RADIATIVE HEAT TRANSFER ON UNSTEADY MHD OSCILLATORY VISCO-ELASTIC FLOW THROUGH POROUS MEDIUM IN A PARALLEL PLATE CHANNEL

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Abstract: The combined effect of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin visco-elastic fluid through a channel filled with saturated porous medium and non-uniform walls temperature has been discussed. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The analytical solutions are obtained for the problem making use of perturbation technique. The effects of the radiation and the magnetic field parameters on velocity profile and shear stress for different values of the visco-elastic parameter with the combination of the other flow parameters are illustrated graphically, and physical aspects of the problem are discussed.

Keywords: Heat transfer, MHD flows, porous medium, parallel plate channels, Radiation effects and visco-elastic fluids,

Introduction: The flow of an electrically conducting fluid has important applications in many branches of engineering science such as magneto hydro dynamics (MHD) generators, plasma studies, nuclear reactor, geothermal energy extraction, electro- magnetic propulsion, and the boundary layer control in the field of aerodynamics. In the light of these applications, MHD flow in a channel has been studied by many authors; some of them are Nigam and Singh [1], Soundalgekar and Bhat [2], Vajravelu [3], and Attia and Kotb [4]. A survey of MHD studies in the technological fields can be found in Moreau [5]. The flow of fluids through porous media is an important topic because of the recovery of crude oil from the pores of the reservoir rocks; in this case, Darcy's law represents the gross effect. Raptis et al. [6] have analysed the hydro magnetic free convection flow through a porous medium between two parallel plates. Aldoss et al. [7] have studied mixed convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Makinde and Mhone [8] have considered heat transfer to MHD oscillatory flow in a channel filled with porous medium. Recently the combined effect of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin visco-elastic fluid through a channel filled with saturated porous medium and non-uniform walls temperature has been discussed by Rita Choudary and Utpal Jyothi Das [12]. In this study, an attempt has been made to extend the problem studied by Makinde and Mhone [8] to the case of visco-elastic fluid characterised by second-order fluid. The constitutive equation for the incompressible second order fluid is of the form

$$\sigma = pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \qquad (1.1)$$

where σ is the stress tensor, *p* is the hydrostatic pressure, *I* is the unit tensor, $A_n (n=1,2)$ are the kinematic Rivlin-Ericksen tensors, μ_1, μ_2 and μ_3 are the material coefficients describing viscosity, elasticity, and cross-viscosity, respectively. The material coefficients μ_1, μ_2 and μ_3 have taken constants with μ_1 and μ_3 as positive and μ_2 as negative (Markovitz and Coleman [9]). The equation (1.1) was derived by Coleman and Noll [10] from that of the simple fluids by assuming that stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

Formulation and solution of the problem: Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer as shown in Fig. 1. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The *x*-axis is taken along the centre of the channel, and the *z*-axis is taken normal to it. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given by

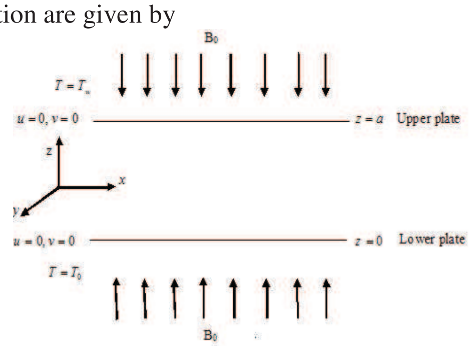

Figure 1: Geometry of the problem.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu_1 \frac{\partial^2 u}{\partial z^2} + \nu_2 \frac{\partial^3 u}{\partial z^2 \partial t} - \frac{\sigma_e B_0^2 u}{\rho} - \nu_1 \frac{u}{K} + g\beta(T - T_0) \qquad (1)$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu_1 \frac{\partial^2 v}{\partial z^2} + \nu_2 \frac{\partial^3 v}{\partial z^2 \partial t} - \frac{\sigma_e B_0^2 v}{\rho} - \nu_1 \frac{v}{K} \qquad (2)$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial z^2}-\frac{1}{\rho C_p}\frac{\partial q_1}{\partial z} \quad (3)$$

