GROUP ACCEPTANCE SAMPLING PLAN – SIZE BIASED LOMAX MODEL

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Abstract: In this paper a group acceptance sampling plan (GASP) is developed when the life time of the items assume the size bi9ased Lomax model. The design parameters such as minimum group size and acceptance number are derived when the consumer's risk and the test termination time are specified. The operating characteristic values at various quality levels are calculated and the minimum ratios of the true mean life to the specified mean life at the specified producer's risk are obtained. The results are discussed through an example. It is concluded that the efficiency of the proposed sampling plan in terms of minimum sample size is required to reach same decision.

Keywords: Consumer's Risk, Group Acceptance Sampling Plan (GASP), Producer's Risk, Size Biased Lomax Model (SBLM).

Introduction: In the present global market different commodities are differentiated on several factors of an end product. One such factor is quality/durability of a product that can be observed through the most statistical quality control techniques. The two important statistical tools for ensuring the quality of a product are (i) Process control and (ii) Product control in the form of acceptance sampling. The group acceptance sampling plan (GASP) also considers the mutual development of both producer's risk and the consumer's risk, denoted by γ and β. The acceptance sampling plans consider a compromise between the producer and consumer risks and only a small number of items are required for inspection. The producer's risk and the consumer's risk are two probabilities of making wrong decisions. The probability of rejecting a good lot is called the producer's risk and the probability of accepting a bad lot is called the consumer's risk. In most of the real life experiments a single item is inspected at a time, known as an ordinary acceptance sampling plan where as in group acceptance sampling plan a multiple number of items are inspected based on the number of testers available to the experimenter for testing. Compared with single acceptance sampli8ng scheme the GASP scheme is more efficient in reducing the cost and the time of the life test.

Many researchers have discussed group acceptance sampling plans by considering the life time random variable of a product as some probability model, a few of them are R. Radhakrishnan and K. Alagirisamy who derived Construction of group acceptance sampling plan using log-logistic distribution and weighted binomial distribution(2011) [6], Muhammad Aslam, munir Ahmad and Abdul Rassaque Mughal proposed Group acceptance sampling plan for lifetime data using generalized Pareto distribution(2010) [5], G. Srinivasa Rao discussed (i). A group acceptance sampling plans based on truncated life tests for Marshall-Olin extended Lomax distribution(2010) [9] (ii). A group acceptance sampling plan for life times following a Marshall-Olkin extended exponential distribution(2011) [10], Single point group acceptance sampling plans for Birnbaum-Saunders distribution studied by Muhammad Aslam, R.R.L. Kantam and Munir Ahmad(2010) [4], M. Aslam and C.H. Jun fhave studied A group acceptance sampling plans for truncated life tests based on the inverse Rayleigh and log-logistic distributions(2009) [1], C.H. Jun, S. Balalmurali and S.H. Lee have studied on Variables sampling plans for weibull distributed lifetimes under sudden death testing(2006) [2], R.R.L. Kantam, K. Rosaiah and G. Srinivasa Rao discussed Acceptance sampling based on life tests: log-logistic models(2001) [3], Group acceptance sampling plans using weighted binomial on truncated life tests for inverse Rayleigh and log-logistic distributions studied by A.R. Sudamani Ramaswamy, Priyah Anburajan(2012) [12], K. Rosaiah, R.R.L. Kantam and R. Subba Rao discussed (i) Pareto distribution in acceptance sampling based on truncated life tests(2009) [8] and (ii) An economic reliability test plan with Pareto distrtibution(2007) [7].

A brief introduction of the size biased Lomax model is given in Section 2. An attempt is made to construct a group acceptance sampling plan for size biased Lomax model in Section 3 of this paper. The operating characteristic values are presented in Section 4. The results are discussed through an example in Section 5. A comparative study of results is given in Section 6.

