ON CHARACTERISTIC POLYNOMIAL OF PRODUCT OF INCIDENCE MATRIX AND ITS TRANSPOSE OF P_n, W_n

V. KALADEVI, V.MANIMOZHI

Abstract: Muddebihal has found that ,the energy of a graph with respect to the product of incidence matrix and its transpose as sum of the degrees of the vertices of a graph. In this paper the product of incident matrix and its transpose of graphs Cn,Kn,K1,n are considered to derive the characteristic polynomial and eigen vectors of such polynomials have applications in many field.

Keywords: Absolute spectrum, Right incidence matrix, left incidence matrix.

Introduction: The spectrum presented in this paper has been classified as the spectrum of product of incidence matrix I(G) and its transpose matrix $I(G)^T$ .Let $X= RR^T$;$Y = R^TR$. The matrix X is known as the right transpose incidence matrix and is denoted by RTI(G). The matrix Y is known as the left transpose incidence matrix and is denoted by LTI(G).The characteristic polynomial of X is called the right transpose incidence characteristic polynomial of G and is denoted by C(RTI(G);λ). The characteristic polynomial of Y is called the left transpose incidence characteristic polynomial of G and is denoted by C(LTI(G);λ).

Definition 1.1: The characteristic polynomial of the right transpose incidence matrix of a graph G is defined as C(G; λ)= det(λI-X) ,where X is the matrix under consideration and I is the identity matrix.

Definition 1.2: The characteristic polynomial of the left transpose incidence matrix of a graph G is defined as C(G; λ)= det(λI-Y) ,whereY is the matrix under consideration and I is the identity matrix.

Definition 1.3: If λ1> λ2>....>λs are the distinct eigen values of X with respective multiplicities m1,m2,...ms ,then the spectrum of G is written as

$$spec(G)=\begin{pmatrix} \lambda_1 & \lambda_2 & ... & \lambda_s \\ m_1 & m_2 & ... & m_s \end{pmatrix}$$
$$=\lambda_1^{m_1}\lambda_2^{m_2}...\lambda_s^{m_s}$$

Note:If p=q,then spec(X) = spec(Y).

2. Characteristic Polynomial Of Rti(P_n), LTI(P_n), RTI (W_n)

Theorem 2.1: The characteristic polynomial of $RTI(P_n)$ is

$$\lambda^n-\frac{(2n-4)}{1!}\lambda^{n-1}+\frac{(2n-3)(2n-4)}{2!}\lambda^{n-2}$$
$$-\frac{(2n-4)(2n-5)(2n-6)}{3!}\lambda^{n-3}+...$$
$$+(-1)^n n$$

and the characteristic polynomial of $LTI(P_n)$ is

$$\lambda^{n-1}-\frac{(2n-4)}{1!}\lambda^{n-2}+\frac{(2n-3)(2n-4)}{2!}\lambda^{n-3}$$
$$-\frac{(2n-4)(2n-5)(2n-6)}{3!}\lambda^{n-4}+...$$
$$+(-1)^{n+1} n$$

Proof:

Let G be a path graph P_n with vertex set $V(G)=\{v_1,v_2,....v_n\}$ and edge set $E(G)=\{v_iv_{i+1};1\le i\le n-1\}$ respectively .

The characteristic polynomial of RTI(P_3) is

$$X=RR^T=\begin{bmatrix}1&1&0\\1&2&1\\0&1&1\end{bmatrix}$$

The characteristic polynomial of RTI(P_4) is

$$X=RR^T=\begin{bmatrix}1&1&0&0\\1&2&1&0\\0&1&2&1\\0&0&1&1\end{bmatrix}$$

In general for P_n

$$\det(X-\lambda I)$$
$$=\begin{vmatrix}1-\lambda&1&0&\dots&0\\1&2-\lambda&1&\dots&0\\0&1&2-\lambda&\dots&0\\\vdots&&&&\\0&0&0&\dots&1-\lambda\end{vmatrix}$$

$$=(1-\lambda)\begin{vmatrix} 2-\lambda & 1 & \dots & 0 \\ 1 & 2-\lambda & \dots & 0 \\ \dots & & & \\ 0 & 0 & \dots & 1-\lambda \end{vmatrix}$$

$$-\begin{vmatrix} 1 & 1 & \dots & 0 \\ 0 & 2-\lambda & \dots & 0 \\ \dots & & & \\ 0 & 0 & \dots & 1-\lambda \end{vmatrix}$$

$$=\lambda^n-\frac{(2n-4)}{1!}\lambda^{n-1}+\frac{(2n-3)(2n-4)}{2!}\lambda^{n-2}$$

$$-\frac{(2n-4)(2n-5)(2n-6)}{3!}\lambda^{n-3}+\dots$$

$$+(-1)_n^n=0$$

There fore, the characteristic polynomial of $RTI(P_n)$ is

$$\lambda^n-\frac{(2n-4)}{1!}\lambda^{n-1}+\frac{(2n-3)(2n-4)}{2!}\lambda^{n-2}$$

