# DECISION MAKING WITH GLOBAL WARMING VIA NANO TOPOLOGY

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**Abstract:** The purpose of this article is that the concept of the nano topology can be used as the basis for decision making and approximate calculation of the data sets. This paper describes about the decision making on data about global warming for the classification purpose.

**Keywords**: Certainity factor, Coverage factor, Lower approximation, Nano topology,, Reducts, Upper approximation.
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**Introduction:** The concept of Nanotopology was originally proposed by Thivagar[6]. Its philosophy is based on the assumption that to every object of the universal set. We associate some information. Objects characterised by some information are indiscernible in view of the available information about them. The indis cernibility relation generated in this way is the mathematical basis of Nano topological theory. The nano topological approach to data analysis[3] has many important advantages such as finding minimal sets of data[5],evaluating significant data[9], generating sets of decision rules from data[10] offering straight forward interpretation of obtained results.

**Preliminaries:** The concept of Nano topology was originally proposed by Thivagar[6]. Its philosophy is based on the assumption that to every object of the universal set. We associate some information. Objects characterised by some information are indiscernible in view of the available information about them. The indiscernibility relation generated in this way is the mathematical basis of Nano topological theory. The nano topological approach to data analysis[3] has many important advantages such as finding minimal sets of data[5],evaluating significant data[9], generating sets of decision rules from data[10] offering straight forward interpretation of obtained results.

**Decision table:** In this section we are going to review the concepts of decision table and decision rules and also we introduce some definitions and theorem related to decision table.

**Definition 3.1:** Let S = (U,A) be an information system, where A is divided into a set C of condition attributes and a set D of decision attributes .Then S is called a decision table

**Definition3.2:** Let S = (U,A) be an information system and B $\subseteq$ A, then

(i) The pair (a,v) is called a formula on B,where a $\in$ B, v $\in V_\alpha$ and $\|(a,v)\|_s$ ={x$\in$ U: a(x) = v}.

(ii)Let $\phi=(a,v)$ and $\psi=(a',v')$ be two formulas , where $a,a'\in B$ and v $\in V_\alpha$ and $v'\in V_\alpha$ ,then $\phi\vee\psi,\phi\wedge\psi$ and u$\phi$ are formulas on B

(a) $\|(\phi\vee\psi)\|_s$ ={x$\in$ U: a(x) = v or $a'(x)=v'$ }.

(b) $\|(\phi\wedge\psi)\|_s$ ={x$\in$ U: a(x) = v and $a'(x)=v'$ }.

(c) $\|\phi\|_s$ ={x$\in$ U: a(x)$\neq$ v}.

The family of all formulas on B is denoted by For(B).

**Theorem 3.3:** Let S=(U,A) be an information system and B$\subseteq$ A. Then for any $\phi$ , $\psi$ $\in$ For (B), we have

1. $\|\phi\vee\psi\|_s=\|\phi\|_s\cup\|\psi\|_s$
2. $\|\phi\wedge\psi\|_s=\|\phi\|_s\cap\|\psi\|_s$
3. $\|u\phi\|_s=U-\|\phi\|_s$

**Proof:** Consider

(i) $\|(\phi\vee\psi)\|_s$ ={x$\in$ U: a(x) = v or $a'(x)=v'$ }
={x$\in$ U: a(x) = v } or {x$\in$ U: $a'(x)=v'$ }.
Hence $\|\phi\vee\psi\|_s=\|\phi\|_s\cup\|\psi\|_s$ .

(ii) $\|(\phi\wedge\psi)\|_s$ ={x$\in$ U: a(x) = v and $a'(x)=v'$ }, which is equal to {x$\in$ U: a(x) = v and {x$\in$ U: $a'(x)=v'$ }.

(iii) $\|u\phi\|_s$ = {x$\in$ U: a(x)$\neq$ v} is equal to {x$\in$ U: x $\notin$ $\|\phi\|_s$ }. Hence $U-\|\phi\|_s$ .

**Definition 3.4:** Let S =(U,C,D) be a decision table, $\phi\in$ For(C) and $\psi\in$ For (D). The expression if $\phi$ then $\psi$ is called a decision rule and is denoted by $\phi\to\psi$ .

**Definition 3.5:** Let S = (U,C,D) be a decision table and $\phi\to\psi$ a decision rule in S, The coefficient of certainity with respect to the rule is defined as

$$\rho(\phi,\psi)=\frac{card[\|\phi\wedge\psi\|_s]}{card\|\phi\|_s}.$$

**Definition 3.6:** Let S=(U,C,D) be a decision table and $\phi\to\psi$ be a decision rule in S, The coefficient of coverage is defined as ,

$$\mathrm{cov}(\phi,\psi)=\frac{card[\|\phi\wedge\psi\|_s]}{card[\|\psi\|_s]}$$

**Definition 3.7:** Let S= (U,C,D) be a decision table and $\phi\to\psi$ a decision rule in S,

1. If $\rho_s(\phi,\psi)=1,$ then $\phi\to\psi$ is called a certain rule.
2. If $0<$ $\rho_s(\phi,\psi)<1$ ,then $\phi\to\psi$ is called an uncertain rule.

