REMARKS ON WEAKLY OPEN SETS IN EXTENDED BITOPOLOGICAL SPACES

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Abstract: The purpose of this paper is to introduce simple extensions of bitopological spaces and define weak form of open sets on it. We also investigate some of their basic properties and establish the relationship between them .

Keywords: Extended bio-topological spaces

Introduction: In 1963, Kelly [2] initiated the study of the bitopological space which is to be a set X equipped with two topologies $\tau_1$and τ_2 on X. The concepts of g-closed sets are introduced in bitopological spaces by Fukutake [1]. Recently, LellisThivagar [3] introduced new bitopological notions of $\tau_1\tau_2$-open sets and $\tau_1\tau_2$-closed sets. After that LellisThivagar et al [4] proved that each (l,2)a-open sets is (1,2) semi open and (1,2) pre open but the converse of each is not true. In 1964, Levin [6] initiated the simple extensions of topologies. In this paper we introduce extended bitopological space by the simple extensions of bitopological spaces and investigate certain properties on it .We introduce weak form of open sets such as $(1,2)^+$ α-open sets, $(1,2)^+$ semi-open sets and $(l,2)^+$ pre-open sets and also discuss relationship between them. We also generalize the weak form of closed sets and discuss their relations.

Preliminaries:

Definition 2.1 [2] A non-empty set X together with two topologies $\tau_1$and $\tau_2$is called a bitopological spaces and is denoted by (X, τ_1, τ_2)

Definition 2.2 [3] A subset S of X is called $\tau_1\tau_2$-open if and only if $S\in\tau_1\cup\tau_2$

The complement of $\tau_1\tau_2$-open sets are called $\tau_1\tau_2$-closed sets. The family of all $\tau_1\tau_2$-open sets is denoted by $\tau_1\tau_2O(X)$.

Note that $\tau_1\tau_2O(X)$ need not necessarily form a topology and $\tau_1O(X), \tau_2O(X)\subseteq\tau_1\tau_2O(X)$.

Remark 2.3 [3] Let A be a subset of X. Then $\tau_1\tau_2$ closure of A is defined as $\tau_1\tau_2cl(A)=\bigcap\{F/A \subseteq F$ and F is $\tau_1\tau_2$ closed$\}$.

We recall the following definitions whichare useful in the sequel.

Definition 2.4 [3] A subset A of X is called

(i) (1,2)α-open if $A\subseteq\tau_1int(\tau_1\tau_2cl(\tau_1int(A))$

(ii) (1,2)semi-open if $A \subseteq\tau_1\tau_2cl (\tau_1int(A))$

(iii)(1,2)pre-open if $A \subseteq\tau_1int(\tau_1\tau_2cl(A))$.

Where $\tau_1int(A)$ is the interior of A with respect to the topology τ_1.

The families of all (1,2)α-open sets, (1,2)semi-open sets and (1,2) pre-open sets of X are denoted by (1,2)αO(X), (1,2)SO(X) and (1,2}PO(X)respectively. In a bitopological space X, it has been proved that (l,2)αO(X) is closed with respect to arbitrary union. If (l,2)α-open sets are closed with respect to finite intersection,then X is called as a (1,2)α-topology or an Ultra space [5].

Definition 2.5 [5] A subset A of X is called

(i) (1,2)αg-closed set if $(l,2)\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)\alpha O(X)$,

(ii) (1,2)sg-closed set if $(l,2)scl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)SO(X)$,

(iii) (1, 2)pg-closed set if $(l,2)pcl(A) \subseteq U$ whenever $A \subseteq U$ and$U \in$ (1,2)PO(X).

The complement of above mentioned closed sets are called their respective open sets.

