STUDY OF (Λ, μ-α)-CLOSED SETS

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Abstract: In this paper, we introduce and study of $\Lambda_{\mu\text{-}\alpha}$ sets and (Λ, μ-α)-closed sets via μ-α-open sets and μ-α-closed sets in generalized topological spaces. Moreover we introduce and characterize some new low separation axioms in generalized topological spaces.

Keywords: $\Lambda_{\mu\text{-}\alpha}$ sets and (Λ, μ-α)-closed sets.

Introduction: In the past few years, different forms of open sets have been studied. Recently, a significant contribution to the theory of generalized open sets was extended by A. Császár. After introduced the notion of μ-α-open sets and μ-α-closed sets in generalized topological spaces, several research papers with interesting results in different respects came to light. In this paper, we define and study some new sets using the notion of μ-α-open sets and μ-α-closed sets.

Throughout the present paper, (X, μ) and (Y, λ) (or X and Y) denote generalized topological spaces in which no separation axiom are assumed unless explicitly stated. Let A be a subset of (X, μ) is called μ-α-open [3] if $A \subseteq i_\mu(c_\mu(i_\mu(A)))$, where $c_\mu(A)$ and $i_\mu(A)$ denote the closure and interior of A respectively. The complement of a μ-α-open set is called μ-α-closed. By $\mu\alpha O(X)$ (resp.$\mu\alpha C(X)$), we denote the family of μ-α-open (resp. μ-α-closed) sets of X. The intersection of all μ-α-closed sets containing A is called μ-α-closure of and is denoted by $\alpha c_\mu(A)$. The μ-α-interior of A is the union of μ-α-open contained in A and is denoted by $\alpha i_\mu(A)$. In this paper, we introduce and investigate (Λ, μ-α)-closed sets.

2. Preliminaries:

Definition 2.1 [4]: Let X be a nonempty set and μ be a collection of subsets of X. Then μ is called a generalized topology (briefly GT) on X if $\phi \in \mu$ and $G_i \in \mu$ for $i \in I \neq \phi$ implies $G = \bigcup_{i\in I} G_i \in \mu$. We say μ is strong if $X \in \mu$, and we call the pair (X, μ) a generalized topological space (briefly GTS) on X.

The elements of μ are called μ-open sets and their complements are called μ-closed sets. The generalized closure of a subset A of X, denoted by $c_\mu(A)$, is the intersection of μ-closed sets including A. And the interior of A, denoted by $i_\mu(A)$, is the union of μ-open sets contained in A.

Definition 2.2 [10]: Let A be a subset of a GTS (X, μ). A subset $\Lambda_\mu(A)$ is defined as follows $\Lambda_\mu(A) = \begin{cases} \cap\{G: G \in \mu, A \subset G\}, & if\ \exists\ G \in \mu\ such\ that\ A \subset G, \\ X & otherwise \end{cases}$

Lemma: 2.3. [10]: For subsets A, B and A_i ($i \in I$) of a GTS (X, μ), the following properties hold:

(i). $A \subset \Lambda_\mu(A)$;

(ii). If $A \subset B$, then $\Lambda_\mu(A) \subset \Lambda_\mu(B)$;

(iii). $\Lambda_\mu(\Lambda_\mu(A)) = \Lambda_\mu(A)$;

(iv). $\Lambda_\mu(\cap\{A_i: i\in I\}) \subset \cap\{\Lambda_\mu(A_i): i \in I\}$;

(v). $\Lambda_\mu(\cup\{A_i: i\in I\}) = \bigcup\{\Lambda_\mu(A_i): i \in I\}$.

Definition: 2.4 [10]: A subset A of a GTS (X, μ), is called a Λ_μ-set if $\Lambda_\mu(A) = A$.

Lemma: 2.5 [10]: For subsets A and A_i ($i \in I$) of a GTS (X, μ), the following properties hold:

(i). $\Lambda_\mu(A)$ is a Λ_μ set;

(ii). If A is μ-open, then A is Λ_μ- set;

(iii). If A_i is a Λ_μ- set for each $i \in I$, then $\cap_{i\in I} A_i$ is a Λ_μ-set;

(iv). If A_i is a Λ_μ- set for each $i \in I$, then $\bigcup_{i\in I} A_i$ is a Λ_μ-set.

Definition: 2.6 [10]: A subset A of a GTS (X, μ), is called a (Λ, μ)-closed set if $A = T \cap C$, where T is a Λ_μ-set and C is a μ-closed set. The complement of a (Λ, μ)-closed set is called (Λ, μ)-open set.

We shall denote the collection of all (Λ, μ)-open sets (resp. (Λ, μ)-closed sets) by Λ_μ-O(X) (resp. Λ_μ-C(X))

Theorem: 2.7 [10]: Let A be a (Λ, μ)-closed subset of a GTS (X, μ). Then, we have

i). $A = T \cap c_\mu(A)$, where T is a Λ_μ-set;

ii). $A = \Lambda_\mu(A) \cap c_\mu(A)$.

Lemma: 2.8 [10]: For a GTS (X, μ),

(i). Every μ-closed set is (Λ, μ)-closed;

(ii). Every Λ_μ-closed set is (Λ, μ)-closed;

(iii). Λ_μ-C(X) (resp. Λ_μ-O(X)) is closed under arbitrary intersection (resp. union).