The Size Biased Lomax Model: The probability density function (p.d.f) and cumulative distribution function (c.d.f) of Lomax model are given by

$$g(t)=\frac{\alpha}{\sigma}\left(1+\frac{t}{\sigma}\right)^{-(\alpha+1)} ;\ t\geq 0,\alpha\succ 0,\sigma\succ 0 \quad (1)$$

$$G(t)=1-\left(1+\frac{t}{\sigma}\right)^{-\alpha} ;\ t\geq 0,\alpha\succ 0,\sigma\succ 0 \quad (2)$$

where α is shape and σ is scale parameter. Mean and variance of Lomax model are given by

$$E(t)=\frac{\sigma}{\alpha-1};\alpha\succ 1$$

$$V(t)=\frac{\alpha\sigma^2}{(\alpha-1)^2(\alpha-2)};\alpha\succ 2 \quad (3)$$

The formula for size biased p.d.f is

$$f(t)=\frac{t.g(t)}{E(t)} \quad (4)$$

In this paer a new GASP model is proposed by considering a size biased Lomax model with known shape parameter. The p.d.f and c.d.f of size biased Lomax model are given below

$$f(t)=\frac{\alpha(\alpha-1)}{\sigma}\frac{t}{\sigma}\left(1+\frac{t}{\sigma}\right)^{-(\alpha+1)} ;t\ge 0,\alpha\succ 1,\sigma\succ 0 \quad (5)$$

$$F(t)=1-\left(1+\frac{\alpha.t}{\sigma}\right)\left(1+\frac{t}{\sigma}\right)^{-\alpha} ;\ t\ge 0,\alpha\succ 1,\sigma\succ 0 \quad (6)$$

where α is shape and σ is scale parameter. Mean and variance of size biased Lomax model are given by

$$E(t)=\frac{2\sigma}{\alpha-2};\alpha\succ 2$$

$$V(t)=\frac{2\alpha\sigma^2}{(\alpha-2)^2(\alpha-3)};\alpha\succ 3 \quad (7)$$

The Proposed Group Acceptance Sampling Plan: Let μ be the true mean life of a product and μ_0 denoted the specified mean life of an item, under the assumption that the life time of an item assumes size biased Lomax model. An item is rejected due to bad value if the true value of the μ is smaller than the specified value μ_0, that is, $\mu\prec\mu_0$, otherwise accepted. The step wise procedure for a GASP is described below:

- Select the number of groups $'g'$ and allocate predefined $'r'$ items to each group so that the sample size for a lot will be $n=g.r$
- Determine the acceptance number c for every group and specify the termination time of the life test t_0
- Implement the life test based on the groups of items, simultaneously and record the number of failures for each group.
- Accept the lot if at most c failed items is found in every group by the termination time.
- Truncate the life test and reject the lot if more than c failures are found in any group.

GASP reduces to the ordinary acceptance sampling plan if for $r=1$. For a specified group size r and termination time t_0 determine the number of group's g for the proposed sampling plan for various values of the acceptance number c. It would be convenient to take the termination time as a multiple of the specified constant $'a'$ (termination ratio). Therefore, the probability p for the size biased Lomax model is

$$p=F(t_0)=1-\left(\frac{2\alpha a}{\left(\frac{\mu_0}{\mu}\right)(\alpha-2)}+1\right)\left(1+\frac{2a}{\left(\frac{\mu_0}{\mu}\right)(\alpha-2)}\right)^{-\alpha} \quad (8)$$

As assumed, that $'\gamma'$ and $'\beta'$ are producer's risk and consumer's risks respectively, for deriving the parameters of the proposed sampling plan, our concentration is only on consumer's risks, it can also express as the consumer's confidence level. If the confidence level is p^*, then the consumer's risk $\beta=1-p^*$. It is a two action decision making process accept or reject on any lot, one can use binomial distribution to develop GASP (Stephens 2001). Therefore the lot acceptance probability

$$L(p)=\left[\sum_{i=0}^{c} r_{c_i} p^i (1-p)^{r-i}\right]^g \quad (9)$$

where p is the function of $F(t_0)$ given in equation (8).

The minimum number of groups required can be determined by considering the consumer's risk when the true mean life equals the specified mean life $(\mu=\mu_0)$ through the following inequality

$$L(p_0)\le\beta \quad (10)$$

where p_0 is the failure probability at $\mu=\mu_0$ and it is given by

$$p_0=1-\left(\frac{2\alpha a}{(\alpha-2)}+1\right)\left(1+\frac{2a}{(\alpha-2)}\right)^{-\alpha} \quad (11)$$

The minimum number of groups for the size biased Lomax model with shape parameter $\alpha=3$, test termination ratio $a=0.7,0.8,1.0,1.2,1.5,2.0$ and the group size $r=2(1)9$, the acceptance number $c=0(1)7$, for consumer's risk $\beta=0.25,0.10,0.05,0.01$ is given in Table 1. The choices of parameters are chosen for the comparison purpose.