**Theorem 3.8 :** Let S = (U,C,D) be a decision table and $\phi \rightarrow \psi$ a decision rule in S, then $\rho_s(\phi,\psi) = \text{cov}_s(\phi,\psi)$ iff $card\|\phi\|_s = card\|\psi\|_s$.

**Proof:** Neccessary: Suppose $\rho_s(\phi,\psi) = \text{cov}_s(\phi,\psi)$, then by definition we have

$$\frac{card[\|\phi \wedge \psi\|_s]}{card\|\phi\|_s} = \frac{card[\|\phi \wedge \psi\|_s]}{card[\|\psi\|_s]}$$

This means that $card\|\phi\|_s = card\|\psi\|_s$ Sufficient: Suppose that $card\|\phi\|_s = card\|\psi\|_s$ and assume that $\|\phi\|_s \neq \phi$ and $\|\psi\|_s \neq \phi$ .If they are empty then the rule $\phi \rightarrow \psi$ does not exist, therefore we have

$$\frac{card[\|\phi \wedge \psi\|_s]}{card\|\phi\|_s} = \frac{card[\|\phi \wedge \psi\|_s]}{card[\|\psi\|_s]}$$

Hence $\rho_s(\phi,\psi) = \text{cov}_s(\phi,\psi)$.

**Theorem: 3.9** Let S = (U,C,D) be a decision table and $\phi \rightarrow \psi$ a decision rule such that $\|\phi\|_s \subseteq \|\psi\|_s$ then $\text{cov}_s(\phi,\psi) \leq 1$.

**Proof:**

By assumption we have $\|\phi\|_s \subseteq \|\psi\|_s$ Therefore $\|\phi \wedge \psi\|_s = \|\phi\|_s$. It follows that $card[\|\phi \wedge \psi\|_s] = card[\|\phi\|_s]$ .This means $\frac{card[\|\phi \wedge \psi\|_s]}{card[\|\psi\|_s]} \leq 1$. Hence we have $\text{cov}_s(\phi,\psi) \leq 1$.

**Theorem: 3.10** Let S=(U,C,D) be a decision table $\phi_1, \phi_2 \in$ For(C) and $\psi \in$ For(D),then

(i) $\text{cov}_s(\phi_1 \wedge \phi_2, \psi) \leq \text{cov}_s(\phi_i, \psi)$ for i = 1,2.............

(ii) $\text{cov}_s(\phi_1 \vee \phi_2, \psi) \geq \text{cov}_s(\phi_i, \psi)$ for i = 1,2................

**Proof:**

We know that $\|\phi_1 \wedge \phi_2\|_s \subseteq \|\phi_i\|_s$ for i = 1,2,................n ..

$\|(\phi_1 \wedge \phi_2) \wedge \psi\|_s = \|\phi_1 \wedge \phi_2\|_s \cap \|\psi\|_s$

$\subseteq \|\phi_i\|_s \cap \|\psi\|_s$

$= \|\phi_i \wedge \psi\|_s$

$\text{cov}_s(\phi_1 \wedge \phi_2, \psi) \leq \text{cov}_s(\phi_i, \psi)$ for i = 1,2............n.

$\|\phi\|_i \subseteq \|\phi_1 \vee \phi_2\|_s$ for i = 1,2,.................n

$\|(\phi_1 \vee \phi_2) \wedge \psi\|_s = \|\phi_1 \vee \phi_2\|_s \cap \|\psi\|_s$

$\supseteq \|\phi_i\|_s \cap \|\psi\|_s$

$\text{cov}_s(\phi_1 \vee \phi_2, \psi) \geq \text{cov}_s(\phi_i, \psi)$ for i = 1,2...............n.

| Fact | Solar energy | Volcanic activity | $CO_2$ | Temperature |
|---|---|---|---|---|
| 1 | High | Active | Less | High |
| 2 | Medium | Active | Less | Low |
| 3 | Medium | Active | Less | High |
| 4 | Below average | Less active | Less | Low |
| 5 | Medium | Less active | More | Low |
| 6 | High | Less active | More | High |

**Example 3.11:** To illustrate, we consider a simple case of information table representing the factors of Global warming that have attributes Solar energy ,Volcanic activity and $CO_2$ .
Here U= {1,2,3,4,5,6} the set of facts. The columns of the table represents the environmental factors of global warming and the rows represents the individual facts. The entries in the table are the attribute values. Solar energy,Volcanic activity, $CO_2$ are the condition attributes.

