SOFT INTERVAL VALUED INTUTIONISTIC FUZZY SEMI-PRE GENERALIZED HOMEOMORPHISM

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Abstract: This paper introduces some new notions of generalized homeomorphism of soft interval valued intuitionistic fuzzy topology along with its characterization.

Keywords: Soft interval valued intuitionistic fuzzy open set (semi-open set, pre-open set, semi-pre open set, α - open set) soft interval valued intuitionistic semi –pre generalized homeomorphisms.

Introduction: The year 1999, gave a new start to a theory which was very convenient and easily applicable to the real life problems. This theory named Soft Set Theory was developed by D.Molodtsov [16]. He paved a way for attaining solution to complicated decision making problems that are complicated due to some uncertainty and incompleteness of information.

In 2003, Maji et al [14] developed several basic notions for this theory. In 2011, Muhammad Shabir and Munazza Naz[17] developed the soft topology.

The soft set theory was pooled with interval valued intuitionistic fuzzy set by Y. Jiang et.al [20] and Jinyan Wang et.al [11] studied interval valued intuitionistic fuzzy soft sets . In 2013, Anjan Mukherjee et. al [4] developed interval valued intuitionistic fuzzy soft topological spaces. This paper aims at generalizing the soft interval valued intuitionistic fuzzy semi-pre homeomorphism along with its characterization.

1. Preliminaries:

Definition:1.1[17] A pair (F,E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

Definition: 1.2[4] Let (ξ_A, E) be an element of interval valued intuitionistic fuzzy set(IVIFS) (U;E), P(ξ_A, E) be the collection of all IVIFS subsets of (ξ_A, E). A sub family τ of P$(\xi_A, E)$is called an interval valued intuitionistic fuzzy soft topology on (ξ_A, E), if the following axioms are satisfied:

$[O_1]$ (ϕ_{ξ_A}, E) ,$(\xi_A, E) \in \tau$

$[O_2]$ $\{(f_A^k, E)/k \epsilon K\} \subseteq \tau \Rightarrow \bigcup_{k\epsilon K}(f_A^k, E) \epsilon \tau$

$[O_3]$

$If\ (f_A, E), (g_A, E) \epsilon \tau, then\ (f_A, E) \cap (g_A, E) \epsilon \tau$

Then the pair $((\xi_A, E), \tau)$ is called an interval valued intuitionistic fuzzy soft topological space. The members of τ are called $\tau -$ open sets, where ϕ_{ξ_A}: A→IVIFS(U) is defined as $\phi_{\xi_A}(e)$= {<x,[0,0],[1,1]>:xϵU }, ∀ e ∈ A}

Definition: 1.3[4] Let $((\xi_A, E), \tau)$ be an interval valued intuitionistic fuzzy soft topological space on (ξ_A, E) and (f_A, E) be an IVIFS in P(ξ_A, E). Then the union of all open IVIFS set of (f_A, E) is called the interior of (f_A, E) and is denoted by int (f_A, E) and defined by int (f_A, E) = $\bigcup$ $\{(g_A, E)/(f_A, E)$ is a neighborhood of $(g_A, E)\}$.

Definition: 1.4[4] Let $((\xi_A, E), \tau)$ be an interval valued intuitionistic fuzzy soft topological space on (ξ_A, E) and (f_A, E) be an IVIFS in P(ξ_A, E). Then the intersection of all closed IVIFS set containing (f_A, E) is called the closure of (f_A, E) and is denoted by cl(f_A, E) and defined by cl(f_A, E) = $\bigcap$ $\{(g_A, E)/(g_A, E)$is a IVIFS-closed set containing$(f_A, E)\}$.

Definition: 1.5[21] Let U be an initial universe and E be the set of parameters . Suppose that A,B $\subseteq$ E, <F,A> and <G,B> are two interval-valued intuitionistic fuzzy soft sets, we say that <F,A> is an interval-valued intuitionistic fuzzy soft subset of <G,B> if and only if (i) A $\subseteq$ B, (ii) ∀ e ∈ A, F(e) is an interval-valued intuitionistic fuzzy soft subset of G(e) , that is for all x∈ U, e ∈ A ,

$$\underline{\mu}_{F(e)}(x) \leq \underline{\mu}_{G(e)}(x),$$

$$\overline{\mu}_{F(e)}(x) \leq \overline{\mu}_{G(e)}(x)$$

$$\underline{\gamma}_{F(e)}(x) \geq \underline{\gamma}_{G(e)}(x),$$

$$\overline{\gamma}_{F(e)}(x) \geq \overline{\gamma}_{G(e)}(x)$$

Definition: 1.6[21] An interval valued intuitionistic fuzzy soft set <F,A> over U is said to be a null interval valued intuitionistic fuzzy soft set denoted by Φ , if ∀ e ∈ A , $\mu_{F(e)}(x) = [0,0], \nu_{F(e)}(x) = [1,1]$ x ∈ U.

Definition: 1.7[21]

An interval valued intuitionistic fuzzy soft set <F,A> over U is said to be an absolute interval valued intuitionistic fuzzy soft set denoted by $\sum$, if ∀ e ∈ A , $\mu_{F(e)}(x) = [0,0], \nu_{F(e)}(x) = [1,1]$ x ∈ U.

2. Soft Interval Valued Intuitionistic Fuzzy Semi-Pre Generalized:

Homeomorphism: In this section we have introduced soft interval valued intuitionistic fuzzy semi-pre generalized homeomorphism and has listed their characterizations.

