EXTENDED BITOPOLOGICAL SPACES VIA NEW OPERATORS

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Abstract: In this paper we introduce Extended Bitopological space by the simple extensions of bitopological spaces and investigate certain properties on it. Then we introduce two classes of sets (1,2)*$^+$-$\wedge_s$-sets and (1,2)*$^+$-$\vee_s$-sets in Extended Bitopological space and also introduce two more classes of sets (1,2)*$^+$-g.$\wedge_s$-sets and (1,2)*$^+$-g.$\vee_s$-sets. Also establish some basic properties and characterizations of these operators is obtained

Keywords: Extended bitopological spaces, (1,2)*$^+$-$\wedge_s$-sets, (1,2)*$^+$-$\vee_s$-sets, (1,2)*$^+$-g.$\wedge_s$-sets, (1,2)*$^+$-g.$\vee_s$-sets

Introduction: J.C. Kelly [1] initiated the study of the bitopological space which is to be a set X on which are defined two arbitrary topologies $\tau_1$and τ_2. The concepts of g-closed sets are introduced in bitopological spaces by Maki in 1986 [5], Levin introduced the concepts of semi-open sets in Topology in 1963 [2]. LellisThivagar et al have introduced the concepts of(1,2)*-semi open sets in bitopological spaces. In 1964, Levin [4] initiated the simple extensions of topologies. In this paper we introduce extended bitopological space by the simple extensions of bitopological spaces and investigate certain properties on it .Then introduce and characterize the concepts of (1,2)*$^+$-$\wedge_s$-sets, (1,2)*$^+$- $\vee_s$-sets, (1,2)*$^+$-g.$\wedge_s$-sets and (1,2)*$^+$-g.$\vee_s$-sets in Extended bitopological space. The purpose of the present paper is defined the operators and also basic properties is obtained.

2. Preliminaries:

Definition 2.1. [7]: A subset *S* of a bitopological space (X, τ_1, τ_2)is said to be $\tau_{1,2}$-open if S = A $\cup$ B, where A $\in \tau_1$ and B $\in \tau_2$.
A subset *S* of *X* is said to be $\tau_{1,2}$-closed if the complement of *S* is $\tau_{1,2}$-open.

Definition 2.2. [7]: Let *S* be a subset of X. Then
(i) The $\tau_{1,2}$- interior of S, denoted by $\tau_{1,2}$- int(S) is defined by $\bigcup${F/F$\subset$S and F is $\tau_{1,2}$-open}
(ii) The $\tau_{1,2}$-closure of S denoted by $\tau_{1,2}$- cl(S) is defined by $\bigcap${F/S $\subset$ F and F is $\tau_{1,2}$-closed}

Remark 2.3:$\tau_{1,2}$-open sets need not form a Topology. We recall the following definitions which are useful in the sequel.

Definition 2.4. [7]: A subset *A* of a bitopological space (X,τ_1,τ_2) is called
(i) (1,2)*-semi open if A $\subseteq\tau_{1,2}$-cl($\tau_{1,2}$-int(A))
(ii) (1,2)*-pre open if *A* $\subseteq\tau_{1,2}$-int($\tau_{1,2}$-cl(A))
(iii)(1,2)*-α-open ifA$\subseteq\tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)))
(iv)(1,2)*-semi closed, (1,2)*-pre closed, (1,2)*-α-closed, if A^{c}is (1,2)*-semi open,(1,2)*-pre open, (1,2)*-α-open, respectively.

Definition 2.5.[7]: (1,2)*-generalized closed (briefly (1,2)*-g-closed) if $\tau_{1,2}$-cl(A) $\subset$ U whenever A $\subset$U and U is $\tau_{1,2}$-open in X.

Definition 2.6. [11]: Let B be a subset of a bitopological space (X,τ_1,τ_2). We define the subsets (1,2)*-$B^{\wedge_s}$ and (1,2)*-$B^{\vee_s}$ as follows:
(i) (1,2)*-$B^{\wedge_s}$ =$\bigcap$ {O : O $\supseteq$B and O is (1,2)*-semi-open}.
(ii) (1,2)-$B^{\vee_s}$ = $\bigcup$ {F:F $\subseteq$ Band F^{c}is (1,2)*-semi-open}

Definition 2.7.[6]: In an bitopological space (X,τ_1,τ_2) a subset B is called a (1,2)*-g.$\wedge_s$-set of (X,τ_1,τ_2) if (1,2)*-$B^{\wedge_s}$ $\subseteq$F whenever B $\subseteq$F and F is (1,2)*-semi closed.