From Table 1, it is noticed that for a given value of r and c as termination ratio (a) is increases, the minimum number of groups (g) is decreasing. More so, the required minimal number of groups in a life test increases as both the acceptance number and the group size decreases.

Operating Characteristics: Once the minimum number of groups is obtained, one may be interested to find the probability of acceptance of a lot when the quality of the product is good enough. The product is considered to be good if $\mu\succ\mu_0$. The probabilities based on equation (9) for various mean life times $(\mu/\mu_0=2,4,6,8,10,12)$ under $\beta=0.25,0.10,0.05,0.01$ with the termination ratio $a=0.7,0.8,1.0,1.2,1.5,2.0$ choosing acceptance number $c=2$, group size $r=4$ and different values of g are presented in Table 2 at $\alpha = 3$. From Table 2, operating characteristic values increase rapidly as the quality increases. For example $\beta = 0.25$, $r = 4$, $c = 2$ and $a = 0.7$, the number of groups required is $g = 2$. If the true mean is twice the specified value $\mu/\mu_0 = 2$, the producer's risk is approximately $\alpha = 0.26962$, while $\alpha = 0.00614$ when the true value of the mean is 6 times than the specified life.

For given producer's risk the minimal ratios of true mean life to the specified mean life can be obtained by using the inequality

$$\left[\sum_{i=0}^{c} r_{c_i} p^i (1-p)^{r-i}\right]^g \geq 1-\alpha \qquad (12)$$

where p is given in equation (8) and for various combinations of consumer's risk β, a and $\gamma = 0.05$, the minimum mean ratio's are found for fixed values of c and r are given in Table 3.

It is apparent from the table that the mean ratios are increasing as the values of termination ratio increases, also except for very few in all most all combinations as consumer's risk increases the minimum mean ratio's decrease.

For construction of proposed sampling plan without loss of generality we consider $\alpha = 3,4,5$ (since for the consider size biased Lomax model $\alpha \succ 2$). Due to space constraints, only for $\alpha = 3$ we presented the values of minimum number of groups required for the proposed sampling plan in Table 1, the values of operating characteristic of the GASP are in Table 2 and the values of minimum ratio of true mean life to specified mean life for the producer's risk of 0.05 in Table 3.

Example: Suppose a bulb manufacturer would like to know whether or not the mean life of the bulb is longer than the specified mean life $\mu_0 = 1000$ hours. Further that the manufacturer wants to run a life experiment in 700 hours by using testers equipped with 4 items each. Assuming that the life time of bulbs follow a size biased Lomax model with $\alpha = 3$. Suppose the manufacturer would like to select the acceptance number $c = 2$ and the multiplier in termination ratio $a = 0.7$ under the consumer's risk $\beta = 0.25$, Table gives the required minimal group size $g = 2$ the GASP $(g,r,c,a) = (2,4,2,0.7)$. In practice, the manufacture needs to draw a random sample of size 8 bulbs from the lot and allocate 4 items to 2 groups on the life test. The lot is accepted if no more than 2 failed bulbs are found in 700 hours in very group. Otherwise the lot is rejected.

The operating characteristic values for the GASP $(g,r,c,a) = (2,4,2,0.7)$ are as follows:

μ/μ_0	2	4	6	8	10	12
P_a	0.7304	0.9678	0.9939	0.9984	0.9994	0.9998

That is, a lot of items will be accepted with probability 0.7304 if the true mean life of items in lot is 2 times more than the specified mean life. The probability of accepting the lot increases up to 0.9939 if the true mean life is 6 times than the specified mean life. For example, when $\beta = 0.10$, $r = 6$, $g = 8$, $c = 4$ and $a = 0.7$, the required ratio is $\mu/\mu_0 = 2.73$.

