have the same features(Values of attributes E,Q,L) but the store two has loss, where as store three has profit .Usually to avoid if this kind of inconsistency a stores is assumed. Also adding more attributes can resolve the inconsistency. In the nano topological theory the approach is different .We try to preserve the data intract and find other ways out, and see what the original data is telling us. To this end we propose instead covering methods (or) reducible covering methods to use nano topological methods of reasoning of the lower and upper approximation space and boundary region of a set.

Let U={1,2,3,4,5,6} and Profit={1,3,6} and Loss={2,4,5} with R= {(1,1), (1,6), (2,2), (2,3), (3,2), (6,1), (4,4), (4,5), (5,5), (5,4), (6,6)}.Then 1R={1,6} ,2R={2,3} ;3R={ϕ} ;4R={4,5}, 5R={5}; 6R={6} also we have $N_r(1)=\{1,6\}$, $N_r(2)=\{2,3\}$, $N_r(3)=\{2,3\}$, $N_r(4)=\{4,5\}$, $N_r(5)=\{5\}$, $N_r(6)=\{6\}$.we can find the covering nano topological space:

X={1,3,6}:(**PROFIT**):

$L_{Rr}(X)=\{1,6\}$ and $U_{Rr}(X)=\{1,2,3,6\}$ and

$B_{Rr}\{X\}=\{2,3\}$.Then $\tau_{Rr}(X)=\{U, \phi, \{1,6\}, \{1,2,3,6\}, \{2,3\}\}$ similarly the fore set in same way.

Y={2,4,5}(**LOSS**):

$L_{Rr}(Y)=\{4,5\}$ and $U_{Rr}(Y)=\{2,3,4,5\}$ and

$B_{Rr}\{Y\}=\{2,3\}$.Then $\tau_{Rr}(Y)=\{U, \phi,\{4,5\},\{2,3,4,5\}, \{4,5\}\}$ similarly the fore set in same way.

Observation:

(i) The set {1,3,6} of all stores having profit.

(a) The set {1,6} of all stores certainly having profit(The lower approximation space of the set)

(b) The set {1,2,3,6} of all stores possibly having profit (the upper approximation of the set).

(c) The set {2,3} of all stores that can be classified as having neither profit nor loss(The boundary region of the set)

(ii) The stores {2,4,5} of all stores having loss.

(a) The set {4,5} of all stores certainly having loss(The lower approximation space of the set)

(b) The set {2,3,4,5} of all stores possibly having loss(The upper approximation space of the set)

(c) The set {2,3} of all stores that can be classified as having neither profit nor loss(The boundary region of the set).

Conclusion: In this paper, we investigate the covering nano topological space based on binary relation by approximation space. The used technique depends basically on a general binary relation to define the covering and also the neighbourhood. Also we generate new methods for computing the nano topology from general binary relation.

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