EDGE - ODD GRACEFUL LABELING OF THE WINDMILL GRAPHS

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Abstract: A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Lo[2] introduced the concept of edge graceful graphs. [5] introduced a new concept of labeling called an edge - odd graceful labeling (EOGL). In this paper, EOGL of the windmill graphs are discussed.

Key words: Edge graceful graph, Edge-odd graceful graph, Graceful graph, Graceful labeling.

Introduction: A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Graph labelings were first introduced in the late 1960's. Most graph labeling methods trace their origin to one introduced by Rosa [6] in 1967. Lo[2] introduced the concept of edge graceful graphs. [5] introduced a new concept of labeling called an edge-odd graceful labeling (EOGL). We say that a labeling is said to be an EOGL if there exists a bijection f from E to {1,3,5,…,2q-1 } so that the induced maping f^+ from V to { 0,1,2,…,2q } given by $f^+(x) = \Sigma$ { f(xy) / xy∈E } (mod | 2q |), the resulting labels are distinct. A graph G with p vertices and q edges is said to be edge-odd graceful if it admits an EOGL. In this paper, EOGL of the windmill graphs are discussed.

2. Main Results:

Definition: 2.1: The **windmill graph** $C_n^{(m)}$ is the family of graphs consisting of m copies of C_n with a vertex in common.

Theorem : 2.2: If m is odd then $C_3^{(m)}$ is an edge – odd graceful graph.

Proof: Let the vertices of $C_3^{(m)}$ be {v, v_i / i= 1 to 2m} and

$E(C_3^{(m)})$ = { vv_i , v_jv_{j+1} /i= 1 to 2m and j= 1,3,…,2m-1}, the edge set of $C_3^{(m)}$.

we first label the edges of $C_3^{(m)}$ as follows

$f(vv_{2i}) = 6i-1$ for i = 1 to m

$f(vv_{2i-1}) = 6i-5$ for i = 1 to m

$f(v_{2i-1}v_{2i}) = 6i-3$ for i = 1 to m

Now the induced vertex labels are $f^+(v) = 0$

$f^+(v_{2i}) = 12i - 4 \pmod{6m}$ for i= 1 to m

$f^+(v_{2i-1}) = 12i - 8 \pmod{6m}$ for i=1 to m

The edge labels are arranged in the following order.

A ={$f(vv_{2i})$ / i = 1 to m}={6i–1/ i=1 to m }
= { 5,11, . . . , 6m-1 }

B={$f(vv_{2i-1})$/ i = 1 to m}={6i–5/ i=1 to m }
= {1,7, . . . , 6m-5 }

C ={$f(v_{2i-1}v_{2i})$/i =1 to m}={6i–3/i=1 to m }
= { 3,9, . . . , 6m-3}

Hence, $f(E(C_3^{(m)}))$ = A∪ B ∪ C = { 1, 3, . . . , 6m-1}

And $f^+(V(C_3^{(m)}))$={$f^+(v_i)$,$f^+(v)$/i =1 to 2m}

Therefore the set of edge labels and the vertex labels are distinct and the edge labels are odd.

So f is an Edge – Odd Graceful Labeling. Hence, $C_3^{(m)}$ is an Edge Odd Graceful graph.

Theorem : 2.3: If m is even then $C_4^{(m)}$ is an edge – odd graceful graph.

Proof: Let the vertices of $C_4^{(m)}$ be {v, v_i / i= 1 to 3m} and

$E(C_4^{(m)})$ = { vv_{3i-2} , vv_{3i} , $v_{3i-2}v_{3i-1}$, $v_{3i}v_{3i-1}$ / i= 1 to m }, the edge set of $C_4^{(m)}$.

we first label the edges of $C_4^{(m)}$ as follows.