Results and Discussion: In order to compare the proposed GASP with that of existing sampling plans we have considered the above example. The GASP for the generalized Pareto distribution studied by Muhammad Aslam, Munir Ahmad and Abdul Razzaque Mughal(2010) requires the values of $(g,r,c,a) = (9,4,2,0.7)$ at $\alpha = 3$, $\delta = 2$, A GASP for lifetimes following a Marshall-Olkin extended exponential distribution derived by G. Srinivasa Rao(2011) requires the values of $(g,r,c,a) = (10,4,2,0.7)$ at $v = 3$ and the proposed GASP for size biased Lomax model requires $(g,r,c,a) = (2,4,2,0.7)$ at $\alpha = 3$. Therefore, proposed GASP needs 8 $(n = r.g)$ items and the existing sampling plans given by Aslam(2010) needs 36 items and that of G. Srinivasa Rao(2011) needs 40 items respectively, to attain the same decision about the submitted items.

In this paper, the design of GASP, developed for the size biased Lomax model, the minimum number of groups, operating characteristic values and the minimum ratio of the true mean life to the specified mean life are derived and tabulated for illustration. It has been observed that the minimum number of groups required decreases as the test termination time multiplier increases and also the operating characteristic values increase more rapidly as the quality improves. Hence if a data is confirmed to follow size biased Lomax model, our proposed sampling plan performs better than the existing sampling plans in terms of minimum sample size required to reach the same decision. The GASP can be used when a multiple number of items at a time are considered for a life test and it would be beneficial in terms of test time and cost because a group of items will be tested simultaneously.

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Table 1Minimum Number of Groups Required for the Proposed Plan using Size Biased Lomax Model with $\alpha = 3$

β	**r**	**c**	**a**					
			0.7	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**
0.25	**2**	**0**	1	1	1	1	1	1
	3	**1**	2	2	1	1	1	1
	4	**2**	2	2	2	1	1	1
	5	**3**	3	3	2	2	1	1
	6	**4**	5	4	2	2	1	1
	7	**5**	7	5	3	2	2	1
	8	**6**	10	7	4	3	2	1
	9	**7**	15	9	5	3	2	1
0.10	**2**	**0**	2	2	1	1	1	1
	3	**1**	3	2	2	2	1	1
	4	**2**	4	3	2	2	2	1
	5	**3**	5	4	3	2	2	1
	6	**4**	8	6	4	3	2	2
	7	**5**	11	8	5	3	2	2
	8	**6**	17	11	6	4	3	2
	9	**7**	24	15	8	5	3	2
0.05	**2**	**0**	2	2	2	1	1	1
	3	**1**	3	3	2	2	2	1
	4	**2**	5	4	3	2	2	2

	5	**3**	7	5	4	3	2	2
	6	**4**	10	7	5	3	3	2
	7	**5**	15	10	6	4	3	2
	8	**6**	21	14	8	5	3	2
	9	**7**	32	19	10	6	4	3
0.01	**2**	**0**	3	3	2	2	2	2
	3	**1**	5	4	3	3	2	2
	4	**2**	7	6	4	3	3	2
	5	**3**	10	8	5	4	3	2
	6	**4**	15	11	7	5	4	3
	7	**5**	22	15	9	6	4	3
	8	**6**	33	21	11	8	5	3
	9	**7**	48	29	15	9	6	4

Table 2 Operating Characteristic values of the Group Acceptance Sampling Plan using Size Biased Lomax Model with $\alpha = 3$, $r = 4$, $c = 2$