Definition 2.8.[6]: In an bitopological space (X,τ_1,τ_2) a subset B is called a (1,2)*-g.$\vee_s$-set of (X,τ_1,τ_2) if B^c is a (1,2)*-g.$\wedge_s$ set of (X,τ_1,τ_2).

Definition 2.9. [4]: Let (X,τ)be a topological space and $\tau\subset\tau^+$. Then τ^+will be termed a simple extension of τifand only if there exists an A $\notin\tau$such that
τ^+ = {O$\cup$(O′$\cap$A) :O,O′$\in\tau$}. We denote $\tau^+ = \tau$(A).

Lemma 2.10. [4]: Let (X,τ) be a topological space and $\tau^+ = \tau$ (A) a simple extension. If B $\subset$ X, then
(i) Int$^+$B = IntB$\cup$IntA(B$\cap$A), where Int$^+$, Int and Int$_A$denote the interior operators relative to τ^+, τ and $\tau\cap$A respectively.
(ii) **cl**$^+$(B) = **cl**(B)$\cap${A$^c\cup$(A$\cap$**cl**(B$\cap$A))} where **cl**and **cl**+ denote the closure operators relative to τ and τ^+ respectively.
(iii). **cl**$^+$(B$\cap$A) = **cl**(B$\cap$A)

3.Extended Bitopological Space: In this section we introduce the new type of extended bitopological spaces by a simple extension on a bitopological space (X,τ_1,τ_2) and is denoted by (X, $\tau^+_{(1,2)}$).

Definition 3.1. Let(X,τ_1,τ_2) be a bitopological space and $\tau_{1,2}\subset\tau^+_{(1,2)}$. Then $\tau^+_{(1,2)}$ will be termed a simple extension of $\tau_{1,2}$if and only if there exists an A $\notin\tau_{1,2}$ such that $\tau^+_{(1,2)}$={G$_1\cup$ (G$_2\cap$ A) : G$_1$, G$_2\in\tau_{1,2}$}. We call (X, $\tau^+_{(1,2)}$) an extended bitopological space on (X, τ_1, τ_2). A subset S of X is said to be $\tau^+_{(1,2)}$ closed if the complement of S is $\tau^+_{(1,2)}$-open.

Example 3.2. Let X = {a,b,c}. Then τ_1= {ϕ,X,{a}} and τ_2 = {ϕ,X,{b}}. Then $\tau_{1,2}$={ϕ, X, {a}, {b}, {a, b}}.
Let A = {a, c} $\notin\tau_{1,2}$.
Then sets are $\tau^+_{(1,2)}$ = {ϕ, X, {a}, {b}, {a, b}, {a, c}} and closed sets are {X, {ϕ}, (a, c}, {b, c}, {b}, {c}}. Then

$(X, \tau^{+}_{(1,2)})$ is an extended bitopological space.

Note: Notice that the set of all $\tau^{+}_{(1,2)}$ sets need not form a Topology.

Definition 3.3. Let $(X, \tau^{+}_{(1,2)})$ be an extended bitopological space and $S \subseteq X$.

(i) The $\tau^{+}_{(1,2)}$-interior of S,denoted by $\tau^{+}_{(1,2)}$-int(S) is defined by $\bigcup$ {F/F $\subset$S and F is $\tau^{+}_{(1,2)}$-open}

(ii) The $\tau^{+}_{(1,2)}$-closure of *S* denoted by $\tau^{+}_{(1,2)}$- cl(S) is defined by $\bigcap$ {F/S $\subset$ F and F is $\tau^{+}_{(1,2)}$ -closed)

Theorem 3.4(X, $\tau^{+}_{(1,2)}$)be an extended bitopological space.

(i) If $S_1 \subseteq S_2 \subseteq$ X, then $\tau^{+}_{(1,2)}$-int(S_1) $\subseteq \tau^{+}_{(1,2)}$-int($S_2$) and $\tau^{+}_{(1,2)}$-cl(S_1) $\subseteq \tau^{+}_{(1,2)}$-cl($S_2$).