Definition 2.6 [6] Let (X,τ) be a topological space and $\tau\subset\tau^+$. Then τ^+will be termed a simple extension of τif and only if there exists an $A \notin \tau$such that $\tau^+= \{O \cup(O'\cap A) : O, O'\in \tau\}$.We denote $\tau^+=\tau^+(A)$

Lemma 2.7 [6] Let (X, τ) be a topological space and $\tau^+ = \tau^+(A)$ a simple extension. If $B \subset X$, then $Int^+(B) = Int\, B \cup Int_A(B \cap A)$, where Int^+, Int and Int_A denote the interior operators relative to τ^+, or τ and $\tau\cap A$ respectively.

Lemma 2.8 [6] Let (X,τ) be a topological pace and $\tau^+ = \tau^+(A)$ be a simple extension. If $B \subset X$,then $cl^+(B) = cl(B) \cap \{A^c\cup (A \cap cl(B \cap A))\}$ where cl and cl^+ denote the closure operators relative to $\tau$and τ^+respectively.

Lemma 2.9 [6] Let (X,τ)be a topological space and $\tau^+ = \tau^+(A)$ be a simple extension. If $B \subset X$, then $cl^+(B \cap A\} = cl(B \cap A)$

Corollary 2.10 [6] Let (X,τ) be a topological space and $\tau^+ = \tau^+(A)$ be a simple extension. Then A is closed in (X,τ) if and only if A is closed in (X, τ^+) (A is always open in (X, τ^+)).

Theorem 2.11 [6] Let (X,τ) be a topological space which is T_0, T_1or T_2 and $A \notin \tau$. Then $(X,\tau^+(A))$ is T_0, T_1orT_2.

3 Extended Bitopological Spaces: In this section we introduce a notion of extended bitopological spaces by a simple extension ofbitopological space(X, τ_1, τ_2).

Definition 3.1 Let(X, τ_1, τ_2) be a bitopologicalspaceand$\tau_1\tau_2O(X)\subset (\tau_1\tau_2)^+$. Then $(\tau_1\tau_2)^+$ will be termed a simple extension of $\tau_1\tau_2O(X)$ if and only if there exists an $A \notin\tau_1\tau_2O(X)$ such that $(\tau_1\tau_2)^+ = (\tau_1\tau_2)^+(A) = \{G_1\cup (G_2\cap A): G_1, G_2\in\tau_1\tau_2O(X)\}$. We call $(X, (\tau_1\tau_2)^+(A))$ an extended bitopological space of (X, τ_1,τ_2) w.r.t A.

Throughout this paper $(X, (\tau_1\tau_2)^+(A))$[or simply X] denote extended bitopological space on which no separation axioms are assumed unless explicitly stated.

Example 3.2 Let X ={a, b, c, d}. Then

$\tau_1= \{\phi, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\phi, X, \{a, b, c\}\}$. Then $\tau_1\tau_2O(X)=\{\phi, X, \{a\}, \{a,b\}, \{a, b, c\}\}=\tau_{1,2}O(X)$.

Let $A = \{b\} \notin \tau_1\tau_2O(X)$. Then $(\tau_1\tau_2)^+(A) =\{\phi,X,\{a\},\{b\},\{a,b\},\{a,b,c\}\}=(\tau_{1,2})^+(A)$.

Remark 3.3 $(\tau_1\tau_2)^+(A)$ is need not necessarily form a topology and $\tau_1^+(A)$, $\tau_2^+(A) \subseteq (\tau_1\tau_2)^+(A)$

Definition 3.4 Let $(X, (\tau_1\tau_2)^+(A))$ be an extended bitopological space and $S \subseteq X$. Then $(\tau_1\tau_2)^+$ closure of S is defined as $(\tau_1\tau_2)^+cl(S)= \cap\{F : S \subseteq F$ and F is $(\tau_1\tau_2)^+$ closed$\}$ and $(\tau_1\tau_2)^+$interior of S is defined as $(\tau_1\tau_2)^+int(S)= \cup\{G : G \subseteq S$ and G is $(\tau_1\tau_2)^+$ open$\}$.

