BOLD SIGNED TOTAL DOMINATION FOR $K_{m,n,p}$, $C_{m,n}$ and W_n GRAPHS

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Abstract: A set D is a subset of V (G) is called dominating or total dominating set in G, if $D \cap N[v] \neq \emptyset$ or $D \cap N(v) \neq \emptyset$, respectively for every vertex v $\in$V(G). The minimum number of vertices of a dominating set or of a total dominating set in G is called the domination number $\gamma(G)$ or the total domination number $\gamma_t(G)$, respectively of G. If v is a vertex of a graph G, then N(v) is its open neighbourhood, (i.e.) the set of all vertices adjacent to v in G. A mapping f : V(G) → { -2,1 }, where V(G) is the vertex set of G, is called a Bold Signed Total Dominating Function (BSTDF) on G, if w(f) = $\sum_{x\in N(v)} f(x) \geq 1$ for each v $\in$V(G). $\min_f\{\sum_{x\in V(G)} f(x)$: f is a BSTDF } is called the bold signed total domination number of G and is denoted by $\gamma_{bst}(G)$. The bold signed total domination number of a graph is a certain variant of the domination number. $\gamma_{bst}(G)$ are found for the graphs $K_{m,n,p}$, $C_{m,n}$ and W_n and independent proofs are obtained.

Keywords: Bold signed total dominating function, Bold signed total domination number, Dominating function, Domination number.

Introduction: In this paper we study the bold signed total domination number of a graph and using the notation as in [2]. We consider finite undirected graphs without loops and multiple edges [1]. The vertex set of a graph G is denoted by V(G). If v $\in$V(G), then the *open neighbourhood* N(v) of v in G is the set of all vertices which are adjacent to v in G. Further, the *closed neighbourhood* of v in G is defined as N[v] = N(v) $\cup$ {v}. Let f be a mapping of V(G) into set of real numbers, let S is a subset of V(G). Then we denote f(S) = $\sum_{x\in S} f(x)$. Futher, the weight of f is w(f) = f(V(G)) = $\sum_{x\in V(G)} f(x)$. We will study the concept, from the definition. A function f : V(G) → { -2,1 } is called a *Bold Signed Dominating Function* (shortly BSDF) of G, if f(N[v]) ≥ 1 for each v $\in$V(G). The minimum of w(f) = f (V(G)) = $\sum_{x\in V(G)} f(x)$, taken over all BSDF of G, is the *bold signed domination number* $\gamma_{bs}(G)$ of G. Similarly, a function f : V(G) → { -2,1 } is called a *Bold Signed Total Dominating Function* (shortly BSTDF) of G, if f(N(v)) ≥ 1 for each v$\in$ V(G). The minimum of w(f) = f(V(G)) = $\sum_{x\in V(G)} f(x)$, taken over all BSTDF of G, is the *bold signed total domination number* $\gamma_{bst}(G)$ of G.

Theorem: Let G be a complete 3-partite graph with tripartition [V_1 : V_2 : V_3]. Let f:V→{-2,1} be a bold signed total dominating function. Let r_1, r_2 and r_3 be the possible number of vertices, assigned with value -2 in the respective sets V_1, V_2, V_3 and whose respective cardinality be n_1, n_2, and n_3 such that $n_1+n_2+n_3 = 3n$. Then
(i)$n_1+n_2 \geq 3(r_1+r_2)+1$,
(ii) $n_1+n_3 \geq 3(r_1+r_3)+1$,
(iii) $n_2+n_3 \geq 3(r_2+r_3)+1$ and (iv) bold signed total domination number is $\gamma_{bst}(G) = n_1+n_2+n_3 - 3 \lfloor r_1+r_2+r_3 \rfloor$

where n is any positive integer.