(ii) $\tau^{+}_{(1,2)}$-int(S) is $\tau^{+}_{(1,2)}$-open and $\tau^{+}_{(1,2)}$-cl(S) is $\tau^{+}_{(1,2)}$-closed for each S $\subset$ X.

(iii) A set S $\subset$ X is $\tau^{+}_{(1,2)}$-open iff S = $\tau^{+}_{(1,2)}$-int(S) and a set S $\subset$ x is $\tau^{+}_{(1,2)}$-closed iff S = $\tau^{+}_{(1,2)}$ cl(S).

(iv)(a). For any S $\subset$ X, we have $\tau^{+}_{(1,2)}$- int($\tau^{+}_{(1,2)}$-int(S)) = $\tau^{+}_{(1,2)}$-int(S);

(b). For any S $\subset$ X, we have $\tau^{+}_{(1,2)}$-cl($\tau^{+}_{(1,2)}$-cl(S)) = $\tau^{+}_{(1,2)}$-cl(S).

(v) (a). $\tau^{+}_{(1,2)}$-int(X-S)=X - $\tau^{+}_{(1,2)}$-cl(S) for any S $\subset$ X.

(b). $\tau^{+}_{(1,2)}$-cl(X-S)=X- $\tau^{+}_{(1,2)}$-int(S) for any S $\subset$X

Theorem 3.5 Let (X, $\tau^{+}_{(1,2)}$) be an extended bitopological space. Then A is closed in (X,τ_1,τ_2) if and only if A is closed in (X, $\tau^{+}_{(1,2)}$(A}}(A. is always open in (X, $\tau^{+}_{(1,2)}$(A))).

Theorem 3.6: Let (X, $\tau^{+}_{(1,2)}$) be an extended bitopological space. A subset S of X is called

(i) (1,2)*$^{+}$-α-open if S $\subset \tau^{+}_{(1,2)}$-int($\tau^{+}_{(1,2)}$-cl $\tau^{+}_{(1,2)}$-int(S)))

(ii) (l,2)*$^{+}$-semi-open if S $\subset \tau^{+}_{(1,2)}$-cl($\tau^{+}_{(1,2)}$-int(S))

(iii)(l,2)*$^{+}$-pre-open if S$\subset \tau^{+}_{(1,2)}$-int($\tau^{+}_{(1,2)}$-cl(S))

The complement of (1,2)*$^{+}$-α-open[resp. (1,2)*$^{+}$-semi-open, (1,2)*$^{+}$-pre-open] set (1,2)*$^{+}$*$^{+}$-α-closed [resp. (l,2)*$^{+}$-Senii-closed, (l,2)*$^{+}$-pre-closed].

Remark 3.7 Every $\tau^{+}_{(1,2)}$ open set is (1,2)*$^{+}$-α-open.

Example 3.8 Let X = {a,b,c}, τ_1 = {X,{ϕ},{a}} and τ_2= {X,{ϕ}, {b}}.Then $\tau_{1,2}$open sets are {X, {ϕ}, {a}, {b}}, {a,b}}. Let A = {a,c} $\notin \tau_{1,2}$. Then Extended bitopological open sets are $\tau^{+}_{(1,2)}$={X,{ϕ},{a},{b}, {a,b},{a,c}} and closed sets are {X, {ϕ}, {b}, {c}, {b,c}, {a,c}}. Then(1,2)*$^{+}$-α-opensetsare{X, {ϕ},{a},{b},{a,b},{a,c}}.

Theorem 3.9 Let (X, $\tau^{+}_{(1,2)}$) be an extended bitopological space. S $\subseteq$ X is a (1,2)*$^{+}$α-open if and only if S is a (1, 2)*$^{+}$ semi-open and a (1, 2)*$^{+}$ pre-open.

Example 3.10: From the Example 3.8, (1,2)*$^{+}$-α-open sets are {X, {ϕ}, {a}, {b}, {a, b}, {a, c}}. Then (1,2)*+-semi open sets are {X, {ϕ}, {a}, {b}, {a,b}, {a c}} and (1,2)*$^{+}$-pre open sets are {X, {ϕ}, {a}, {b}, {a, b}, {a, c}}.

Definition 3.11: A subset A of an extended bitopological space (X, $\tau^{+}_{(1,2)}$) is called (1, 2)*$^{+}$-generalized closed if $\tau^{+}_{(1,2)}$-cl(A) $\subset$U whenever A $\subset$U and U is $\tau^{+}_{(1,2)}$-open in X.