Lemma 3.5 Let $(X, (\tau_1\tau_2)^+(A))$ be an extended bitopological space and $S \subseteq X$. Then $(\tau_1\tau_2)^+int(S)=\tau_1\tau_2int(S)\cup Int_{(\tau1\tau2\cap A)}(S \cap A)$.

Proof : Let $x \in (\tau_1\tau_2)^+int(S)=\cup\{G : G \subseteq S$ and G is $(\tau_1\tau_2)^+$ open$\}$ $\Leftrightarrow$ there exists a $(\tau_1\tau_2)^+$ open subset G of S such that $x \in G \Leftrightarrow x \in G = O\cup(O'\cap A\}$,where O, $O'\in \tau_1\tau_2O(X)\Leftrightarrow x\in O\subseteq S$ or $x\subseteq O'\cap A\subseteq S\Leftrightarrow x \in \tau_1\tau_2int(S)\cup Int_{(\tau1\tau2\cap A)}(S\cap A)$.
Thus $(\tau_1\tau_2)^+int(S)=\tau_1\tau_2int(S)\cup Int_{(\tau1\tau2\cap A)}(S\cap A)$.

Theorem 3.6 Let X be an extended bitopological space.

(i) If $S_1\subseteq S_2\subseteq X$, then $(\tau_1\tau_2)^+int(S_1) \subseteq(\tau_1\tau_2)^+int(S_2)$ and $(\tau_1\tau_2)^+cl(S_1) \subseteq (\tau_1\tau_2)^+cl(S_2)$.

(ii) $(\tau_1\tau_2)^+int(S)$ is $(\tau_1\tau_2)^+$open and $(\tau_1\tau_2)^+$ cl(S) is $(\tau_1\tau_2)^+$closed, for each $S \subset X$

(iii)A set $S \subset X$ is $(\tau_1\tau_2)^+$open iff$S = (\tau_1\tau_2)^+int(S)$ and a set $S \subset X$ is $(\tau_1\tau_2)^+$ closed iff$S = (\tau_1\tau_2)^+cl(S)$.

(iv)(a) For any $S \subset X$, we have
$(\tau_1\tau_2)^+ int((\tau_1\tau_2)^+int(S))=(\tau_1\tau_2)^+int(S)$
(b)For any $S \subset X$, we have
$(\tau_1\tau_2)^+cl((\tau_1\tau_2)^+cl(S))=(\tau_1\tau_2)^+cl(S)$

(v) (a)$(\tau_1\tau_2)^+int(X-S)=X-(\tau_1\tau_2)^+cl(S)$for any $S \subset X$;
(b)$(\tau_1\tau)^+cl(X-S)=X-(\tau_1\tau_2)^+int(S)$ for any $S \subset X$

(vi)For any family $\{S_i/i\in I\}$ of subsets of X we have

(a) $\bigcup_i (\tau_1\tau_2)^+int(S_i)\subset (\tau_1\tau_2)^+int(\bigcup_i S_i)$;

(b)$\bigcup_i (\tau_1\tau_2)^+cl(S_i) = (\tau_1\tau)^+cl(\bigcup_i S_i)$;

(c)$(\tau_1\tau_2)^+int(\bigcap_i S_i) = \bigcap_i (\tau_1\tau)^+int(S_i)$;

(d)$(\tau_1\tau_2)^+cl(\bigcap_i S_i)\subset\bigcap_i (\tau_1\tau_2)^+cl(S_i)$;

Proof: (i) Since $S_2\subseteq (\tau_1\tau_2)^+cl(S_2)$, $S_1\subseteq(\tau_1\tau_2)^+cl(S_2)$. Since $(\tau_1\tau_2)^+cl(S_1)$ is the smallest $(\tau_1\tau_2)^+$ closed set containing S_1. Therefore, $(\tau_1\tau_2)^+cl(S_1) \subseteq (\tau_1\tau_2)^+cl(S_2)$. Similarly we can prove $(\tau_1\tau_2)^+int(S_1) \subseteq (\tau_1\tau_2)^+int(S_2)$.