**4. Attribute Reduction:** In this section we are finding the reducts to frame the decision rules.
The equivalence classes for the set of condition attributes are {{1}, {2,3}, {4}, {5} ,{6}}. Not all condition attributes in an information system are necessary to depict the decision attribute before decision rules are generated and hence we are interested to find the reduct of this information system.

**Algorithm:** Given a finite universe U, a finite set A of attributes, where A= C ∪ D, C-condition attributes and D-decision attributes, an equivalence relation R on U corresponding to C and a subset X of U, represent the data as an information table, columns of attributes and rows with objects.

1) Find the lower approximation, upper approximation and the boundary region of X with respect to C.
2) Generate the nano topology $\tau_C(X)$ on U and its basis $\beta_C(X)$ corresponding to C.

3) Remove an attribute X from C .
4) Generate the nano topology $\tau_C(X)$ on U and its basis $\beta_C(X)$.
5) Repeat steps (3) and (4) for all attributes in C.
6) Those attributes in C for which $\beta_{C(x)}(X) = \beta_C(X)$ forms the reduct.

**Case(i) : High Temperature**

Let X={1,3,6},when. temperature is high. Let R be the equivalence relation on U with respect to the condition attributes. The family of equivalence classes determined by R and corresponding to C is givenby U/R(C)={{1},{2,3},{6},{4},{5}}. The upper and lower approximation of X with respect to R are given by $L_C(X)=\{1,6\}, U_C(X)=\{1,2,3,6\}$. Therefore the nano topology on U with respect R are given by $\tau_C(X)=$ {U,$\phi$,{1,6},{1,2,3,6},{2,3}}. The basis for this topology is given by $\beta_C(X)=$ {U,$\phi$,{1,6},{2,3}}.

**Step1:** If we remove the attribute solar energy from the set of condition attributes then the family of equivalence classes corresponding to the resulting set of attributes is given by

U/R(C-{S})={{1,2,3},{4},{5,6}}

**Step2:** Therefore $\tau_{C-\{S\}}(X)=$ {U,$\phi$,{1,2,3,5,6}}.

$\beta_{C-\{S\}}(X)=$ {U,$\phi$,{1,2,3,5,6}} $\neq \beta_C(X)$.

**Step3:** If we remove the volcanic activity from the set of condition attributes then the family of equivalence classes corresponding to the resulting set of attributes is given by

U/R(C-{V}) = {{1},{2,3},{4},{5},{6}}.

**Step4:** Then the $\tau_{C-\{V\}}(X)=\{U,\phi$,{1,6},{2,3}} and

$\beta_{C-\{V\}}(X)=\{U,\phi,\{1,6\},\{2,3\}\}=\beta_C(X)$.

If we remove the attribute $CO_2$ from the set of attributes then the family of equivalence classes corresponding to the resulting attributes is given by

$U/R(C$ -$O_2$})={{1},{2,3},{4},{5},{6}} and then $\tau_{C-\{CO_2\}}(X)=\{U,\phi,$ {1,6}, {1,2,3,6}, {2,3}} and its basis is given by

$\beta_{C-\{CO_2\}}(X)=\{U,\phi,$ {1,6},{2,3}} $=\beta_C(X)$

**Case2:** X={2,4,5} ,when temperature is low. Then U/R(C)={{1},{2,3},{4},{5}, {6}}. Therefore the nano topology on U with respect to X is given by $\tau_C(X)=\{U,\phi$,{4,5},{2,3,4,5},{2,3}} and its basis is given by $\beta_C(X)=\{U,\phi$,{4,5},{2,3}}. If we remove the attribute solar energy from the set of condition attributes the family of equivalence classes corresponding to the resulting set of attributes is given by U/R(C-{S}) ={{1,2,3}, {4}, {5,6}}.

$\beta_{C-\{s\}}(X)=\{U,\phi,$ {4},{1,2,3,5,6}} $\neq \beta_C(X)$. If we remove the attribute volcanic activity from the set of condition attributes.

U/R(C-{V})={{1},{2,3},{4},{5},{6}}.

Therefore $\tau_{C-\{v\}}(X)=\{U,\phi$,{4,5},{2,3,4, 5},{2,3}} and its basis is given by

$\beta_{C-\{v\}}(X)=\{U,\phi$,{4,5},{2,3}} = $\beta_C(X)$. If we remove the attribute $CO_2$ from the set of condition attributesU/R[C-{$CO_2$}]={{1},{2,3},{4},{5},{6}}.

Therefore $\tau_{C-\{Co_2\}}(X)=$ {U,$\phi$, {4,5},{2,3,4,5}{2,3}

and its basis is given by

$\beta_{C-\{CO_2\}}(X)=\{U,\phi,$ {4,5},{2,3}}= $\beta_C(X)$. Since $\beta_{C-\{V\}}=\beta_C$, $\beta_{C-\{CO_2\}}=\beta_C$ and C-{$CO_2$}={S,V} and C-{V}={S, $CO_2$} are the two reducts.