$f(vv_{3i-2}) = 4i-3$ for i = 1 to m

$f(vv_{3i}) = 4m+4i-1$ for i = 1 to m

$f(v_{3i-1}v_{3i}) = 4i-1$ for i = 1 to m

$f(v_{3i-2}v_{3i-1}) = 4m+4i-3$ for i = 1 to m

Now the induced vertex labels are

$f^+(v) = 0$

$f^+(v_{3i}) = (f(vv_{3i}) + f(v_{3i-1}v_{3i})) \pmod{8m}$ for i= 1 to m

$f^+(v_{3i-1})=(f(v_{3i-2}v_{3i-1}) + f(v_{3i-1}v_{3i}))\pmod{8m}$ for i=1 to m

$f^+(v_{3i-2})=(f(vv_{3i-2})+ f(v_{3i-2}v_{3i-1})) \pmod{8m}$ for i= 1 to m

The edge labels are arranged in the following order

A={$f(vv_{3i-2})$/i = 1 to m}={4i – 3/i=1 to m }
= { 1,5, . . . , 4m-3 }

B={$f(vv_{3i})$/i=1 to m}={4m+4i -1/i=1 to m}
= {4m+3,4m+7, . . . , 8m-1 }

C={$f(v_{3i-1} v_{3i})$/i =1 to m}={4i–1/i=1 to m }
= { 3,7, . . . , 4m-1}

D ={$f(v_{3i-2} v_{3i-1})$ / i = 1 to m}={4m+4i – 3 / i=1 to m }
= {4m+1,4m+5, . . . , 8m-3}

Hence $f(E(C_4^{(m)}))$ = A∪ B ∪ C ∪ D
= { 1, 3, . . . , 8m-1}

And $f^+(V(C_4^{(m)}))$={$f^+(v_i)$, $f^+(v)$/i=1to 3m}

Therefore the set of edge labels and the vertex labels are distinct and the edge labels are odd.

So f is an Edge – Odd Graceful Labeling. Hence, $C_4^{(m)}$ is an Edge Odd Graceful graph.

Theorem: 2.4: If m is odd then $C_5^{(m)}$ is an edge – odd graceful graph.

Proof: Let the vertices of $C_5^{(m)}$ be {v, v_i / i= 1 to 4m} and

$E(C_5^{(m)})$ = { vv_{4i-3} ,vv_{4i} , $v_{4i-3}v_{4i-2}$, $v_{4i-2}v_{4i-1}$, $v_{4i-1}v_{4i}$ / i= 1 to m }, the edge set of $C_5^{(m)}$.

we first label the edges of $C_5^{(m)}$ as follows.

$f(vv_{4i-3}) = 10i-9$ for i = 1 to m

$f(vv_{4i}) = 10i-1$ for i = 1 to m

$f(v_{4i-3}v_{4i-2}) = 10i-7$ for i = 1 to m

$f(v_{4i-2}v_{4i-1}) = 10i-5$ for i = 1 to m

$f(v_{4i-1}v_{4i}) = 10i-3$ for i = 1 to m

Now the induced vertex labels are $f^+(v) = 0$

$f^+(v_{4i-3}) = 20i - 16 \pmod{10m}$ for i= 1 to m

$f^+(v_{4i-2}) = 20i - 12 \pmod{10m}$ for i=1 to m

$f^+(v_{4i-1}) = 20i - 8 \pmod{10m}$ for i= 1 to m

$f^+(v_{4i}) = 20i - 4 \pmod{10m}$ for i= 1 to m

The edge labels are arranged in the following order

A ={$f(vv_{4i-3})$/i =1 to m}={10i–9/i=1 to m }

= { 1,11, . . . , 10m-9 }
$B=\{f(v_{4i-3}v_{4i-2})/i=1 \text{ to } m\}=\{10i–7/i=1\text{to } m\}$
= {3,13, . . . , 10m-7}
$C=\{f(v_{4i-2}v_{4i-1})/i=1\text{to } m\}=\{10i–5/i=1\text{to } m\}$
= {5,15, . . . , 10m-5}
$D=\{f(v_{4i-1}v_{4i})/i=1\text{to } m\}=\{10i–3/i=1\text{to } m\}$
= { 7,17, . . . , 10m-3}
$K=\{f(vv_{4i})/i=1 \text{ to } m\}=\{10i – 1/ i=1 \text{ to } m\}$
= {9,19, . . . , 10m-1 }
Hence $f(E(C_5^{(m)})) = A\cup B \cup C \cup D\cup K$
= { 1, 3, . . . , 10m-1}
And $f^+(V(C_5^{(m)})) =\{f^+(v_i), f^+(v)/i=1\text{to } 4m\}$

Therefore the set of edge labels and the vertex labels are distinct and the edge labels are odd.
So f is an Edge – Odd Graceful Labeling. Hence, $C_5^{(m)}$ is an Edge Odd Graceful graph.

Theorem : 2.5: If both n and m are odd or both n and m are even then $C_n^{(m)}$ is an edge – odd graceful graph.

Proof: By above theorems(2.2),(2.3) and (2.4),
We conclude that, $C_n^{(m)}$ is an edge odd graceful graph if either both n and m are odd or both n and m are even.

Conclusion: Motivated by the definition of graceful labeling of the graphs[1], [5] introduced an edge – odd graceful labeling of the graph. In this paper, edge – odd graceful labeling for windmill graphs are discussed.

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