(ii). It follows from the definition of $(\tau_1\tau_2)^+$closure and $(\tau_1\tau_2)^+$interior.

(iii). Assume that S is $(\tau_1\tau_2)^+$-closed, S is the smallest $(\tau_1\tau_2)^+$-closed set containing S and hence $S = (\tau_1\tau_2)^+cl(S)$. Conversely, $S = (\tau_1\tau_2)^+cl(S)$, then S is the smallest $(\tau_1\tau_2)^+$closed set containing S. Hence S is $(\tau_1\tau_2)^+$closed. Similarly we can prove $(\tau_1\tau_2)^+$open iff$S = (\tau_1\tau_2)^+int(S)$.

(iv)By (ii) and (iii) we get the result (iv).

(v)If $x \in X-(\tau_1\tau_2)^+int(S)$,$x \notin (\tau_1\tau_2)^+int(S)$.Then $G \subseteq S$, for every $(\tau_1\tau_2)^+$-open set G containing x . That is $G \cap(X - S) \neq\phi$, for every $(\tau_1\tau_2)^+$-open set G containing x.Then $x \in (\tau_1\tau)^+cl(X-S)$. On the other hand, let $x \in (\tau_1\tau_2)^+cl(X-S)$, then $G\cap(X-S\} \neq\phi$,for every $(\tau_1\tau_2)^+$-open set G containing x.That is $G \not\subset S$, for every $(\tau_1\tau_2)^+$-open set G containing x. Then $x \notin (\tau_1\tau_2)^+int(S)$ implies $x \in X-(\tau_1\tau_2)^+int(S)$. Hence $(\tau_1\tau_2)^+cl(X-S)=X - (\tau_1\tau_2)^+int(S)$ for any $S \subset X$. Similarly we prove the first part.

(vi). Since $S_i\subseteq\bigcup_i S_i$for each $i\in I$, then $(\tau_1\tau)^+cl(S_i) \subseteq (\tau_1\tau_2)^+cl(\bigcup_i S_i)$ for each $i\in I$. Then $\bigcup_i (\tau_1\tau_2)^+cl(S_i) \subseteq (\tau_1\tau_2)^+cl(\bigcup_i S_i)$. On the other hand, since $(\bigcup_i S_i) \subseteq\bigcup_i (\tau_1\tau_2)^+(S_i)$,by definition and $(\tau_1\tau_2)^+cl(\bigcup_i S_i)$ is the smallest $(\tau_1\tau)^+$-closed set containing $\bigcup_i S_i$, $(\tau_1\tau)^+cl(\bigcup_i S_i) \subseteq\bigcup_i (\tau_1\tau_2)^+cl(S_i)$. Hence the result of part (b). Since $\bigcap_i S_i\subset S_i$, for each $i\in I$, then $(\tau_1\tau_2)^+cl(\bigcap_i S_i) \subset\bigcap_i (\tau_1\tau_2)^+cl(S_i)$. This proves part (d). Similarly we can prove part (a) and (c).

Theorem 3.7: Let $(X, (\tau_1\tau_2\}^+(A))$ be an extended bitopological space and $B \subset X$.
Then$(\tau_1\tau_2)^+cl(B)=\tau_1\tau_2cl(B)\cap\{A^c\cup(A\cap\tau_1\tau_2cl(B\cap A\}\}\}$.

Proof:$(\tau_1\tau)^+cl(B)=[(\tau_1\tau_2)^+int(B^c)]^c=[\tau_1\tau_2int(B^c) \cup Int_{(\tau1\tau2\cap A)}(B^c\cap A)]^c=[\tau_1\tau_2int(B^c)]^c\cap[Int_{\tau l\tau 2\cap A}(B\cap A)^{cA}]^c= \tau_1\tau_2cl(B)\cap\{A^c\cup [Int_{\tau l\tau 2\cap A}(B \cap A)^{cA}]^{cA}$
$= \tau_1\tau_2cl(B) \cap \{A^c\cup cl_{\tau 1\tau 2\cap A}(B\cap A)\}=\tau_1\tau_2cl(B)\cap \{A^c\cup (A\cap\tau_1\tau_2cl(B\cap A))\}$.