β	***g***	***a***	***μ/μ₀***					
			2	**4**	**6**	**8**	**10**	**12**
0.25	**2**	**0.7**	0.73038	0.96778	0.99386	0.99835	0.99944	0.99977
	2	**0.8**	0.64104	0.94763	0.98910	0.99693	0.99893	0.99956
	2	**1.0**	0.47265	0.89085	0.97324	0.99172	0.99693	0.99869
	1	**1.2**	0.57926	0.90336	0.97346	0.99112	0.99652	0.99846
	1	**1.5**	0.44056	0.82811	0.94384	0.97899	0.99112	0.99585
	1	**2.0**	0.27743	0.68750	0.87160	0.94384	0.97346	0.98652
0.10	**4**	**0.7**	0.53345	0.93660	0.98776	0.99671	0.99888	0.99955
	3	**0.8**	0.51325	0.92249	0.98370	0.99540	0.99839	0.99934
	2	**1.0**	0.47265	0.89085	0.97324	0.99172	0.99693	0.99869
	2	**1.2**	0.33555	0.81606	0.94763	0.98231	0.99306	0.99693
	2	**1.5**	0.19409	0.68576	0.89085	0.95842	0.98231	0.99172
	1	**2.0**	0.27743	0.68750	0.87160	0.94384	0.97346	0.98652
0.05	**5**	**0.7**	0.45590	0.92140	0.98473	0.99589	0.99861	0.99944
	4	**0.8**	0.41094	0.89801	0.97833	0.99387	0.99786	0.99913
	3	**1.0**	0.32495	0.84082	0.96013	0.98760	0.99540	0.99804
	2	**1.2**	0.33555	0.81606	0.94763	0.98231	0.99306	0.99693
	2	**1.5**	0.19409	0.68576	0.89085	0.95842	0.98231	0.99172
	2	**2.0**	0.07697	0.47265	0.75968	0.89085	0.94763	0.97324
0.01	**7**	**0.7**	0.33298	0.89171	0.97869	0.99425	0.99805	0.99922
	6	**0.8**	0.26343	0.85099	0.96767	0.99082	0.99679	0.99869
	4	**1.0**	0.22340	0.79361	0.94719	0.98350	0.99387	0.99739
	3	**1.2**	0.19437	0.73720	0.92249	0.97359	0.98960	0.99540
	3	**1.5**	0.08551	0.56788	0.84082	0.93829	0.97359	0.98760
	2	**2.0**	0.07697	0.47265	0.75968	0.89085	0.94763	0.97324

Table 3Minimum Ratio of True Mean Life to Specified Mean Life for the Producer's risk of 0.05 using Size Biased Lomax Model with $\alpha = 3$

β	r	c	a					
			0.7	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**
0.25	**2**	**0**	13.35	15.26	19.09	22.9	28.63	38.15
	3	**1**	5.96	6.81	6.67	8	10	13.33
	4	**2**	3.55	4.06	5.07	4.98	6.23	8.3
	5	**3**	2.84	3.25	3.69	4.42	4.63	6.17
	6	**4**	2.49	2.72	2.94	3.53	3.74	4.99
	7	**5**	2.2	2.36	2.68	2.96	3.7	4.23
	8	**6**	2	2.16	2.45	2.78	3.22	3.7
	9	**7**	1.87	1.98	2.25	2.47	2.86	3.31
0.10	**2**	**0**	19.6	22.41	19.09	22.9	28.63	38.15
	3	**1**	6.82	6.81	8.51	10.21	10	13.33
	4	**2**	4.27	4.52	5.07	6.08	7.6	8.3
	5	**3**	3.18	3.46	4.06	4.42	5.53	6.17
	6	**4**	2.73	2.95	3.4	3.85	4.41	5.87
	7	**5**	2.38	2.57	2.95	3.22	3.7	4.93
	8	**6**	2.17	2.32	2.63	2.93	3.48	4.29
	9	**7**	2	2.13	2.42	2.7	3.08	3.81
0.05	**2**	**0**	19.6	22.41	28	22.9	28.63	38.15
	3	**1**	6.82	7.8	8.51	10.21	12.76	13.33
	4	**2**	4.51	4.87	5.65	6.08	7.6	10.14
	5	**3**	3.42	3.64	4.33	4.87	5.53	7.37
	6	**4**	2.84	3.04	3.56	3.85	4.81	5.87
	7	**5**	2.5	2.67	3.05	3.4	4.02	4.93
	8	**6**	2.24	2.41	2.75	3.05	3.48	4.29
	9	**7**	2.08	2.21	2.51	2.78	3.24	4.11
0.01	**2**	**0**	24.41	27.9	28	33.6	42	56
	3	**1**	8.04	8.56	9.74	11.69	12.76	17.02
	4	**2**	4.9	5.39	6.09	6.78	8.48	10.14
	5	**3**	3.68	4.02	4.55	5.19	6.08	7.37
	6	**4**	3.06	3.31	3.8	4.27	5.1	6.47
	7	**5**	2.66	2.86	3.28	3.66	4.25	5.36
	8	**6**	2.4	2.56	2.9	3.3	3.81	4.64
	9	**7**	2.19	2.34	2.67	2.96	3.47	4.32

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