Remark 3.12: Every $\tau^{+}_{(1,2)}$ closed sets are (1,2)*$^{+}$-g.closed sets.

Example 3.13: Let X = {a,b,c}, τ_1= {X,{ϕ},{b,c}} and τ_2= {X, {ϕ}, {b}}.Then $\tau_{1,2}$ = {X, {ϕ}, {b}, {b, c}}. Let A = {a, b} $\notin \tau_{1,2}$. Then the sets in {X, {ϕ}, {b}, {a, b}, {b, c}} are $\tau^{+}_{(1,2)}$-open sets and {X, {ϕ}, {a}, {c}, {a,c}} are $\tau^{+}_{(1,2)}$- closed sets.We have (1,2)*$^{+}$-g.closed sets are {X,{ϕ}, {a}, {c}, {a, c}}.

4.(1,2)*$^{+}$-$\wedge_S$ -Sets and (1,2)*$^{+}$-$\vee_s$- Sets

In this section we introduce the new types of operators in Extendedbitopological spaces and some characterizations are derived.

Definition 4.1: Let B be a subset of an Extended bitopological space (X, $\tau^{+}_{(1,2)}$).we define the subsets (1,2)*$^{+}$- $B^{\wedge_s}$ and (l,2)*$^{+}$- $B^{\vee_s}$ as follows:

(i) (l,2)*$^{+}$- $B^{\wedge_s}$ = $\bigcap$ {O : O $\supseteq$B and 0 is (1,2)*$^{+}$-semi-open}

(ii) (1,2)*$^{+}$- $B^{\vee_s}$ = $\bigcup$ {F:F $\subseteq$B and $F^c$is (1,2)*$^{+}$-semi-open}

(l,2)*$^{+}$- $B^{\wedge_s}$ is called the (l,2)*$^{+}$-semi-kernal of B.

Proposition 4.2: Let A,B and{B_λ : λ is λλΩ} be subsets of an extended bitopological space (X, $\tau^{+}_{(1,2)}$). Then the following properties are valid.

(i)B $\subset$ (1,2)*$^{+}$ - $B^{\wedge_s}$

(ii) If A $\subset$ B then (1,2)*$^{+}$- $A^{\wedge_s}$ $\subset$(1,2)*$^{+}$- $B^{\wedge_s}$

(iii)(l,2)*$^{+}$-((l,2)*$^{+}$- $B^{\wedge_s}$)$^{\wedge_s}$ = (1,2)*+- $B^{\wedge_s}$

(iv)(1,2)*$^{+}$-$(\bigcup_{\lambda\in\Omega} B_\lambda)^{\wedge_s} = \bigcup_{\lambda\in\Omega} (1,2)^{*+} - B_\lambda^{\wedge_s}$

(v) If A $\in$ (l,2)*$^{+}$-SO(X, $\tau^{+}_{(1,2)}$), then (1,2)*$^{+}$- $A^{\wedge_s}$ = A

(vi)(1,2)*$^{+}$- $(B^c)^{\wedge_s}$ = ((1,2)*$^{+}$- $B^{\vee_s}$)^c

(vii)(1,2)*$^{+}$- $B^{\vee_s}$ $\subseteq$B (viii)If B ∈ (1,2)*+-SC(X, $\tau^+_{(1,2)}$) then (1,2)*+- $B^{\vee_s}$ = B

(ix)(1,2)*+-($\bigcap_{\lambda\in\Omega} B_\lambda)^{\wedge_s} \subseteq \bigcap_{\lambda\in\Omega}$ (1,2)*+- $B_\lambda^{\wedge_s}$

(x) (1,2)*+-($\bigcup_{\lambda\in\Omega} B_\lambda)^{\vee_s} \supseteq \bigcup_{\lambda\in\Omega}$ (1,2)*+- $B_\lambda^{\vee_s}$

Proof:

(i) It is clear by definition 4.1.

(ii) Suppose that x∉ (1,2)*+- $B^{\wedge_s}$. Then there exists a subset O ∈ (1,2)*+-SO(X, $\tau^+_{(1,2)}$) such that O ⊃B with x ∉ O. Since B⊇ A, then x ∉ (1,2)*+- $A^{\wedge_s}$, and thus (1,2)*+- $A^{\wedge_s}$ ⊆ (1,2)*+- $B^{\wedge_s}$.