Theorem 3.8 Let $(X, (\tau_1\tau_2)^+(A))$ be an extended bitopological space and $B \subset X$.
Then $(\tau_1\tau_2)^+cl(B\cap A) = \tau_1\tau_2cl(B\cap A)$.

Proof: $(\tau_1\tau_2)^+cl(B\cap A) = \tau_1\tau_2cl(B\cap A)\cap\{A^c\cup (A\cap\tau_1\tau_2cl(B\cap A))\}$(by theorem 3.7) $= \tau_1\tau_2cl(B \cap A)$.

Theorem3.9: Let $(X, (\tau_1\tau_2)^+(A))$ be an extended bitopological space. Then A is closed in (X, τ_1,τ_2) if and only if A is closed in $(X, (\tau_1\tau_2)^+(A))$(A is always open in$(X,(\tau_1\tau_2)^+(A))$).

Proof: By theorem 3.8, $(\tau_1\tau_2)^+cl(A)=\tau_1\tau_2cl(A)$, and we have $A=(\tau_1\tau_2)^+cl(A)$if and only if $A = \tau_1\tau_2cl(A)$.

Theorem 3.10 Let (X,τ_1,τ_2) be a bitopological space which is T_0, T_1or T_2 and $A\notin \tau_1\tau_2O(X)$.
Then $(X, (\tau_1\tau_2)^+(A))$ is T_0, T_1 or T_2.

Proof: It follows from the fact that $\tau_1\tau_2O(X) \subset(\tau_1\tau_2)^+(A)$.

Definition3.11: Let $(X,(\tau_1\tau_2)^+(A))$ be an extended bitopological space.A subset S of X is called

(i) $(l,2)^+\alpha$-open if $S \subset\tau_1^+ int((\tau_l\tau_2)^+cl(\tau_1^+ int(S)))$

(ii) $(l,2)^+$semi-openif $S \subset (\tau_1\tau)^+cl(\tau_1^+ int(S))$ and

(iii)$(1,2)^+$pre-open if $S \subset\tau_1^+ int((\tau_1\tau_2)^+cl(S))$.

The Complements of the sets mentioned above from (i) to (iii) are called their respective closed sets.
The collection of all $(1,2)^+\alpha$-open,(resp. $(1,2)^+$semi-open and $(1,2)^+$pre-open) sets of X are denoted by $(l,2)^+\alpha O(X,A)$, [resp. $(1,2)^+SO(X,A)$, $(1,2)^+PO(X,A)$].

The collection of all $(1,2)^+\alpha$-closed,[resp. $(1,2)^+$ semi-closed and $(1,2)^+$pre-closed] sets of X are denoted by $(1,2)^+\alpha C(X,A)$,[resp. (1,2)+ SC(X,A), $(1,2)^+PC(X,A)$].

Example 3.12 Let X = {a, b, c,d}. Then τ_1={ϕ,X, {a}, {a, b},} and τ_2 = {ϕ, X, {a, b, c}}. Then $\tau_1\tau_2O(X)$ ={ϕ, X, {a}, {a,b}, {a, b, c}}. Let A = {b} $\notin \tau_1\tau_2O(X)$. Then $(\tau_1\tau_2)^+(A)$ = {ϕ,X, {a}, {b}, {a,b}, {a, b, c}}. $[(\tau_1\tau_2)^+(A)]^c$= {ϕ, X, {d}, {c,d}, {a, c, d}, {b, c, d}} and τ_1^+ (A) = {ϕ, X, {a}, {b}, {a, b}}.Then $(1,2)^+\alpha O(X,A)$= {ϕ, X, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}}=$(1,2)^+PO(X,A)$, $(1,2)^+SO(X,A)$= {ϕ,X,{a},{b}, {a,b}, {a,c}, {a,d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}.

