A STUDY ON JUST EXCELLENT AND VERY EXCELLENT WEAKLY CONNECTED SET DOMINATION

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Abstract: In this paper we introduce just excellent graphs and very excellent graphs with respect to the parameter Weakly connected set domination (wcsd) and obtained certain results. In a wcsd-just excellent graphs the distinct γ_{wcsd} - sets are disjoint. But in wcsd- very excellent graphs, we find distinct γ_{wcsd} - sets $S_1 \& S_2$ such that $S_1 \bigcap S_2$ contains $\gamma_{wcsd}(G)-1$ vertices. Throughout this paper we consider only simple connected graphs.

Keywords: weakly connected set dominating (wcs) sets, Dominating weakly connected set dominating(wcsd) sets, wcsd- just excellent sets , wcsd-very excellent sets and wcsd domatic number.

Introduction: Sampath kumar and Pushpa Latha[3] have defined Set Dominating sets in graphs. Hedetniemi et all[2] defined Weakly Connected Domination in graphs. We define the concept of Weakly connected Set Dominating Sets, Weakly connected Point Set Dominating Sets and elucidate some results in our earlier paper. We extend these to new class of excellent graphs with respect to the parameter weakly connected set domination.

2. Preliminaries:

Definition2.1[2]: A subset $S \subseteq V$ is said to be a dominating set in G if for every vertices in V-S is adjacent to some vertex in S

Definition2.2[7]: The domatic number of a graph G is defined to be the maximum number of elements in a partition of V(G) into dominating sets

Definition2.3[3]: Let G be a connected graph. $S \subseteq V$ is called a set domination set if for every $T \subseteq V-S$ there exists $R \subseteq S$ such that $< T \cup R >$ is connected

Definition2.4[5]: Let G be a connected graph. $S \subseteq V$ is called a weakly connected set if the sub graph$<S>_w$ whose vertex set is N[S] and whose a edge set consists of those edges in E(G) with at least one vertex, and possibly both in S is connected.

3. Class of wcsd-just excellent and wcsd- very excellent graphs

Definition3.1: Let G= (V, E) be a connected graph. $S \subseteq V$ is called a **weakly connected set dominating set** if $\forall T \subseteq V-S$,there exists $R \subseteq S$ such that $< T \cup R >$ is weakly connected and is denoted by **wcs-set.** A dominating wcs-set is denoted by **wcsd-set** and its minimal cardinality is denoted by $\gamma_{wcsd}(G)$

Definition3.2: A connected graph G is said to **wcsd-excellent** if every vertex belong to some γ_{wcsd} - set of G

Definition 3.3: A connected graph G is said to **wcsd- just excellent** if every vertex belong to unique γ_{wcsd} - set of G.

Definition3. 4: A connected graph G is said to **wcsd-very excellent** if there is a γ_{wcsd} - set S of G such that $\forall u \in V-S$ there exist a vertex $v \in S$ such that $(S-v) \cup \{u\}$ is a γ_{wcsd} - set of G and S is called a very excellent γ_{wcsd} - set of G.

Definition 3.5: Private neighbor set of a vertex $v \in S$ with respect to a set S denoted PN(v, S) is N(v)-N(S-v) and $\forall u \in PN(v,S)$ is called a private neighbor of 'v' with respect to S.

Definition 3.6: A vertex $u \in V(G)$ is called a γ_{wcsd} **- level vertex** of G if $\gamma_{wcsd}(G-u) = \gamma_{wcsd}(G)$

Definition 3.7: A vertex $u \in V(G)$ is called a γ_{wcsd} **- non level vertex** of G is $\gamma_{wcsd}(G-u) = \gamma_{wcsd}(G)-1$.

Definition3.8: The **wcsd- domatic number** $d_{wcsd}(G)$ is defined to be the maximum number of elements in a partition of V(G) into γ_{wcsd} - sets.

Theorem3.9: If G is a wcsd-just excellent graph then $|PN(u,S)| \geq 2 \, \forall u \in S$ where S is any γ_{wcsd} -set of G

Proof:

Let G be a wcsd-just excellent graph and S be a γ_{wcsd} - set of G. let $u \in S$

then PN(u, S)=N(u)-N(S-u). We claim that $|PN(u,S)| \geq 2 \, \forall u \in S$

suppose that $|PN(u,S)| < 2 \, \forall u \in S$

case 1: let $|PN(u,S)| = 0 \, \forall u \in S$

then $PN(u,S) = \phi \;\forall u \in S$

Define $S_1 = (S-u) \cup \{w\}$ where 'w' is in N(u)

Now S_1 is a γ_{wcsd} -set of G and every element of S is in S_1 other than 'u'

That is $S \cap S_1 \neq \phi$ thus G is not a wcsd- just excellent graph which is a contradiction

Therefore $|PN(u,S)| \neq 0$

Case 2: let $|PN(u,S)| = 1 \forall u \in S$

let $PN(u,S)=\{w\}\forall u\in S$

Define $S_2=(S-u)\cup\{w\}$

Now S_2 is a γ_{wcsd}-set of G and every element of S is in S_2 other than 'u'

