ON SEMI PREOPEN SETS IN NANO TOPOLOGICAL SPACES

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Abstract: The purpose of this paper is to propose a new class of sets called nano semi pre open sets in nano topological spaces and derive some of their characterizations.

Keywords: nano semi pre open set, nano semi pre closed set.

Introduction: The notion of nano topology was introduced by Lellis Thivagar [1] in 2013 which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He has also defined nano open sets and nano closed sets using nano interior and nano closure operators respectively. Dimitrije Andrijevic [3] introduced semi pre open sets in topological spaces in 1985 together with its corresponding semi pre closure and semi pre interior operators. In this paper we introduce the notion of semi pre open sets in nano topological spaces and establish some of their properties.

1. Preliminaries:

Definition 1.1[1]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X $\subseteq$ U

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$.

That is $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$,

where R(x) denotes the equivalence class determined by x$\in$U.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is

$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \Phi\}$

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is

$B_R(X) = U_R(X) - L_R(X)$.

Definition 1.2[2]: Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let X$\subseteq$U. Let $\tau_R(X)$ = {U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$ }. Then $\tau_R(X)$ is a topology on U, called as the nano topology with respect to X. Elements of the nano topology are known as the nano-open sets in U and (U, $\tau_R(X)$) is called the nano topological space.

$[\tau_R(X)]^c$ is called as the dual nano topology of $\tau_R(X)$.

Elements of $[\tau_R(X)]^c$ are called as nano closed sets.

Definition 1.3[2]: If (U, $\tau_R(X)$) is a nano topological space with respect to X where X$\subseteq$U and if A$\subseteq$U, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by Nint(A). That is, Nint(A) is the largest nano open subset of A.

The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A). That is, Ncl(A) is the smallest nano closed set containing A.

Throughout this paper, U is a non-empty finite universe and U/R denotes the family of equivalence class by equivalence relation R on U.

Definition 1.4[2]: Let (U, $\tau_R(X)$) be a nano topological space and A$\subseteq$U. Then A is said to be

(i) nano semi-open if A$\subseteq$Ncl(Nint(A))

(ii) nano pre-open if A $\subseteq$ Nint(Ncl(A))

(iii) nano α-open if A $\subseteq$ Nint(Ncl(Nint(A)))

(iv) nano regular open if A = Nint(Ncl(A))

NSO(U,X), NPO(U,X) and NαO(U,X) respectively denote the families of all nano semi-open, nano pre-open, nano α-open and nano regular open subsets of U.

Let (U,$\tau_R(X)$) be a nano topological space and A $\subseteq$ U. A is said to be nano semi closed, nano pre-closed, nano α-closed, and nano regular closed if its complement is respectively nano semi-open, nano pre-open, nano α-open and nano regular open.

2. Nano Semi Pre Open Sets:

Definition 2.1: A subset A of a nano topological space (U,$\tau_R(X)$) is said to be nano semi pre open if A $\subseteq$ Ncl(Nint(Ncl(A))).

Example 2.2: Let U = {a, b, c, d} with U/R = {U, ϕ, {a}, {c}, {b, d} } and X = {a, b}. Then the nano topology defined as $\tau_R(X)$ = {U, ϕ, {a}, {a, b, d}, {b, d}}. The nano closed sets are: {U, ϕ, {c}, {a, c}, {b, c, d}}. NSO(U, $\tau_R(X)$) = {U, ϕ, {a}, {a, c}, {b, d}, {a, b, d}, {b, c, d}}. NPO(U, $\tau_R(X)$) = {U, ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}}. N αO (U, $\tau_R(X)$) = {U, ϕ, {a}, {b, d}, {a, b, d}}. NSPO (U, $\tau_R(X)$) = {U, ϕ, {a}, {b}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}.

Remark 2.3: From the example we note that nano semi open, nano pre open and nano semi pre open sets of (U, $\tau_R(X)$) does not form a topology but nano α-open sets forms a topology.

Theorem 2.4: If A is nano-open, then it is nano semi pre open in (U, $\tau_R(X)$).