Theorem 3.13 Let X be an extended bitopological space. S $\subseteq$ X is a $(1,2)^+\alpha$-open if and only if S is a $(1,2)^+$semi-open and a $(1, 2)^+$pre-open

Proof:Assume S is $(1,2)^+\alpha$-open set, then S$\subset \tau_1^+$ int($(\tau_1\tau_2)^+$cl(τ_1^+ int(S))) $\subset$ $(\tau_1\tau_2)^+$cl(τ_1^+ int(S)). Thus S is $(1,2)^+$semi-open.Since τ_1^+ int(S)$\subset$S, then S $\subset \tau_1^+$ int($(\tau_1\tau_2)^+$cl(τ_1^+ int(S)))$\subset \tau_1^+$ int($(\tau_1\tau)^+$cl(S)). Thus S is (1,2)+pre-open. Conversely , if S is a $(1,2)^+$semi-open and a $(1,2)^+$pre-open, then $(\tau_1\tau)^+$cl(S) $\subset$ $(\tau_1\tau_2)^+$ cl(τ_1^+ int(S)) and S$\subset \tau_1^+$ int($(\tau_1\tau_2)^+$cl(S)).Then S $\subset \tau_1^+$ int($(\tau_1\tau_2)^+$cl(τ_1^+ int(S))). Hence S is $(1, 2)^+\alpha$-open set.

Remark 3.14 Let (X,τ_1,τ_2) be a bitopo-logical space, if $(1,2)\alpha O(X)$ is form a topology,then $(1,2)^+\alpha O(X,A)$ is also form a topology. We call the topology $(1,2)^+\alpha O(X,A)$ as a $(1,2)^+\alpha$-topology or an Ultra$^+$ space.

Definition 3.15 $(1,2)^+\alpha$cl(S) [resp. $(1,2)^+$scl(S) and $(1,2)^+$pcl(S)] is defined as the intersection of all $(1,2)^+\alpha$-closed [resp.$(1,2)^+$ semi-closed and $(1,2)^+$pre-closed] sets containing A. Also $(1,2)^+\alpha$int(S) [resp. $(1,2)^+$sint(S) and $(1,2)^+$ pint(S)] is defined as the union of all $(1,2)^+\alpha$-open [resp. $(1,2)^+$semi-open and $(1,2)^+$pre-open] sets contained in A.

Theorem 3.16 Let X be an extended bitopologicalspace .

(i) If $S_1 \subseteq S_2 \subseteq$ X , then $(1,2)^+\alpha$int(S_1) $\subseteq(1,2)^+\alpha$int(S_2) and $(1,2)^+\alpha$cl(S_1)$\subseteq(1,2)^+\alpha$cl(S_2).

(ii) $(1,2)^+\alpha$int(S) is $(1,2)^+\alpha$-open and $(1,2)^+\alpha$cl(S) is $(1,2)^+\alpha$-closed, for each S $\subset$ X .

(iii) A set S $\subset$ X is $(1,2)^+\alpha$-open iff S = $(1,2)^+\alpha$int(S) and a set S $\subset$ X is $(1,2)^+\alpha$-closed iff S = $(1,2)^+\alpha$cl(S).

(iv) (a) For any S $\subset$ X,
we have $(1,2)^+\alpha$int($(1,2)^+\alpha$int(S))=$(1,2)^+\alpha$int(S)
(b) For any S $\subset$ X,
we have $(1,2)^+\alpha$cl($(1,2)^+\alpha$cl(S))=$(1,2)^+\alpha$cl(S)

(v) (a)$(1,2)^+\alpha$int(X-S)=X-$(1,2)^+\alpha$cl(S) for any S $\subset$ X;
(b)$(1,2\}^+\alpha$cl(X-S)=X-$(1,2)^+\alpha$int(S)for any S $\subset$ X

