STATUS SEQUENCE AND DETOUR STATUS SEQUENCE OF TREES

DR.K.CHITHRA, P.BACKIALAKSHMI

Abstract: Let G=(V,E) be a simple undirected connected graph. The status of a vertex $v \in V$ of a graph G is the sum of shortest distance from v to the other vertices of G. The status sequence of G is the list of its status value in non-decreasing order. The detour status of a vertex $v \in V$ of a graph G is the sum of longest distance from v to the other vertices of G. The detour status sequence of G is the list of its detour status values in non-decreasing order. In this paper a status sequence (SS) and detour status sequence (DSS) of trees are investigated.

Keywords: detour status sequence. status sequence, detour status values, status values.

Introduction: Let G=(V,E) be a simple undirected connected graph. For vertices u and v in a connected graph G, the distance d(u,v) is the length of a shortest u-v path in G. A u-v path of length d(u,v) is called u-v geodesic. The status of a vertex of a graph G is the sum of the shortest distances from v to the other vertices of G. The status sequence(SS) of G is the list of its status value in non-decreasing order. The length of a longest u-v path between two vertices u and v in G is called a detour distance D(u,v) between u and v. A u-v path of length D(u,v) is a u-v detour. The detour status of a vertex v□V of a graph G is the sum of longest distance from v to the other vertices of G. The detour status sequence (DSS) of G is the list of its detour status values in non-decreasing order. In this paper, a status sequence (SS) and detour status sequence (DSS) of trees are investigated with some examples.

2. Main Results:

Theorem:2.1: The status sequence of a star graph $SS(k_{1,n}) = \{SS(u_0) = n, SS(u_j) = 2n-1 \text{ for } j=1 \text{ to } n-1\}$and Detour status sequence of a star graph is $DSS(k_{1,n}) = \{SS(u_0) = n, SS(u_j) = 2n-1 \text{ for } j=1 \text{ to } n-1\}$

Proof: Let $V(K_{1,n}) = \{u_0,u_1,u_2,\ldots\ldots u_{n-1}\}$ be the vertex set of $K_{1,n}$.
and Let $E(K_{1,n}) = \{u_0u_j ; 1 \leq j \leq n-1\}$, the edge set $K_{1,n}$.
If n=3, the SS & DSS of $K_{1,3}$ is
$SS(k_{1,3})=\{3,5,5,5\}$and $DSS(K_{1,3})= \{3,5,5,5\}$The result is true for k=3.
By mathematical induction,Let us assume that the result is true for k= n & n ≥3.
To prove that, this result is true for k=n+1
The SS & DSS of $K_{1,n+1}$ is defined by
$SS(K_{1,n+1}) = \{SS(u_0) = n+1, SS(u_j) = 2n+1 \text{ for } j=1 \text{ to } n\}$
$DSS(K_{1,n+1}) = \{SS(u_0) = n+1, SS(u_j) = 2n+1$
for j=1 to n$\}$
So, the result is true for all k≥3Therefore the SS & DSS of $k_{1,n}$ is given by
$SS(k_{1,n}) = \{SS(u_0) = n, SS(u_j) = 2n-1 \text{for } j=1 \text{ to } n-1\}$
$DSS(k_{1,n}) = \{SS(u_0) = n, SS(u_j) = 2n-1 \text{ for } j=1 \text{ to } n-1\}$

Theorem:2.2: The status sequence of a bistar graph $SS(B(n,n))=\{u_i=5n+1, i=1\text{to}2n; u_j=3n+1, j=2n+1,2n+2\}$& Detour status sequence of a bistar graph is $DSS(B(n,n))=\{u_i=5n+1,i=1\text{to}2n; u_j=3n+1;j=2n+1,2n=2\}$

Proof: Let $V(B(n,n)) = \{u_i ; i=1 \text{ to } 2n+2\}$ be the vertex set of B(n,n).
and Let $E(B(n,n)) = \{u_iu_{2n+1}, u_ju_{2n+2}, u_{2n+1}u_{2n+2} \text{ for } i, j =1 \text{ to } n\}$, the edge set of B(n,n)
If n=3, the SS & DSS of B(3,3).
SS(B(3,3))={10,10,16,16,16,16,16,16 }
DSS(B(3,3))= {10,10,16,16,16,16,16,16}
This result is true for k=3.
By mathematical induction,Let us assume that the result is true for k= n & n ≥3.
To prove that this result is true for k=n+1
The SS & DSS of B(n+1,n+1) is
$SS(B(n+1,n+1)=\{u_j=3n+4 \text{ for } 2n+1,2n+2; u_i=5n+6$
for i=1 to 2n$\}$ and
$DSS(B(n+1,n+1)=\{u_j=3n+4 \text{ for } j=2n+1,2n+2; u_i=5n+6$
for i=1 to 2n$\}$
So, the result is true for all k≥3
Therefore the SS & DSS of B(n,n) is given by
$SS(B(n,n)) =\{u_j=3n+1\text{for } j=2n+1,2n+2 ; u_i=5n+1$
for i=1 to 2n$\}$
$DSS(B(n,n)=\{u_j=3n+1 \text{ for } j=2n+1,2n+2; u_i=5n+1$
for i=1 to 2n$\}$