Proof: Since A is nano-open in U, Nint(A) = A. Then A $\subseteq$ Nint(Ncl(A)). Therefore Ncl(A) $\subseteq$ Ncl(Nint(Ncl(A))). That is A $\subseteq$ Ncl(Nint(Ncl(A))). Thus A is nano semi pre-open.

But the converse of the above theorem need not be true can be seen from the example 2.2. Here the sets {b}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {a, c, d}, {b, c, d} are nano semi pre open but not nano open.

Theorem 2.5: Let (U, $\tau_R(X)$) be a nano topological space and A $\subseteq$ U, then

(i) Every nano pre open set is nano semi pre open.
(ii) Every nano regular open set is nano semi pre open.
(iii) Every nano α-open set is nano semi pre open.
(iii) Every nano semi open is nano semi pre open.

can be proved in a similar way.

The reverse implications of the above theorem need not be true is seen from the following example.

Example 2.6: Let U = {a, b, c, d} with U/R = {U, ϕ, {a}, {c}, {b, d} } and X = {a, b}. Then the nano topology defined as $\tau_R(X)$ = {U, ϕ, {a}, {a, d}, {b, d}}. Then the sets {a, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d} are nano semi pre open but not nano pre open. The sets {b}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d} are nano semi pre open but not nano regular open. The sets {b}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {a, c, d}, {b, c, d} are nano semi pre open but not nano α- open. And the sets {b}, {d}, {a, b}, {a, d}, {b, c}, {c, d}, {a, b, c}, {a, c, d} are nano semi pre open but not nano semi open.

Theorem 2.7: Union of all nano semi pre open sets is nano semi pre open in a nano topological space.

Proof: Let {A_s: s $\in$ S} be a family of semi pre open sets in a nano topological space. Then

$\bigcup_s A_s \subset \bigcup_s(\mathrm{Ncl}(\mathrm{Nint}(\mathrm{Ncl}(A)))) \subset$
$\mathrm{Ncl}(\bigcup_s(\mathrm{Nint}(\mathrm{Ncl}(A_s)))) \subset$
$\mathrm{Ncl}(\mathrm{Nint}(\bigcup_s(\mathrm{Ncl}(A_s)))) \subset$
$\mathrm{Ncl}(\mathrm{Nint}(\mathrm{Ncl}(\bigcup_s(A_s))))$.

Remark 2.8: The intersection of nano semi pre open sets need not be nano semi pre open in general is proved by the following example.

From example 2.2 we see that the sets {a, b} and {b, c} are nano semi pre open but {a, b} $\cap${b, c} = {c} is not nano semi pre open.

Definition 2.9: A is said to be nano semi pre open if there exists pre open set F such that

F $\subseteq$ A $\subseteq$ Ncl(A).

Theorem 2.10: NSO(X)$\cup$NPO(X) $\subset$ NSPO(X).

Example 2.2 shows that the equality in the above theorem does not hold in general.

Theorem 2.11: For any subset of a nano topological space (U,$\tau_R(X)$) the following conditions are equivalent:

(a) A $\in$ NSPO(X)
(b) A $\subset$ Ncl(Nint(Ncl(A)))
(c) Ncl(A) $\in$ NRcl(X)

Proof: (a) $\Rightarrow$ (b): Let A be nano semi pre open set. Then F$\subset$A$\subset$ Ncl(F) for a nano pre open set F. This implies Ncl(A) = Ncl(F), and hence Ncl(Nint(Ncl(A))) = Ncl(Nint(Ncl(F))). Since F is Nano pre open we have, F $\subset$ N(int(Ncl(F))) and finally A$\subset$ Ncl(F) $\subseteq$ Ncl(Nint(Ncl(F))) = Ncl(Nint(Ncl(A))).

(b) $\Rightarrow$ (c) : If A $\subseteq$ Ncl(Nint(Ncl(A))). Then Ncl(A) =Ncl(Nint(Ncl(A))). That is,

Ncl(A) $\in$ NRcl(X).