(iii)By (i) (1,2)*+- $B^{\wedge_s}$ ⊆ (1,2)*+-((1,2)*+- $B^{\wedge_s}$)$^{\wedge_s}$. Since (1,2)*+- $B^{\wedge_s}$ by definition 4.1, is a (1,2)*+-semi-open set containing ((1,2)*+- $B^{\wedge_s}$)$^{\wedge_s}$, (1,2)*+-((1,2)*+- $B^{\wedge_s}$) ⊆ (1,2)*+- $B^{\wedge_s}$ .Hence (iii) holds.

(iv)Suppose that there exists a point x such that x ∉ (1,2)*+-($\bigcup_{\lambda\in\Omega} B_\lambda)^{\wedge_s}$ .Then there exists a (1,2)*+-semi-open set O such that $\bigcup_{\lambda\in\Omega} B_\lambda \subset$ O with x ∉ O. Thus for eachλ∈Ω, we have x ∉ (1,2)*+- $B_\lambda^{\wedge_s}$. This implies x∉ $\bigcup_{\lambda\in\Omega}$ (1,2)*+- $(B_\lambda)^{\wedge_s}$.

Conversely, Suppose that there exists a point x ∈X such that x ∉ $\bigcup_{\lambda\in\Omega}$ (1,2)*+- $(B_\lambda)^{\wedge_s}$, then by definition 4.1, there exists a (1,2)*+-semi-open sets O_λ,for each λ∈Ωsuchthat x ∉ O_λ,$B_\lambda \subset O_\lambda$.Let O = $\bigcup_{\lambda\in\Omega} O_\lambda$. Then O is a (1,2)*+-semi-open set of X suchthat x ∉ O and $\bigcup_{\lambda\in\Omega} B_\lambda \subseteq$O. This implies that x ∉ $\bigcup_{\lambda\in\Omega} B_\lambda$. Thus (iv) holds

(v) If A is a (1, 2)*+-semi-open set in X. Then by definition 4.1, (1, 2)*+- $A^{\wedge_s}$ ⊆ A, By (i) we have A = (1, 2)*+- $A^{\wedge_s}$.

(vi)((1,2)*+ $B^{\vee_s}$)^c = ∩{F^c : F^c⊇B^c, F^c is (1,2)*+-semi-open}= (1,2)*+(B^c)$^{\wedge_s}$.

(vii) Obvious by definition 4.1.

(viii) If B is (1,2)*+-semi-closed then $B^c$is (1,2)*+-semi-open) therefore by (v) and (vi) B^c = ((1,2)*+-B^c)$^{\wedge_s}$ = ((1,2)*+- ($B^{\vee_s}$)^c. Hence B = (1,2)*- $B^{\vee_s}$.

(ix) Suppose that there exists a point x ∈Xsuch that x ∉ $\bigcap_{\lambda\in\Omega}$ (1,2)*+-B_λ)$^{\wedge_s}$. Thenthere exists a λ∈Ω such that x ∉ (1,2)*+- $B_\lambda^{\wedge_s}$ .This implies that there exists a λ∈Ω and a (1,2)*+-semi-open set O in X such that O ⊇B_λ and x ∉ O Thus x ∉ (1,2)*+-($\bigcap_{\lambda\in\Omega} B_\lambda)^{\wedge_s}$.

(x) By (vi) (1,2)*+-($\bigcup_{\lambda\in\Omega} B_\lambda)^{\vee_s}$ = ((1,2)*+-(($\bigcup_{\lambda\in\Omega} B_\lambda)^c)^{\wedge_s}$)^c=((1,2)*+-($\bigcap_{\lambda\in\Omega} (B_\lambda)^c)^{\wedge_s}$)^c⊇ (($\bigcap_{\lambda\in\Omega}$ (1,2)*+-$(B_\lambda)^c)^{\wedge_s}$)^c = ($\bigcap_{\lambda\in\Omega}$ ((1,2)*+-($B_\lambda^{\vee_s}$)^c)^c = $\bigcap_{\lambda\in\Omega}$ (1,2)*+- $B_\lambda^{\vee_s}$.

Remark 4.3: (1,2)*+-(B_1∩B_2)$^{\wedge_s}$ and (1,2)*+-(B_1)$^{\wedge_s}$ ∩ (1,2)*+-(B_2)$^{\wedge_s}$ are not equal as per the following example.