Theorem:2.3: The status sequence of a path P_n is
$SS(P_n) = \{u_k = u_{n-k+1}=n(n-\lceil n/2\rceil)-k(n-(k+1)),$
$u_{\lceil n/2\rceil} =\lceil n/2\rceil(n-\lceil n/2\rceil)\}$
for $k=1,2,3,\ldots\lfloor n/2\rfloor$,n≥3 & n is odd and $SS(P_n)=[n(n-1)/2]-k(n-(k+1))$ for k=0,1,2,3,..[n-1/2], n≥2 & n is even and the Detour status sequence
$DSS(P_n)=\{u_k=u_{n-k+1}=n(n-\lceil n/2\rceil)-k(n-k-1),$
$u_{\lceil n/2\rceil}=\lceil n/2\rceil(n-\lceil n/2\rceil)\}$ for $k=1,2,3,\ldots\lfloor n/2\rfloor$, n≥3& n is odd and
$DSS(P_n)=\{[n(n-1)/2]-k(n-(k+1))\}$
for k=0,1,2,3,..[n+1/2], n≥2 & n is even

Proof: Let $V(P_n) = \{u_i ; i=1 \text{ to } n\}$ be the vertex set of P_n and Let $E(P_n)=\{u_iu_{i+1} ; i=1\text{to } n-1\}$, the edge set of P_n.
case(i) If n is odd
If n=3, the SS & DSS of P_3.
$SS(P_3)=\{3,2,3\}DSS(P_3)= \{3,2,3\}$
This result is true for k=3.
By mathematical induction,Let us assume that the result is true for k= n & n ≥3.when n is odd
Clearly the result is true for k=n+2 by using our assumptions.
So,the result is true for all k≥3 and n is oddTherefore the SS & DSS of P_n is given by
$SS(P_n) =\{u_k=u_{n-k+1}=n(n-\lceil n/2\rceil)-k(n-(k+1)); k=1,2,3,\ldots\lfloor n/2\rfloor,$
n≥3 &n is odd$\}$
$DSS(P_n)=\{u_k=u_{n-k+1}=n(n-\lceil n/2\rceil)-k(n-(k+1)); k=1,2,3,\ldots\lfloor n/2\rfloor,$
n≥3 &n is odd$\}$

case(ii) if n is even

If n=2, the SS & DSS of P_2.

$SS(P_2)=\{1,1\}$

$DSS(P_2)=\{1,1\}$

This result is true for k=2.

By mathematical induction,Let us assume that the result is true for k= n & n ≥2.when n is even

Clearly the result is true for k=n+2 by using our assumptions.

So, the result is true for all k≥2 and n is evenTherefore the SS & DSS of P_n is given by

$SS(P_n)=\{(n(n-1)/2)-k(n-(k+1))\}$; k=0,1,2,3,..[(n+1)/2], n≥2 & n is even}

$DSS(P_n)=\{(n(n-1)/2)-k(n-(k+1))\}$; k=0,1,2,3,..[(n+1)/2], n≥2 & n is even}

Observation:2.4: Status sequence(SS) and detour status sequence(DSS) of trees are same.

Conclusion: Motivated by the definition of Status sequence and detour status sequence of a graph, we have discussed the SS and DSS of trees. The Problem of SS and DSS of connected graphs are under investigation.

References:

1. G. Chartrand, F. Harary, Distance in Graphs, Addison Wesley, (1990).
2. Indra Rajasingh, Bharathirajan,S.Prabhu, Detour Distance Sequence,ICMCS,25-26,july 2008, pp.276-280.
3. Indra Rajasingh, Bharathirajan,S. Prabhu, Maximum Distance Matrix of the Complete Bipartite Graph and its Distance Sequence, ICMCS,5- 6,Janauary 2009, pp.148-152.
4. F. Harary, Graph Theory, Addision Wesley,Publishing Company,1969.
5. E.Sampath kumar,V.Swaminathan, P.Vishvanathan , G.Prabhakaran, Detour Graphs,ICMCS2007,March1-3, pp.247-248
6. Vijayan and T. Binuselin, An introduction to Geodesic Polynomial of a graph, Bulletin of Pure and applied Sciences,Vol.31E,No.1(2012),pp.25-32.

* * *

Dr.K.Chithra/ Head & Assistant Professor of Mathematics(SF)/ Nehru Memorial College/ Puthanampatti, Trichy Dt /rasaimailsu@yahoo.co.in

P.Backialakshmi/ Assistant Professor of Mathematics/ Nehru Memorial College/ Puthanampatti/ Trichy Dt /bakya21@yahoo.com)