Corresponding boundary conditions

$$u=0, v=0, T=T_W \text{ on } z=1 \quad (4)$$

$$u=0, v=0, T=T_0 \text{ on } z=0 \quad (5)$$

Where *u* is the axial velocity, *t* is the time, *T* is the fluid temperature, *P* is the pressure, *g* is the gravitational force, q_1 is the radiative heat flux, β is the coefficient of volume expansion due to temperature, C_p is the specific heat at constant pressure, *k* is the thermal conductivity, *K* is the porous medium permeability co-efficient, $B_0(=\mu_e H_0)$ is electromagnetic induction, μ_e is the magnetic permeability, H_0 is the intensity of the magnetic field, σ_e is the conductivity of the fluid, ρ is fluid density, and $\nu_i=\mu_i/\rho,(i=1,2)$. It is assumed that both walls of temperature T_0, T_w are high enough to induce radiative heat transfer. Following Cogley et.al [11], it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q_1}{\partial z}=4\alpha_1^2(T_0-T), \quad (6)$$

Where α_1 is the mean radiation absorption co-efficient. Combining equations (1) and (2) and let $q=u+iv$ and $\xi=x+iy$, we obtain

$$\frac{\partial q}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial \xi}+\nu_1\frac{\partial^2 q}{\partial z^2}+\nu_2\frac{\partial^3 q}{\partial z^2\partial t}-\frac{\sigma_e B_0^2 q}{\rho}$$
$$-\nu_1\frac{q}{K}+g\beta(T-T_0) \quad (7)$$

The following non-dimensional quantities are introduced:

$$x^*=\frac{x}{a},\ y^*=\frac{y}{a},\ \xi^*=\frac{\xi}{a},\ z^*=\frac{z}{a},$$
$$u^*=\frac{u}{U},\ v^*=\frac{v}{U}, q^*=\frac{q}{U},$$
$$t^*=\frac{tU}{a},\ p^*=\frac{ap}{\rho\nu_1 U},\ \theta=\frac{T-T_0}{T_w-T_0}$$

Making use of non-dimensional variables, the dimensionless governing equations together with appropriate boundary conditions (dropping asterisks) are

$$\mathrm{Re}\frac{\partial q}{\partial t}=-\frac{\partial p}{\partial \xi}+\frac{\partial^2 q}{\partial z^2}+\alpha\frac{\partial^3 q}{\partial z^2\partial t}$$
$$-(M^2+S^2)q+GrT \quad (8)$$

$$Pe\frac{\partial \theta}{\partial t}=\frac{\partial^2\theta}{\partial z^2}+R^2\theta \quad (9)$$

with

$$q=0,\ \theta=1 \text{ on } z=1 \quad (10)$$

$$q=0,\ \theta=0 \text{ on } z=0 \quad (11)$$

Where

$\mathrm{Re}=\frac{Ua}{\nu_1}$ is the Reynolds number, $M^2=\frac{\sigma_e B_0^2 a^2}{\nu_1 U}$ is the Hartmann number, $D=\frac{k}{a^2}$ is the Darcy number (or) $S=\frac{1}{D}$ is the porous medium shape factor parameter,

$\alpha=\frac{\nu_2\mathrm{Re}}{a^2}$ is the visco-elastic parameter,

$Gr=\frac{g\beta(T_w-T_0)a^2}{\nu_1 U}$ is the Grashoff number,

$Pe=\frac{Ua\rho C_p}{k}$ is Peclet parameter and $R^2=\frac{4\alpha_1^2 a^2}{k}$ is the Radiation parameter.

Solving the equations (8) and (9) for purely oscillatory flow, Let

$$-\frac{\partial P}{\partial \xi}=\lambda e^{i\omega t},\ q(z,t)=q_0(z)e^{i\omega t},$$
$$\theta(z,t)=\theta_0(z)e^{i\omega t} \quad (12)$$

Where, λ is constant and ω is the frequency of oscillation.