Example 4.4 Let X = {a,b,c}, τ_1= {ϕ,X, {b}}, τ_2= {ϕ,X,{c},{b,c}}. Then $\tau_{1,2}$ = {X,{ϕ}, {b},{c},{b,c}} and let A = {a,b} ∉ $\tau_{1,2}$.

Then the sets in {ϕ,X,{b},{c}, {a,b},{b,c}} are $\tau^+_{1,2}$ sets and the sets {ϕ,X, {a},{c},{a,c}, {a,b}}. Let B_1= {a} and B_2 = {c} Then (1,2)*+-(B_1∩ B_2)$^{\wedge_s}$ = ϕ,

But (1,2)*+-(B_1)$^{\wedge_s}$ ∩ (1,2)*+-(B_2)$^{\wedge_s}$ = {b}.

Definition4.5. In an extended bitopological Space (X, $\tau^+_{1,2}$) a subset B is a (1,2)*+-$\wedge_s$-set if B = (1,2)*+- $B^{\wedge_s}$.

Remark 4.6: By proposition 4.2 (v) and (viii) we have that

(i) If B is (1,2)*+-semi-open,then B is a (1, 2)*+-$\wedge_s$-set

(ii) If B is (1,2)*+-semi-closed, then B is a (1,2)*+-$\vee_s$-set

Proposition 4.7 Let (X, $\tau^+_{(1,2)}$)be an extended bitopological Space

(i) The subsets ϕand X are (1,2)*+-$\wedge_s$-sets and (1,2)*+-V_s-sets.

(ii) Arbitrary union of (1,2)*+-$\wedge_s$-sets is a (1,2)*+-$\wedge_s$-sets.

(iii)Arbitrary intersection of (1,2)*+-$\wedge_s$-sets is a (1,2)*+-$\wedge_s$-sets.

(iv)A subset B is a (1,2)*+-$\wedge_s$-set iff$B^c$is a (1,2)*+-$\vee_s$-set.

5. (1,2)*+-g.$\wedge_s$-Sets and (1,2)*+-g.$\vee_s$- sets: In this section, by using the (1,2)*-$\wedge_s$-sets and (1,2)*+-$\vee_s$ sets, we introduce the classes of (1,2)*+-generalized.$\wedge_s$ (briefly (1,2)*+-g.$\wedge_s$) and (1,2)*+-generalized $\vee_s$ (briefly (1,2)*+-g.$\vee_s$).

Definition 5.1: In an extended bitopological space (X, $\tau^+_{(1,2)}$) be a subset B is called a (1,2)*+-g.$\wedge_s$-set of (X, $\tau^+_{(1,2)}$) if (1,2)*+- $B^{\wedge_s}$ ⊆ F whenever B ⊆ F is (1,2)*+-semi closed.

Definition 5.2: In an extended bitopological space (X, $\tau^+_{(1,2)}$) a subset B is called a (1,2)*+-g.$\vee_s$-set of (X, $\tau^+_{(1,2)}$) if B^c is a (1,2)*+-g.$\wedge_s$-set of (X, $\tau^+_{(1,2)}$).

Remark 5.3: By (1,2)*+-D^{A*} we will denote the family of all (1,2)*+-p.A_s-sets of (X, $\tau^+_{(1,2)}$).

Proposition 5.4 Let (X, $\tau^+_{(1,2)}$) be an extended bitopological space. Then

(i) Every (1,2)*+-$\wedge_s$-set is a (1,2)*+-g.$\wedge_s$-set.

(ii) Every (1,2)*+-$\vee_s$-set is (1,2}*+-g.$\vee_s$-set.

(iii)If $B_\lambda \in$ (1,2)*+- $D^{\wedge_s}$ for all $\lambda \in \Omega$ then $\bigcup_{\lambda\in\Omega}$ -$B_\lambda \in$ (1,2)*+- $D^{\wedge_s}$

(iv)If $B_\lambda \in$ (1,2)*+- $D^{\vee_s}$ for all $\lambda \in \Omega$ then $\bigcap_{\lambda\in\Omega}$ -$B_\lambda \in$ (1,2)*+- $D^{\vee_s}$

Proof:

(i) Follows from the definition of 5.1 and 5.2.