(vi)For any family {S_i /i$\in$I} of subsets of X we have
(a) $\bigcup_i (1,2)^+\alpha\text{int}(S_i) \subset (1,2)^+\alpha\text{int}(\bigcup_i S_i)$;
(b) $\bigcup_i (1,2)^+\alpha\text{cl}(S_i) = (1,2)^+\alpha\text{cl}(\bigcup_i S_i)$;
(c) $(1,2)^+\alpha\text{int}(\bigcap_i S_i) = \bigcap_i (1,2)^+\alpha\text{int}(S_i)$;
(d) $(1,2)^+\alpha\text{cl}(\bigcap_i S_i) \subset \bigcap_i (1,2)^+\alpha\text{cl}(S_i)$;

Proof: Proof is similar as Theorem 3.6.

4. $(1,2)^+\alpha$-Generalized closed sets: In this section, we introduce new types of generalized closed sets and study some of their properties.

Definition 4.1 Let X be an extended bitopological space. A subset S of X is called a

(i) $(1,2)^+\alpha$g-closed set if $(1,2)^+\alpha$cl(S) $\subseteq$U whenever S $\subseteq$U and U$\in(1,2)^+\alpha O(X,A)$,

(ii) $(1,2)^+$sg-closed set if $(1,2)^+$scl(S) $\subseteq$ U whenever S$\subseteq$U and U$\in(1,2)^+SO(X,A)$,

(iii)$(1,2)^+$pg-closed set if $(1,2)^+$pcl(S) $\subseteq$U wheneverS$\subset$U and U$\in(1,2)^+PO(X, A)$.

The complement of above closed sets are called their respective open sets.

Example 4.2 Let X = {a, b, c, d}, τ_1= {ϕ,X,{a, b}}, τ_2 = {ϕ,X,{b},{a, b, c}}, then $\tau_1\tau_2O(X)$={ϕ,X,{b},{a,b},{a,b,c}}.If A = {a} $\notin \tau_1\tau_2O(X)$.
Then $(\tau_1\tau_2)^+(A)$ = {ϕ,X,{a},{b},{a,b},{a,b,c},}.
τ_1^+ (A)={ϕ,X,{a},{a,b}}.Then$(1,2)^+\alpha O(X,A)$={ϕ,X,{a}, {a,b},{a,b,c},{a,b,d}}=$(1,2)^+PO(X,A)$ and$(1,2)^+SO(X,A)$={ϕ,X,{a},{a,b},{a,c}, {a,d},{a,b,c},{a,b,d},{a,c,d}}.$(1,2)^+\alpha C(X,A)$={ϕ,X,{c}, {d},{c,d},{b,c,d}}=$(1,2)^+PC(X,A)$and $(1,2)^+SC(X,A)$={ϕ,X,{b},{c},{d},{b,c},{b,d},{c,d},{b,c, d}}.Also$(1,2)^+\alpha GCL(X,A)$={ϕ,X,{c},{d},{c,d},{a,c,d}, {b,c,d}}=$(1,2)^+PGCL(X,A)$and$(1,2)^+SGCL(X,A\}$ ={ϕ,X,{b},{c},{d},{b,c},{b,d},{c,d},{b,c,d}}.

Remark 4.3 Every $(1,2)^+\alpha$-closed set is a $(1,2)^+\alpha$g-closed set but the converse need not be true as could be seen from the example 4.2. Here {a,c,d} is a $(1,2)^+\alpha$g-closed set but not a $(1, 2)^+\alpha$-closed set.

Theorem 4.4 In an extended bitopological space X, a set S of X is called a $(1,2)^+\alpha$g-closed set iff [$(1, 2)^+\alpha$cl(S)-S] contains no nonempty $(1, 2)^+\alpha$-closed set.