Substituting the above expressions (12) into the equations (8) and (9), and making use of the corresponding boundary conditions (10) and (11), we obtain

$$(1+i\alpha\omega)\frac{d^2 q_0}{\partial z^2}-m_1^2 q_0=-\lambda-Gr\theta_0 \quad (13)$$

$$\frac{d^2\theta_0}{\partial z^2}+m_2^2\theta_0=0 \quad (14)$$

Subjected to the boundary conditions

$$q_0=0,\ \theta_0=1 \text{ on } z=1 \quad (15)$$

$$q_0=0,\ \theta_0=0 \text{ on } z=0 \quad (16)$$

Where $m_1=\sqrt{M^2+S^2+i\omega\mathrm{Re}}$ and $m_2=\sqrt{R^2-i\omega Pe}$

Equations (13) and (14) are solved; we obtained the solution for the fluid velocity and temperature as follows

$$q(z,t)=\left\{a_1 e^{(m_1 z)/b}+\left(a_1-\frac{\lambda}{m_1^2}\right)e^{-(m_1 z)/b}\right.$$

$$+\frac{\lambda}{m_1^2}+\frac{Gr\sin(m_2 z)}{(m_2^2 b+m_1^2)\sin(m_2)}\Bigg\}\ e^{i\omega t} \quad (17)$$

$$\theta(z,t)=\frac{\sin(m_2 z)}{\sin(m_2)}\ e^{i\omega t} \quad (18)$$

Where $a_1=\dfrac{\left(\left(\dfrac{\lambda}{m_1^2}\right)e^{-m_1/b}-\left(\dfrac{\lambda}{m_1^2}\right)-\dfrac{Gr}{(m_2^2 b+m_1^2)}\right)}{\left(e^{m_1/b}-e^{-m_1/b}\right)}$,

$b=1+i\alpha\omega$

The non-dimensional shear stress σ at the wall $z=0$ is given by

$$\sigma=\frac{\sigma^*}{(\mu_1 U/a)}=\left(\frac{\partial u}{\partial z}+\alpha\frac{\partial^2 u}{\partial z\partial t}\right)_{z=0}$$

$$=\left[\frac{a_1 m_1}{b}-\left(a_1-\frac{\lambda}{m_1^2}\right)+\frac{Gr}{(m_2^2 b+m_1^2)\sin(m_2)}\right]$$

$$(1+i\omega\alpha)\ e^{i\omega t} \quad (19)$$

The rate of heat transfer across the channel wall $z=1$ is given as

$$Nu=-\left(\frac{\partial\theta}{\partial z}\right)_{z=1}=\frac{m_2\cos(m_2)}{\sin(m_2)}\ e^{i\omega t} \quad (20)$$

Results and Discussion: The flow governed by the non dimensional parameters, Re is the Reynolds number, M^2 is the Hartmann number, D is the Darcy number (or) S is the porous medium shape factor parameter, α is the visco-elastic parameter, R is the Radiation parameter with fixed values of Gr the Grashoff number, Pe Peclet parameter. We have considered the real and imaginary parts of the results *u* and *v* throughout for numerical validation. The velocity profiles for the components against *z* is plotted in Figures (2–11) while figure (12-16) to observe temperature profiles on the visco-elastic effects and other parameters for various sets of values of Hartmann number *H*, porous parameter *S* and radiation parameter *R*, with fixed values of other flow parameters, namely, Pe = 0.7, t = 0.1, Gr = 2 and λ = 1.

It is evident from Figures (2–15) that the velocity profiles is parabolic in nature, and the magnitude of velocity *u* and *v* increase with the increasing values of the Reynolds number Re, Porous parameter *S*, the visco-elastic parameter $|\alpha|$, Radiation parameter R and the frequency of oscillation ω (Figure 2, 3, 6-13). The magnitude of the velocity *u* increases and *v* decreases with increase in Grashoff number *Gr* (14-15). It is also noted from the figures (4-5) that the magnitude of the velocity component *u* experiences retardation and the behaviours of the velocity component *v* remains the same with the increasing values of the Hartmann number. We observe that lower the permeability of the porous medium lesser the fluid speed in the entire fluid region. The resultant velocity *q* enhances with increasing the parameters Re, *D*, α, *R* and experiences retardation with increasing the intensity of the magnetic field.