(ii) Let B be a (1,2)*+-$\vee_s$-set. Then B = (1,2)*+- $B^{\vee_s}$ by proposition 5.2

(vi) (1,2)*+- $(B^c)^{\wedge_s}$ =((1,2)*+- $B^{\vee_s})^c$ = B^c.

Therefore by(i) and definition 5.2, B is a (1, 2)*+-g.$\vee_s$-set.

(iii)Let $B_\lambda \in$ (1,2)*+- $D^{\wedge_s}$ for all for all $\lambda \in \Omega$ then by proposition 5.2

(iv) (1,2)*+- $\bigcup_{\lambda\in\Omega}(B_\lambda^{\wedge_s})$ = $\bigcup_{\lambda\in\Omega}((1,2)*^+ - B_\lambda^{\wedge_s})$. Hence by hypothesis and definition 5.1 $\bigcup_{\lambda\in\Omega}$ $B_\lambda \in$ (1,2)*+- $D^{\wedge_s}$.

(iv)follows from (iii) and the definition 5.2.

Remark 5.5

In general the intersection of two (1,2)*+-g.$\wedge_s$ sets is not a (1,2)*+-g.$\wedge_s$-set.

Example 5.6

$(X, \tau^+_{(1,2)})$ is an Extended biopological space as the Example 4.4. The sets A={a,b} and B={a,c} are (1,2)*+-g.$\wedge_S$ sets, but A$\cap$ B = {a} is not a (1,2)*+-g.$\wedge_s$-set.

Remark 5.7

(i) If A $\in$ (1,2)*+-SO(X, $\tau^+_{(1,2)}$) then A is a (1,2)*+-g.$\wedge_s$-set.

(ii) If A $\in$ (1,2)*+-SC(X, $\tau^+_{(1,2)}$) then A is a (1,2)*+-g.$\vee_s$-set.

Proposition 5.8 Let $(X, \tau^+_{(1,2)})$ be an extended bitopological space. Then

(i) For each x $\in$X, {x} is a (1,2)*+-semi-open set or $\{x\}^c$ is a (1,2)*+-g.$\wedge_S$ set of (X, $\tau^+_{(1,2)}$)

(ii) For each x $\in$X, {x} is a (1,2)*+-semi-open set or {x} is a (1,2)*+-g.$\vee_s$-set of (X, $\tau^+_{(1,2)}$)

Proof:

(i) Suppose that {x} is not (1,2)*+-semi-open set. Then the only (1,2)*+-semi-closed set containing $\{x\}^c$ is X. Therefore $\{x\}^c$ is a (1,2)*+-g.$\wedge_s$ set of (X, $\tau^+_{(1,2)}$).

(ii) Follows from (i) and definition 5.2.

Conclusion: This paper has attempt to establish Extended bitopological space and characterize the operators (1,2)*+-g.$\wedge_s$ -Sets (1,2)*+-g.$\vee_s$-Sets in Extended bitopological spaces. It also aims to state that the several definitions and results in this paper will result in obtaining several characterizations and also enable to study various properties. The future scope of study is the extension of extended bitopological spaces to some new operators.

References:

1. Kelley.J.C, *Bitopological Spaces,* Proc.London .Math.Soc (3), 13, 71-89
2. Levine.N, Semi open sets and semi continuity in topological spaces, Amer.Math. Monthly, 70 (1963), 36-41.
3. Levine.N, *Generalized closed sets in topology,* Rend.Circ. Math.Polarmo (2),1970, 89-96.
4. Levine.N, *Simple Extensions of Topologies,* The American Mathe-matical Monthly, Vol.71,No.l (1964), 22-25
5. Maki, H., Generalized Asets and the Associated closure Operator, The special Issue in Commemoration of Prof.Kazuada, IKEDA's Retirement (1986), 139-146
6. Nirmala Mariappan, LellisThivagar.M, and Navalagi.G.B, (1,2)*-g-$\wedge_s$-Setsand (1,2)*-g-$\vee_s$-sets, Int.jour.App.Mat.Anal and App. Vol.5, No.l, Jan-Jun (2005), 35-44
7. Ravi.O, and LellisThivagar.M, *On Stronger Forms of (1,2)*-Quotient Mappings in Bitopological Spaces,* Int.J.Math Game theory and Algebra, 14,No.6(2004),481-492.

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