Definition:2.1:A mapping $\varphi_\psi : (X, \tau, E) \to (Y, \sigma, K)$ is called a soft interval valued intuitionistic fuzzy semi pre generalized continuous(SIVIFSPG continuous) mapping if $\varphi_{\psi}^{-1}(V,K)$ is a soft interval valued intuitionistic fuzzy semi pre generalized closed set in (X,τ,E) for every soft interval valued intuitionistic fuzzy closed set (V,K) of (Y,σ,K).

Definition:2.2: A mapping $\varphi_{\psi}:(X,\tau,E)\to(Y,\sigma,K)$ is called a soft interval valued intuitionistic fuzzy semi pre generalized open (SIVIFSPGO) mapping if $\varphi_{\psi}(F,E)$ is a soft interval valued intuitionistic fuzzy semi pre generalized open in Y for each soft interval valued intuitionistic fuzzy open set (F,E) of X.

Definition: 2.3 Let $\varphi_{\psi}:(X,\tau,E)\to(Y,\sigma,K)$ be a soft bijective mapping. Then φ_{ψ} is said to be a soft interval valued intuitionistic fuzzy semi pre generalized homeomorphism (SIVIFSPGHM) if φ_{ψ} is both a SIVIFSPG continuous mapping and a SIVIFSPGO mapping.

Theorem: 2.4 Let $\varphi_{\psi}:(X,\tau,E)\to(Y,\sigma,K)$ be a soft bijective mapping. If φ_{ψ} is a SIVIFSPG continuous mapping , then the following statements are equivalent:

(i) φ_{ψ} is a SIVIFSPGO mapping.

(ii) φ_{ψ} is a SIVIFSPGHM.

(iii) φ_{ψ} is a SIVIFSPGC mapping.

Proof: Straight forward from the definition.

References

Definition2.5:A mapping $\varphi_{\psi}:(X,\tau,E)\to(Y,\sigma,K)$ is called a soft interval valued intuitionistic fuzzy M- semi pre generalized open (SIVIFMSPGO) mapping if $\varphi_{\psi}(F,E)$ is a soft interval valued intuitionistic fuzzy semi pre generalized open in Y for each soft interval valued intuitionistic fuzzy semi pre generalized (SIVIFSPG)open set (F,E) of X.

Definition2.6:A mapping $\varphi_{\psi}:(X,\tau,E)\to(Y,\sigma,K)$ is called a soft interval valued intuitionistic fuzzy semi pre generalized irresolute (SIVIFSPG) mapping if $\varphi_{\psi}^{-1}(V,K)$ is a soft interval valued intuitionistic fuzzy semi pre generalized closed in Y for every SIVIFSPGCS (V,K) of Y.

Definition: 2.7 Let $\varphi_{\psi}:(X,\tau,E)\to(Y,\sigma,K)$ be a soft bijective mapping. Then φ_{ψ} is said to be a soft interval valued intuitionistic fuzzy M-semi pre generalized homeomorphism (SIVIFMSPGHM) if φ_{ψ} is both a SIVIFSPG irresolute mapping and a SIVIFMSPGO mapping.

Theorem: 2.8 Every SIVIFMSPGHM is a SIVIFSPGHM but not conversely.

Proof: Assume that $\varphi_{\psi}:(X,\tau,E)\to(Y,\sigma,K)$ is SIVIFMSPGHM. Let $(V,K)\,\tilde{\subseteq}\,(Y,K)$ be a SIVIFCS. Then (V,K) is a SIVIFSPGCS in Y. By hypothesis $\varphi_{\psi}^{-1}(V,K)$ is a SIVIFSPGCS in X. Hence φ_{ψ} is a SIVIFSPG continuous mapping. Let $(F,E)\,\tilde{\subseteq}\,X$ be a SIVIFOS. Then (F,E) is a SIVIFSPGOS in X . By hypothesis, $\varphi_{\psi}(F,E)$ is a SIVIFSPGOS in Y . Hence φ_{ψ} is a SIVIFSPGO mapping.

Thus φ_{ψ} SIVIFSPGHM.

Remark: 2.9 The composition of two SIVIFMSPGHM is a SIVIFMSPGHM.

Theorem: 2.10 Let $\varphi_{\psi}:(X,\tau,E)\to(Y,\sigma,K)$ be a soft bijective mapping. If φ_{ψ} is a SIVIFSPG irresolute mapping, then the following statements are equivalent:

(i) φ_{ψ} is a SIVIFMSPGO mapping.

(ii) φ_{ψ} is a SIVIFMSPGHM.

(iii) φ_{ψ} is a SIVIFMSPGCM.

Proof: Straight forward from the definition.

Theorem: 2.11 Let $\varphi_{\psi}:(X,\tau,E)\to(Y,\sigma,K)$ be a SIVIFMSPGHM, then $spgcl\left(\varphi_{\psi}^{-1}(V,K)\right)\tilde{\subseteq}\,\varphi_{\psi}^{-1}(spcl(V,K))$for every SIVIFS (V,K) in .

Proof: Let $(V,K)\,\tilde{\subseteq}\,Y$. Then $spcl(V,K)$ is a SIVIFSPGCS in Y. Since φ_{ψ} is a SIVIFSPG irresolute mapping, $\varphi_{\psi}^{-1}(spcl(V,K))$ is a SIVIFSPGCS in X. This implies

$spgcl\left(\varphi_{\psi}^{-1}(V,K)\right)=\varphi_{\psi}^{-1}(V,K)$. Hence

$spgcl\left(\varphi_{\psi}^{-1}(V,K)\right)\tilde{\subseteq}\,spgcl(\varphi_{\psi}^{-1}(spgcl(V,K)))=\varphi_{\psi}^{-1}(spgcl(V,K))$.

Conclusion : The present work is devoted to study the behavior of soft interval valued intuitionistic fuzzy semi-pre generalized homeomorphism and has established results connected with it.

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