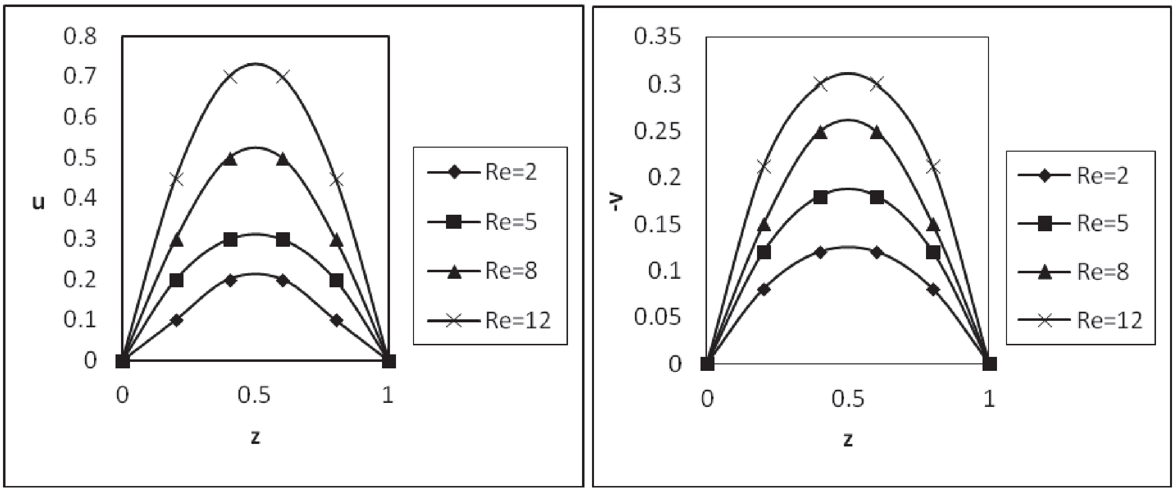

Fig. 2 & 3: The velocity profile for *u* and *v* on Reynolds number Re with $M=2,\ S=1,\ \alpha=-0.1,\ R=1.5,\ \omega=\pi/4, Gr=2,\ Pe=0.7$