Proof: Let $V_i = \{v_{ij}\}$, j=1 to n_i , i = 1,2,3. Now $|V_i| = n_i$, i = 1,2,3. Let f : V →{-2,1} be a bold signed total domin a tingfunction in G. r_1, r_2 and r_3 are the possible number of vertices in G takes the value -2 in the respective vertex sets V_1, V_2, V_3. Further we have $\sum_{v\in N(u)} f(v) \geq 1$, for all u$\in$ V.------[1].

Case (i): u$\in V_1$. : Then u has n_2+n_3 adjacent vertices in G (Since all the vertices of both V_2 and V_3 are adjacent to every vertex of V_1). Therefore N(u) contains n_2+n_3 vertices. Now u is either -2 or 1 and further, r_2 vertices are assigned with -2 in V_2 and r_3 vertices are assigned with -2 in V_3. Therefore $n_2-r_2+n_3-r_3-2r_2-2r_3 \geq 1$ (since by eqn.[1]).
(i.e.) $n_2+n_3-3r_2-3r_3 \geq 1$.
(i.e.) $n_2+n_3 \geq 3(r_2+r_3)+1$. -------[2]

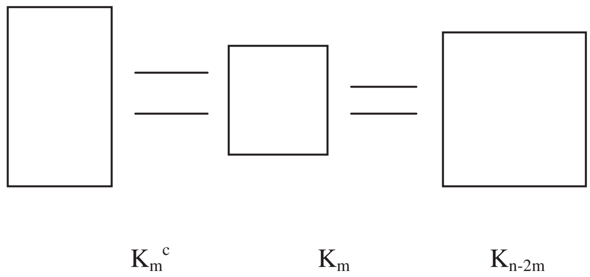
Case (ii): u$\in V_2$.: Then u has n_1+n_3 adjacent vertices in G (Since all the vertices of both V_1 and V_3 are adjacent to every vertex of V_2). Therefore N(u) contains n_1+n_3 vertices. Now u is either -2 or 1 and further, r_1 vertices are assigned with -2 in V_1 and r_3 vertices are assigned with -2 in V_3. Therefore $n_1-r_1+n_3-r_3-2r_1-2r_3 \geq 1$ (since by eqn.[1]).
(i.e.) $n_1+n_3-3r_1-3r_3 \geq 1$.
(i.e.) $n_1+n_3 \geq 3(r_1+r_3)+1$. --------[3]

Case (iii): u$\in V_3$.: Then u has n_1+n_2 adjacent vertices in G (Since all the vertices of both V_1 and V_2 are adjacent to every vertex of V_3). Therefore N(u) contains n_1+n_2 vertices. Now u is either -2 or 1 and further, r_1 vertices are assigned with -2 in V_1 and r_2 vertices are assigned with -2 in V_2. Therefore $n_1-r_1+n_2-r_2-2r_1-2r_2 \geq 1$ (since by eqn.[1]).
(i.e.) $n_1+n_2-3r_1-3r_2 \geq 1$.
(i.e.) $n_1+n_2 \geq 3(r_1+r_2)+1$. ---------[4]

By solving the equations [2], [3], and [4], we get the values for r_1, r_2, and r_3 in fractions. Take the integral values of r_1 , r_2 and r_3 such that any 2 of equations [2], [3] and [4] are satisfied and $\lfloor r_1+r_2+r_3 \rfloor$ also satisfied.

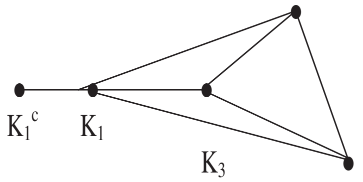
Hence $\gamma_{bst}(G) = n_1+n_2+n_3 - 3 \lfloor r_1+r_2+r_3 \rfloor$.

Definition: For $1 \leq m \leq (n/2)$, let $C_{m,n}$ [1] denote the graph K_m V (K_m^c + K_{n-2m}), depicted in Figure.