Proof: Let S be a $(1, 2)^+\alpha$g-closed set. Sothere exists a $(1, 2)^+\alpha$-open set U such thatS $\subseteq$U and $(1,2)^+\alpha$cl(S) $\subseteq$U. Let F be anon-empty $(1,2)^+\alpha$-closed set such that F $\subseteq$[$(1,2)\alpha$cl(S)-S], which implies that S $\subseteq$ F^c.Hence $(1,2)^+\alpha$cl(S) $\subseteq F^c$and so F $\subseteq$ $[(1,2)^+\alpha\text{cl}(S)]^c$. Then we have F$\subseteq(1,2)^+\alpha$cl(S) $\cap[(1,2)^+\alpha\text{cl}(S)]^c$. Hence F = ϕ. Conversely, let S$\subseteq$U and U $\in$ $(1,2)^+\alpha O(X,A)$ suchthat S is not a $(1,2)^+\alpha$g-closed set.So $(1,2)^+\alpha$cl(S) is not a subset of U, which implies that $(1,2)^+\alpha$cl(S) $\subseteq U^c$. Hence $(1,2)^+\alpha$cl(S) $\cap U^c$ is a non-empty $(1,2)^+\alpha$-closedsubset of [$(1,2)^+\alpha$cl(S)-S], which is a con-tradiction.

Theorem 4.5: A $(1, 2)^+\alpha$g-closed set is $(1,2)^+\alpha$-closed iff [$(1,2)^+\alpha$cl(S)-S] is $(1,2)^+\alpha$-closed.

Proof: If S is $(1,2)^+\alpha$-closed, then [$(1,2)^+\alpha$cl(S)-S] = ϕ. Conversely [$(1,2)^+\alpha$cl(S)-S]itself is a subset of it. By theorem 4.4, it is equal to ϕ. Hence S is $(1, 2)^+\alpha$-closed.

Theorem4.6: If S and T are $(1,2)^+\alpha$g-closed, Then $S \cup T$ is also $(1,2)^+\alpha$g-closed.

Proof: Let $S \cup T \subseteq U$, where $U \in (1, 2)^+\alpha O(X)$. Since $(1,2)^+\alpha cl(S\cup T) = (1,2)^+\alpha cl(S) \cup (1,2)^+\alpha cl(T)$ and S and T are $(1,2)^+\alpha$g-closed sets, $(1,2)^+\alpha cl(S \cup T) \subseteq U$. Hence the result

Remark 4.7 The intersection of two $(1,2)^+\alpha$g-closed sets need not always be a $(1,2)^+\alpha$g-closed set as could be seen from the following example

Example4.8:

Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$, $\tau_2=\{\phi,X,\{b\},\{a,b\}\}$,Then$\tau_1\tau_2O(X)=\{\phi, X,\{b\},\{a,b\}\}$.Let $A=\{a\}\notin \tau_1\tau_2O(X)$. Then$(\tau_1\tau_2)^+(A)=\{\phi,X,\{a\},\{b\},\{a,b\}\}$.

$\tau_1^+ =\{\phi,X ,\{a\}\}$.

Then$(1,2)^+\alpha O(X,A)=\{\phi,X,\{a\}\}$,$(1,2)^+\alpha C(X,A)= \{\phi,X,\{b,c\}\}and(1,2)^+\alpha GCL(X,A) =\{\phi,X,\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$.Here$\{a,b\}and\{a,c\}$are $(1,2)^+\alpha$g-closed set but $\{a\}$ not $(1,2)^+\alpha$g-closed set .

Lemma 4.9 For an extended bitopological space X, every singleton set $\{x\}$ is either $(1,2)^+\alpha$-closed or $\{x\}^c$is $(1, 2)^+\alpha$g-closed.

Proof: If $\{x\}$ is not $(1,2)^+\alpha$-closed, then the only$(1,2)^+\alpha$-open set containing $X-\{x\}$ is X . Hence $\{x\}^c$is $(1, 2)^+\alpha$g-closed.

Conclusion: In this paper, we have extended the bitopological space and established the relationship between the bitopological spaces and the extended bitoplogical spaces. Also we characterized the properties of weak form of open sets and generalized closed sets. In future, we can develop some new separation axioms and also we can apply it into many fields such as fuzzy topology, digital topology, etc.

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