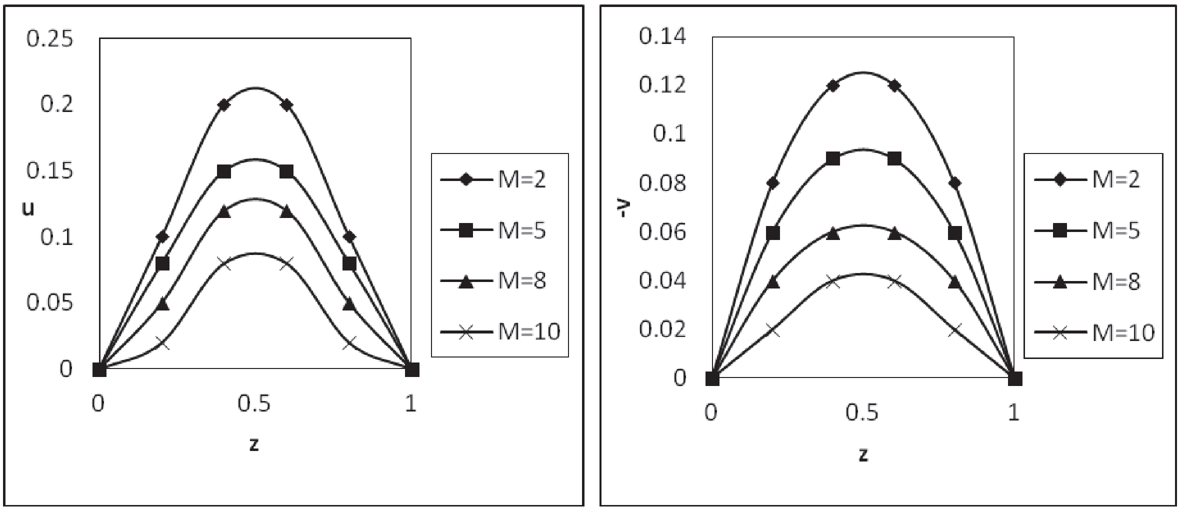

Fig. 4 & 5: The velocity profile for *u* & *v* on Hartmann number M with

$$\mathrm{Re}=2,\ S=1,\ \alpha=-0.1,\ R=1.5,\omega=\pi/4,Gr=2,\ Pe=0.7$$

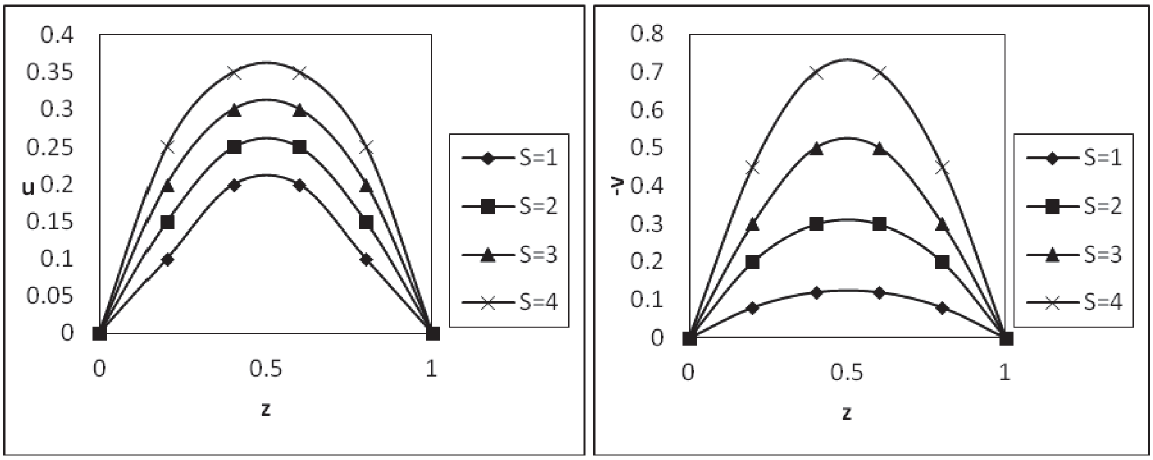

Fig. 6 & 7: The velocity profile for *u* & *v* on Porous parameter S with

$$\mathrm{Re}=2,\ M=2,\ \alpha=-0.1,\ R=1.5,\omega=\pi/4,Gr=2,\ Pe=0.7$$

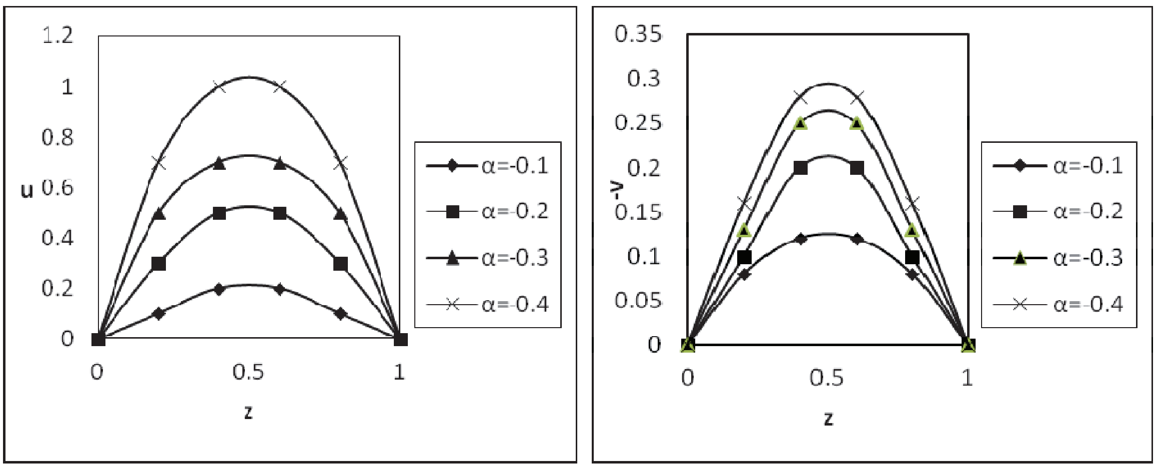

Fig. 8 & 9: The velocity profile for *u* and *v* on visco-elastic parameter α with