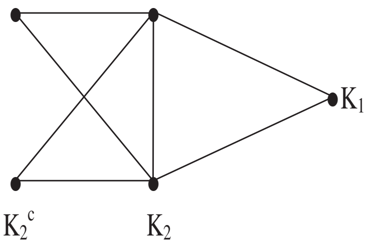


Example:

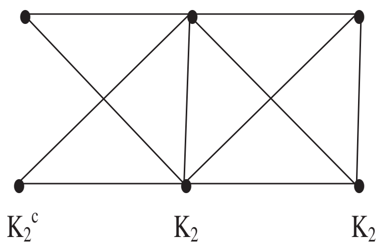
1. $C_{1,5} = K_1 \vee (K_1^c + K_3)$



2. $C_{2,5} = K_2 \vee (K_2^c + K_1)$



3. $C_{2,6} = K_2 \vee (K_2^c + K_2)$



Theorem:

Prove that $\gamma_{bst}(C_{m,n}) = \begin{cases} 2 & \text{if } n = 3s+2 \\ 4 & \text{if } n = 3s+1 \\ 3 & \text{if } n = 3s, \end{cases}$

for all $n \geq 3$ and s is any positive integer.

Proof: We know that $C_{m,n} = K_m \vee (K_m^c + K_{n-2m})$.

Every vertex in the part K_m is adjacent to every other vertices of $C_{m,n}$. Let r be the number of vertices assigned with -2 in $C_{m,n}$.

We know that

$w(f) = \sum_{x \in V(G)} f(x)$ and $\sum_{x \in N(v)} f(x) \geq 1$

for all $v \in V(G)$.

Therefore $(n-1-r)-2r \geq 1$. (i.e.) $3r \leq n-2$. (i.e.) $r \leq (n-2)/3$.

Since r is an integer,

$$r \leq \begin{cases} (n-2)/3 & \text{if } n = 3s+2 \\ (n-2)/3 - 2/3 & \text{if } n = 3s+1 \\ (n-2)/3 - 1/3 & \text{if } n = 3s. \end{cases}$$

Therefore $w(f) = \sum_{v \in V(G)} f(v)$

$= m+m+n-2m-r-2r = n-3r.$

That is,

$$w(f) \geq \begin{cases} n - 3[(n-2)/3] = 2 & \text{if } n = 3s+2 \\ n - 3[(n-2)/3 - 2/3] = 4 & \text{if } n = 3s+1 \\ n - 3[(n-2)/3 - 1/3] = 3 & \text{if } n = 3s. \end{cases}$$

Therefore

$$\gamma_{bst}(C_{m,n}) = \min w(f) = \begin{cases} 2 & \text{if } n = 3s+2 \\ 4 & \text{if } n = 3s+1 \\ 3 & \text{if } n = 3s, \end{cases}$$

for all $n \geq 3$ and s is any positive integer.

Theorem: For a wheel W_n , $n \geq 4$, we have $\gamma_{bst}(W_n) = n$.

Proof:

Let W_n be a wheel, $n \geq 4$. Let r be the number of vertices assigned with -2.(i.e.) n-r vertices assigned with 1.

Case (i): Any vertex v other than the centre vertex.

Since N(v) contains only 3 vertices in W_n , $f(N(v)) = 3-r-2r \geq 1$. (i.e.) $3-3r \geq 1$. (i.e.) $3r \leq 2$. (i.e.) $r \leq 2/3$.

Since r is an integer, r = 0.

Therefore all the vertices are assigned with 1. Hence $\gamma_{bst}(W_n) = \min w(f) = n$.

Case (ii): Vertex v is the centre vertex.

Here N(v) contains n-1 vertices in W_n.

Therefore $f(N(v)) = (n-1-r)-2r \geq 1$. (i.e.) $3r \leq n-2$. (i.e.) $r \leq (n-2)/3$.

Since W_n has atleast 4 vertices, $r \leq 2/3$ and $r \leq (n-2)/3$.

Hence r = 0.

Therefore all the vertices are assigned with 1. Hence $\gamma_{bst}(W_n) = \min w(f) = n$.

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