$$-\frac{(2n-4)(2n-5)(2n-6)}{3!}\lambda^{n-3}+\dots+(-1)^n n$$

The characteristic polynomial of LTI(P_2) is

$$Y=R^TR=(1\ \ 1)\begin{pmatrix}1\\1\end{pmatrix}=(2)$$

The characteristic polynomial of LTI(P_3) is

$$X=RR^T=\begin{bmatrix}1 & 1 & 0\\1 & 2 & 1\\0 & 1 & 1\end{bmatrix}$$

The characteristic polynomial of LTI(P_4) isI

$$Y=R^TR=\begin{bmatrix}2 & 1 & 0\\1 & 2 & 1\\0 & 1 & 2\end{bmatrix}$$

In general ,

$$\det(Y-\lambda I)$$

$$=(2-\lambda)\begin{vmatrix} 2-\lambda & 1 & \dots & 0 \\ 1 & 2-\lambda & \dots & 0 \\ \vdots & \vdots & & \\ 0 & 0 & \dots & 2-\lambda \end{vmatrix}$$

$$-\begin{vmatrix} 1 & 1 & \dots & 0 \\ 0 & 2-\lambda & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & 2-\lambda \end{vmatrix}$$

$$=\lambda^{n-1}-\frac{(2n-4)}{1!}\lambda^{n-2}+\frac{(2n-3)(2n-4)}{2!}\lambda^{n-3}$$

$$-\frac{(2n-4)(2n-5)(2n-6)}{3!}\lambda^{n-4}+\dots$$

$$+(-1)_n^{n+1}=0$$

Therefore,the characteristic polynomial of $LTI(P_n)$ is

$$\lambda^{n-1}-\frac{(2n-4)}{1!}\lambda^{n-2}+\frac{(2n-3)(2n-4)}{2!}\lambda^{n-3}$$

$$\frac{(2n-4)(2n-5)(2n-6)}{3!}\lambda^{n-4}+\dots$$

$$+(-1)^{n+1}n$$

Theorem 2.2:

The characteristic polynomial of $RTI(w_n)$ is

$$(\lambda-2)^{(n-2)}(\lambda-3)(\lambda-(n-1))$$

$$-(\lambda-2)^{(n-2)}$$

$$-(n-2)(\lambda-2)^{(n-2)}(\lambda-(n-2))$$

Proof:

Let G be the wheel graph W_n with vertex set $V(G)=\{v_1,v_2,\dots,v_n\}$ and edge set $E(G)=\{v_0v_i;1\le i\le n\}\cup\{v_1v_2,\dots,v_nv_1\}$ respectively.

The characteristic polynomial of RTI(W_4) is

$$\det(X-\lambda I)=\begin{vmatrix} 3-\lambda & 1 & 1 & 1 \\ 1 & 3-\lambda & 1 & 1 \\ 1 & 1 & 3-\lambda & 1 \\ 1 & 1 & 1 & 3-\lambda \end{vmatrix}$$

The characteristic polynomial of RTI(W_5) is

$$X=\begin{bmatrix} 4 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 1 & 3 \end{bmatrix}$$

$$\det(X-\lambda I)$$

$$=\begin{vmatrix} 4-\lambda & 1 & 1 & 1 & 1 \\ 1 & 3-\lambda & 1 & 1 & 1 \\ 1 & 1 & 3-\lambda & 1 & 1 \\ 1 & 1 & 1 & 3-\lambda & 1 \\ 1 & 1 & 1 & 1 & 3-\lambda \end{vmatrix}$$

The characteristic polynomial of RTI(W_6) is

$$(X-\lambda I)$$

$$=\begin{bmatrix} 5-\lambda & 1 & 1 & 1 & 1 & 1 \\ 1 & 3-\lambda & 1 & 1 & 1 & 1 \\ 1 & 1 & 3-\lambda & 1 & 1 & 1 \\ 1 & 1 & 1 & 3-\lambda & 1 & 1 \\ 1 & 1 & 1 & 1 & 3-\lambda & 1 \\ 1 & 1 & 1 & 1 & 1 & 3-\lambda \end{bmatrix}$$

In general for W_n,

$$\det(X-\lambda I)$$

$$=\begin{vmatrix} (n-1)-\lambda & 1 & 1 & \cdots & 1 \\ 1 & 3-\lambda & 1 & \cdots & 1 \\ 1 & 1 & 3-\lambda & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 & 3-\lambda \end{vmatrix}$$

$$=(\lambda-2)^{(n-2)}(\lambda-3)(\lambda-(n-1))$$
$$-(\lambda-2)^{(n-2)}$$
$$-(n-2)(\lambda-2)^{(n-2)}(\lambda-(n-2))$$

Conclusion:

The Graph energies have special significance in chemical graphs. In this paper the characteristic polynomials of RTI (P_n) , LTI(P_n) and RTI(W_n) are found out.

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V.Kaladevi/Professor Emirates,Bishop Heber College/kaladevi1956@gmail.com

V.Manimozhi/ Assistant Professor/Chidambaram Pillai College for Women/v.manimozhi@yahoo.in