$$\mathrm{Re}=2,\ M=2,\ S=1,\ R=1.5,\omega=\pi/4,Gr=2,\ Pe=0.7$$

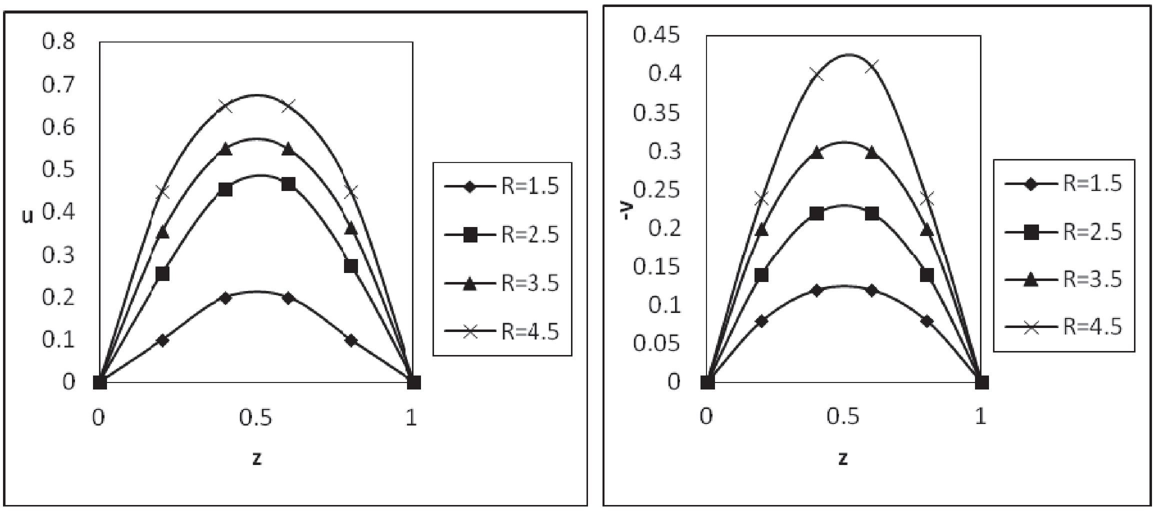

Fig. 10 & 11: The velocity profile for *u* and *v* on Radiation parameter R with

$$\mathrm{Re}=2,\ M=2,\ S=1,\ \alpha=-0.1, \omega=\pi/4,\ Gr=2,\ Pe=0.7$$

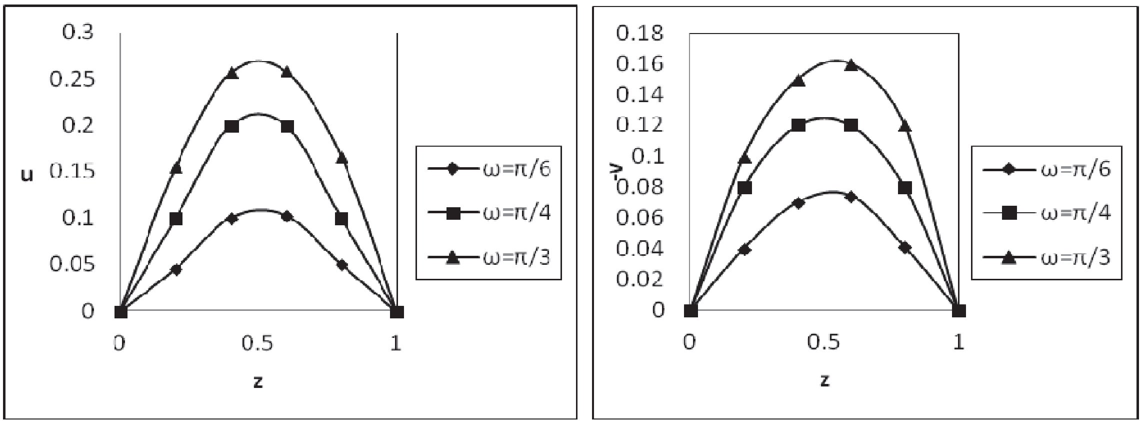

Fig. 12 & 13: The velocity profile for *u* and *v* on frequency of oscillation ω with