That is $S\cap S_2\neq\phi$ thus G is not a wcsd- just excellent graph which is a contradiction

$\left|PN(u,S)\right|\neq 1\forall u\in S$

Thus $\left|PN(u,S)\right|\geq 2\forall u\in S$

Theorem 3.10: In a wcsd-just excellent graph G, every vertex 'u' is a γ_{wcsd}-level vertex

Proof:

Let G be a wcsd-just excellent graph and $u\in G$

Then there exists a γ_{wcsd}-set of G not containing 'u'

Hence $\gamma_{wcsd}(G-u)\leq\gamma_{wcsd}(G)$

We claim that $\gamma_{wcsd}(G-u)=\gamma_{wcsd}(G)$

If possible assume that $\gamma_{wcsd}(G-u)<\gamma_{wcsd}(G)$

Let S be a γ_{wcsd}-set of G-u

Then $S\cup\{v\}$ is a γ_{wcsd}-set of G for all 'v' in N(u)

Since G is connected N(u) contains at least two vertices. Let it be 'u' & 'v'

Now $S\cup\{u\}$ & $S\cup\{v\}$ are γ_{wcsd}-sets of G with every element of S is common to both sets

That implies G is not a wcsd-just excellent graph which is a contradiction.

Therefore $\gamma_{wcsd}(G-u)=\gamma_{wcsd}(G)$

Thus 'u' is a γ_{wcsd} -level vertex of G

Theorem 3.11: If G is a wcsd- just excellent graph with $\gamma_{wcsd}(G)=\frac{n}{3}$, then G is a Hamiltonian

Proof: Let G be a wcsd- just excellent graph with $\gamma_{wcsd}(G)=\frac{n}{3}$

Then the wcsd –domatic number $d_{wcsd}(G)=3$

Let $S_1,.S_2$ & S_3 be the distinct γ_{wcsd}-sets of G

Then $V(G)=S_1\cup S_2\cup S_3$

Define

$A=\{e\in E(G)/e\in<S_i>\ for some\, i=1,2\ \&\ 3\}$ &

H=G-A

Then $\gamma_{wcsd}(G)\leq\gamma_{wcsd}(H)$

As $S_1,.S_2$ & S_3 be the distinct γ_{wcsd}-sets of H, H is a wcsd-just excellent graph

If H is Hamiltonian then G is so. It is enough to prove the result by assuming that each S_i is an independent set in G

By theorem 3.9, $\left|PN(u,S_i)\right|\geq 2\forall u\in S_i$

$\forall v(\neq u)\in S_i, PN(u,S_i)\ \&\ PN(v,S_i)$ are disjoint

$$\left|\bigcup_{u\in S_i}PN(u,S_i)\right|\geq\bigcup_{u\in S_i}\left|PN(u,S_i)\right|\geq 2|S_i|\forall u\in S_i$$

$$\left|\bigcup_{u\in S_i}PN(u,S_i)\right|\geq 2\gamma_{wcsd}(G)\rightarrow 1$$

Consider $|V-S_i|=n-\gamma_{wcsd}(G)$

$|V-S_i|=3\gamma_{wcsd}(G)-\gamma_{wcsd}(G)$

$|V-S_i|=2\gamma_{wcsd}(G)$

Thus 1 becomes $\left|\bigcup_{u\in S_i}PN(u,S_i)\right|\geq|V-S_i|$

Hence $\bigcup_{u\in S_i}PN(u,S_i)=V-S_i$

then $\forall u\in S_i$ deg(u)=2, i=1,2 &3

As G is connected and deg(u)=2, it contains a cycle C_n

Thus G is Hamiltonian

Theorem 3.12: A grapb G is a wcsd-very excellent if and only if there exists a γ_{wcsd} set S of G such that $\forall u\in V-S\ \exists\ v\in S\ \ni: PN[v,S]\subset N[u]$

Proof:

Let G be a wcsd-very excellent graph. Then G has a very excellent γ_{wcsd} set S.

Then $\forall u\in V-S\ \exists v\in S\ \ni:(S-v)\cup\{u\}$ is a γ_{wcsd} set of G

PN[v, S]=N[v]-N[S-v]

Then S-v does not dominate any vertex of PN[v, S]

But $(S-v)\cup\{u\}$ is a γ_{wcsd} set of G

Therefore 'u' dominates every vertex in PN[v, S]

Thus $PN[v,S]\subset N[u]$

Converse part:

Let G be a graph with a γ_{wcsd} set S such that

$\forall u\in V-S\ \exists\ v\in S\ \ni: PN[v,S]\subset N[u]$

$\forall u\in V-S\ \exists\ v\in S\ \ni: N[v]-N[S-v]\subset N[u]$

Then (S-v)U{u}is a γ_{wcsd} set of G

Thus G is a wcsd- very excellent graph.

Conclusion: This paper has attempt to establish new class of excellent graphs with respect to the parameter weakly connected set domination and enable to study various properties of such graphs. The future scope of study is to make new class of excellent graphs with respect to the parameter weakly connected point set domination.

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