(c) $\Rightarrow$ (a): Suppose that Ncl(A)=Ncl(Nint(Ncl(A))), and put

F = A $\cap$ Nint(Ncl(A)). F is nano pre open since Npint(A) = A $\cap$ Nint(Ncl(A)). Now we have,

Nint(Ncl(A)) = Ncl(A) $\cap$ Nint(Ncl(A)) $\subset$
Ncl(A $\cap$ Nint(Ncl(A))) = Ncl(F). Hence
A $\subset$ Ncl(A) = Ncl(Nint(Ncl(A))) $\subset$Ncl(F)

and therefore A $\in$ NSPO(X).

Theorem 2.12: If V is nano open and A is nano semi pre open then V$\cap$A is nano semi pre open.

Proof: Since A is nano semi pre open V$\cap$A $\subset$ V$\cap$ Ncl(Nint(Ncl(A))) $\subset$ Ncl(V$\cap$Nint(Ncl(A)))
= Ncl(Nint(V$\cap$ Ncl(A))) $\subset$ Ncl(Nint(Ncl(V$\cap$A))).

Definition 2.13: A subset A of a nano topological space (U, $\tau_R(X)$) is nano pre semi closed if X-A is nano semi pre open.

The following four theorems are dual to 2.7, 2.10, 2.11, 2.12. Thus the proof is omitted.

Theorem 2.14: The intersection of nano semi pre closed sets is nano semi pre closed.

Remark 2.15: Union of two nano semi pre closed sets need not be nano semi pre closed can be seen from the example 3.2. Here the sets {a}, {b, d} are nano semi pre open but {a}$\cup${b, d} = {a, b, d} is not nana semi pre closed.

Theorem 2.16: NScl(X)$\cup$NPcl(X) $\subset$ NSPcl(X).

Remark 2.17: The equality of the above theorem does not hold in general can be seen from the example 2.2 where NScl(X)$\cup$NPcl(X) = {U, ϕ, {a}, {b},{c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, c, d}, {b, c, d}} and NScl(X)$\cup$NPcl(X) = NSPO (U, $\tau_R(X)$) = {U, ϕ, {a}, {b}, {c}, {d}, {a, c}, {b, c}, {c, d}, {a, b, c}, {a, c, d}, {b, c, d}}. Therefere NScl(X)$\cup$NPcl(X) $=$ NSPcl(X).

Theorem 2.18: For any subset of a nano topological space (U,$\tau_R(X)$) the following conditions are equivalent

(a) A $\in$ NSPcl(X)
(b) Nint(Ncl(Nint(A))) $\subset$ A
(c) Nint(A) $\in$ NRO(X)

Theorem 2.19: If A is nano closed and B is nano semi pre closed then A$\cup$B is nano semi pre closed.

We observe that NSPcl(A) is nano semi pre closed by theorem 2.14. Our next result expresses the semi preclosure of a set in terms of interior and closure.

Theorem 2.20: Let A be a subset of X. Then NSPcl(A) = A ∪ Nint(Ncl(Nint(A))).

Theorem 2.21: NSPcl(A)⊂ NScl(A) ∩ NPcl(A)

Remark 2.22: The equality of the above theorem does hold in general which is shown from the following example.

Example 2.23: Let U = {a, b, c, d} with U/R = {U, ϕ, {a}, {c}, {b, d} } and X = {b,d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {b, d}}. Here NScl(A) ∩ NPcl(A) = {U, ϕ,{a}, {c}, {a,c}}and NSpcl(A) = {U, ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {a, c, d}}. Hence NSPcl(X) = NScl(X) ∩ NPcl(X).

Theorem 2.24: Let A be asubset of X.

Then NSPint(A) = A ∩ Ncl(Nint(A))

Corollary 2.25:

(i) NSPint(X-A) = X- NSPcl(A)

(ii) NSPcl(X-A) = X - NSPint(A)

Theorem 2.26: If A is nano semi pre open (resp.nano semi pre closed) and nano semi closed (resp. nano semi open) then A is nano semi open (resp. nano semi closed).

Corollary 2.27: NSPO(X) NScl(X) = NSOcl(X) = NSPcl(X) NSO(X).

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