$$\mathrm{Re}=2,\ M=2,\ S=1,\ \alpha=-0.1, R=1.5,\ Gr=2,\ Pe=0.7$$

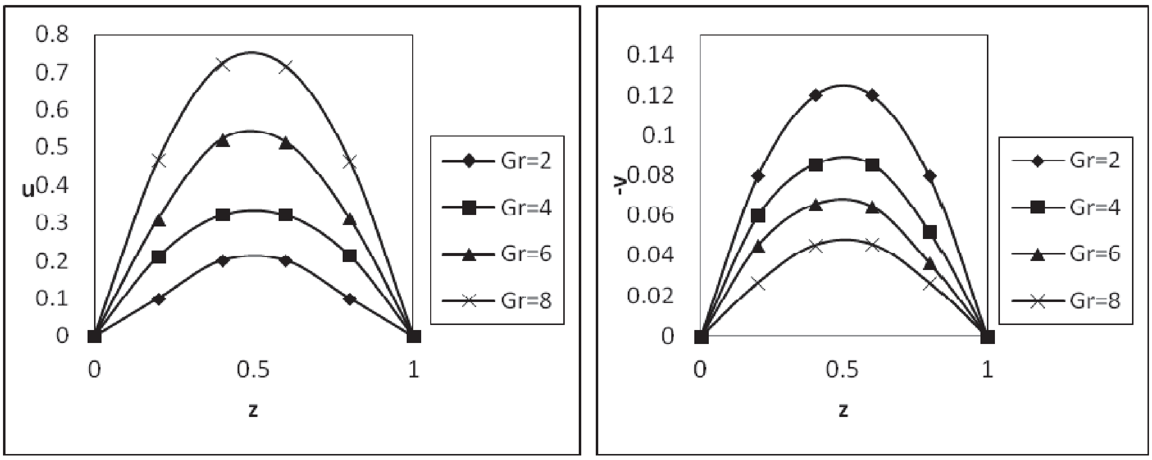

Fig. 14 & 15: The velocity profile for *u* and *v* on Grashof number *Gr* with

$$\mathrm{Re}=2,\ M=2,\ S=1,\ \alpha=-0.1, R=1.5,\ Gr=2, Pe=0.7$$

It is evident that the temperature profiles exhibit the nature of the flow on governing parameters. The magnitude of the temperature increases with increase in frequency of oscillation ω and radiation parameter *R* and experiences retardation with increasing Peclet number *Pe* (Figures 16-18). The shear stress on the wall and the rate of heat transfer evaluated analytically and computationally discussed with reference to governing parameters (Tables I-II). It is evident that the shear stress enhances with increasing Re, *M* and α and reduces with increase in S and ω. We also noted that it is increases firstly and then decreases with the increasing values of radiation parameter R (Table. I). Table. (II) depict that the Nusselt number (*Nu*) or the rate of heat transfer increases with increasing *Pe* retardation with radiation parameter *R* and ω.

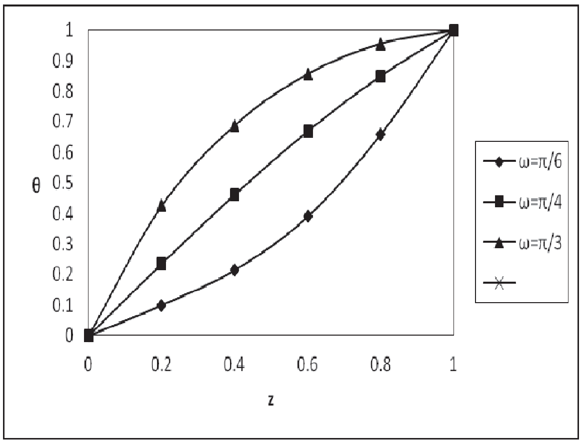

Fig. 16: The temperature profile for θ on frequency of oscillation ω with $Pe = 0.7,\ R = 1.5$

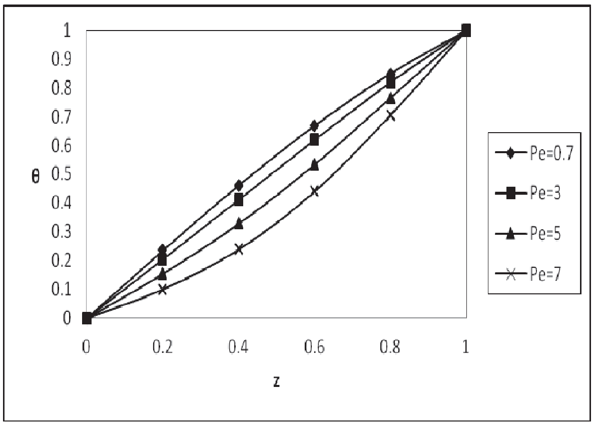

Fig. 17: The temperature profile for θ on Peclet number *Pe* with $R = 1.5,\ \omega = \pi / 2$