**5. Decision Rule:**

(i) If (Solar energy, high) $\longrightarrow$(temp, high)

(ii) If (Solar energy, medium) $\wedge$(volcanic activity, active) $\longrightarrow$(temp, high)

(iii) If (Solar energy, below average) or (Solar energy, medium)$\wedge$(volcanic activity, less active)$\longrightarrow$ (temp, low).

(iv) If (Solar energy, medium) $\wedge$ (volcanic activity, active)$\longrightarrow$(temp, low)

**Decision Rules and Approximations:**

There is an interesting relationship between decision rules and approximations certain rules describe the lower approximations of the set of facts pointed out by the conclusion of the rule where as the uncertain decision rules describe the boundary region of set of facts pointed out by the conclusion of the rule.

Certain rules describing temperature high [The lower approximations of the set of facts{1,6}] as follows:

1. If [Solar energy, high] then [temperature, high].
2. Uncertain rules describing temperature high [The boundary region {2,3} of the set of facts {1,3,6}]
3. If [Solar energy, medium] $\wedge$[Volcanic activity, active] then [temp, high].
4. Certain rules describing temperature low [lower approximation of the set of facts]{4,5}:
5. If [Solar energy, below average] or [Solar energy, medium]$\wedge$[volcanic activity, less active] then [temperature, low]
6. Uncertain rule S describing temperature low (the boundary region {2,3} of the set of facts {2,4,5})
7. If [Solar energy, medium]$\wedge$ [volcanic activity, active] then [temperature, low].
8. Thus certain and uncertain rules regarding the temperature high represents the basic of nano topology with respect to X={1,3,6} temp high,which is given by $\beta_C(X_H)=\{U,\phi,$ {1,6},{2,3}}

**6. Inverse Decision Rule:**

1. If [temperature, high] then [Solar energy, high].
2. If [temperature, high] then (Solar energy, medium) and [Volcanic activity, active].
3. If [temperature, low] then (solar energy, below

average) or (solar energy, medium) and (volcanic activity, less active)
4. If [temperature, low] then (Solar energy, medium) and (volcanic activity, active).

**Certainty and Coverage factors:**

| Rule | Certainity | Coverage |
|---|---|---|
| 1 | 1 | 0.66 |
| 2 | 0.5 | 0.33 |
| 3 | 1 | 0.66 |
| 4 | 0.5 | 0.33 |

**7. What the data tell us:**

The certainty factor can be interpreted as a degree of truth of the decision rule. On the contrary, the coverage factor can be viewed as a degree of truth of the "inverted decision rule". From the certainty factors of decision rules, we can draw the following conclusion of global warming are as follows :

1. If solar energy is high,then certainty temperature is high.
2. If solar energy is medium and volcanic is active then the probability temperature is high equals to 0.5.
3. If solar energy is below average or solar energy is medium and volcanic is less active then certainty and the temperature is low.
4. If solar energy is medium and volcanic is active then the probability temperature is low is equal to 0.5.

**Remark7.1:** For the decision rules, the certainty factor and coverage factor can be mutually exchanged.

The coverage factors of decision rule leads us to the following explanation of global warming as follows:

1. If the temperature is high, then the probability of the solar energy is high is equal to 0.67
2. If the temperature is high, then the probability of the solar energy is medium and the volcanic is active is equal to 0.33
3. If the temperature is low, then the probability that the solar energy is below average or medium and the less active volcanic is equal to 0.66
4. If the temperature is low, then the probability of the solar energy is **medium** and active volcanic is equal to 0.33.

Summing up, from the data we can conclude that

1. **Medium** solar energy and high volcanic activity (or) high solar energy certainty cause **high** temperature.
2. Medium solar energy and low volcanic activity cause

- **High** temperature with probability (0.5)
- **Low** temperature with probability(0.5)

The reason for high temperature

- **High** solar energy with probability(0.67)
- Solar energy medium and Volcanic **active** with probability is equal to 0.33.

The reason for low temperature

- Solar energy below averge or **medium** and volcanic is less active with probability is equal 0.66
- Solar energy medium and volcanic is **active** with probability is equal to 0.33

**Observation:**

- The most probable reason for high temperature is **high** solar energy.
- The most probable reason for low temperature is **low** solar energy.

**Conclusion:** In the present study the theory of nano topology is applied to evaluate the importance of attributes , learn common decision making rules, reduce all redundant objects and attributes so as to attain satisfactory classification. It is concluded that the decision rules based on the reducts not only provide a new global insight but also are useful for the experts to analyse the problem effectively.

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