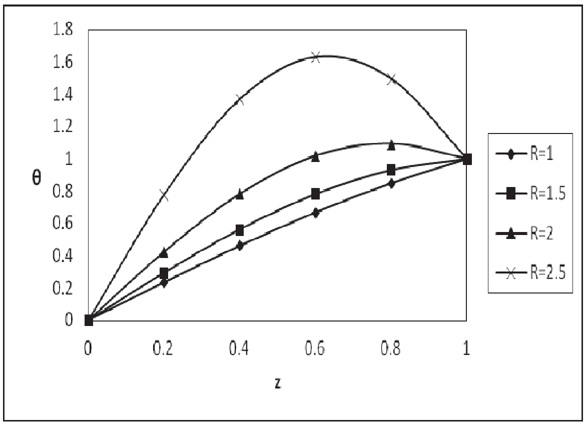

Fig. 18: The temperature profile for θ on Radiation parameter R with $Pe = 0.7,\ \omega = \pi / 2$

Table I: The shear stresses (τ) at the wall $z=0$									
	M	*S*	α	*R*	ω	$Re=2$	$Re=4$	$Re=6$	$Re=8$
I	**2**	**1**	**-0.1**	**1.5**	$\pi/4$	0.255466	0.350645	0.458556	0.569985
II	**5**	1	-0.1	1.5	$\pi/4$	0.288554	0.399685	0.482857	0.596048
III	**8**	1	-0.1	1.5	$\pi/4$	0.322562	0.421152	0.524655	0.622545
IV	2	**2**	-0.1	1.5	$\pi/4$	0.222514	0.322541	0.422589	0.526336
V	2	**3**	-0.1	1.5	$\pi/4$	0.155284	0.255652	0.360022	0.569966
VI	2	1	**-0.2**	1.5	$\pi/4$	0.366526	0.566326	0.758459	1.225566
VII	2	1	**-0.3**	1.5	$\pi/4$	0.566022	0.966585	1.221589	1.889562
VIII	2	1	-0.1	**2**	$\pi/4$	0.322566	0.458878	0.588495	0.966584
IX	2	1	-0.1	**2.5**	$\pi/4$	0.141522	0.266526	0.355669	0.655208
X	2	1	-0.1	1.5	$\pi/6$	0.362245	0.465654	0.566589	0.679985
XI	2	1	-0.1	1.5	$\pi/3$	0.055855	0.150021	0.200545	0.322565

Table. II: Rate of heat transfer (Nusselt number) at the wall $z=1$							
R	I	II	III	IV	V	VI	VII
1	0.632579	0.728221	0.92545	1.17385	0.637865	0.625179	0.60403
1.5	0.099432	0.259275	0.552611	0.89258	0.103291	0.094012	0.07843
2	-0.91347	-0.55317	-0.00877	0.52589	-0.91447	-0.91218	-0.9091
2.5	-3.28321	-1.92831	-0.61817	0.27059	-3.31803	-3.23613	-3.1115

	I	II	III	IV	V	VI	VII
Pe	0.7	3	5	7	0.7	0.7	0.7
ω	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/6$	$\pi/3$	$\pi/2$

Conclusions: The magnitude of velocity *u* and *v* increase with the increasing values of the Reynolds number Re, Porous parameter *S*, the visco-elastic parameter α, Radiation parameter *R* and the frequency of oscillation ω. The magnitude of the velocity component *u* experiences retardation and the behaviours of the velocity component *v* remains the same with the increasing values of the Hartmann number. Lower the permeability of the porous medium lesser the fluid speed in the entire fluid region. The magnitude of the velocity *u* enhances and *v* reduces with increase in *Gr*. The resultant velocity *q* enhances with increasing the parameters Re, *D*, α, ω, *R* and experiences retardation with increasing the intensity of the magnetic field and *Gr*. The magnitude of the temperature increases with increase in frequency of oscillation ω and radiation parameter *R* and experiences retardation with increasing Peclet number *Pe* The shear stress enhances with increasing Re, *M* and α and reduces with increasing *S* and ω. We also noted that it is increases firstly and then decreases with the increasing values of radiation parameter *R*.

The Nusselt number (*Nu*) or the rate of heat transfer increases with increasing *Pe* retardation with radiation parameter *R* and ω. It has also been observed that the temperature field is not significantly affected by the visco-elastic parameter.

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