3. (Λ, μ-α)-closed sets:

Definition: 3.1: Let A be a subset of a generalized topological space (X, μ). A sub set $\Lambda_{\mu\text{-}\alpha}(A)$ is defined as follows$\Lambda_{\mu-\alpha}(A) = \begin{cases} \cap\{B \in \alpha(\mu): B \subset A\}, if\ \exists\ B \in \alpha(\mu)\ and\ B \subset A \\ X, \quad Otherwise \end{cases}$

Lemma: 3.2: For subsets A, B and A_i ($i \in I$) of a GTS (X, μ), the following properties hold:

(i). $A \subset \Lambda_{\mu\text{-}\alpha}(A)$;

(ii). If $A \subset B$, then $\Lambda_{\mu\text{-}\alpha}(A) \subset \Lambda_{\mu\text{-}\alpha}(B)$;

(iii). $\Lambda_{\mu\text{-}\alpha}(\Lambda_{\mu\text{-}\alpha}(A)) = \Lambda_{\mu\text{-}\alpha}(A)$;

(iv). $\Lambda_{\mu\text{-}\alpha}(\cap\{A_i : i \in I\}) \subset \cap\{\Lambda_{\mu\text{-}\alpha}(A_i): i \in I\}$;

(v). $\Lambda_{\mu\text{-}\alpha}(\bigcup\{A_i : i \in I\}) = \bigcup\{\Lambda_{\mu\text{-}\alpha}(A_i): i \in I\}$.

Proof: We prove only the statement (iv) and (v)

(iv). Suppose $x \notin \cap\{\Lambda_{\mu\text{-}\alpha}(A_i): i \in I\}$. There exist $i_o \in I$ such that $x \notin \Lambda_{\mu\text{-}\alpha}(A_{io})$ and there exist an μ-α-open set B such that $x \notin B$ and $A_{io} \subset B$, we have $\cap_{i\in I} A_i \subset A_{Io} \subset B$ and $x \notin B$. Therefore, $x \notin \Lambda_{\mu\text{-}\alpha}(\cap\{A_i : i\in I\})$.

(v) First $A_i \subset \Lambda_{\mu\text{-}\alpha}(A_i) \subset \Lambda_{\mu\text{-}\alpha}(\cup_{i\in I}\Lambda_{\mu\text{-}\alpha}(A_i))$ and hence $\Lambda_{\mu\text{-}\alpha}(A_i) \subset \Lambda_{\mu\text{-}\alpha}(\cup_{i\in I}(A_i))$. Therefore, we obtain $\cup_{i\in I}\Lambda_{\mu\text{-}\alpha}(A_i) \subset \Lambda_{\mu\text{-}\alpha}(\cup_{i\in I}(A_i))$. Conversely, suppose that $x \notin \cup_{i\in I}\Lambda_{\mu\text{-}\alpha}(A_i)$. Then $x \notin \Lambda_{\mu\text{-}\alpha}(A_i)$ for each $i \in I$ and hence there exists $B_i \in \mu\alpha O(X)$ such that $A_i \subset B_i$ and $x \notin B_i$ for each i ∈ I. We have $\cup_{i\in I}A_i \subset \cup_{i\in I}B_i$ and $\cup_{i\in I}B_i$ is an μ-α-open set which does not contain x. Therefore x ∉ $\Lambda_{\mu\text{-}\alpha}(U_{i\in I}(A_i))$. This show $\Lambda_{\mu\text{-}\alpha}(U_{i\in I}(A_i)) \subset U_{i\in I}\Lambda_{\mu\text{-}\alpha}(A_i)$.

Remark: 3.3: In lemma 3.2 (iv), he converse is not always true as the following example shows

Example:3.4: Let X = {a, b, c, d}, and μ = {φ, {a}, {a, b}, {a, b, c}}. Now put A = {c} and B = {d},
$\Lambda_{\mu\text{-}\alpha}(A \cap B) = \Lambda_{\mu\text{-}\alpha}(\varphi) = \varphi$, $\Lambda_{\mu\text{-}\alpha}(A) \cap \Lambda_{\mu\text{-}\alpha}(B)$ = {a, c} ∩ X = {a, c}.

Definition: 3.5: A subset A of a GTS (X, μ), is called a $\Lambda_{\mu\text{-}\alpha}$-set if $\Lambda_{\mu\text{-}\alpha}(A) = A$.

Lemma: 3.6: For subsets A and A_i (i ∈ I) of a GTS (X, μ), the following properties hold:
(i). $\Lambda_{\mu\text{-}\alpha}(A)$ is a $\Lambda_{\mu\text{-}\alpha}$- set;
(ii). If A is μ-α-open, then A is $\Lambda_{\mu\text{-}\alpha}$- set;
(iii). If A_i is a $\Lambda_{\mu\text{-}\alpha}$- set for each i ∈ I, then $\cap_{i\in I} A_i$ is a $\Lambda_{\mu\text{-}\alpha}$-set;
(iv). If A_i is a $\Lambda_{\mu\text{-}\alpha}$- set for each i ∈ I, then $\cup_{i\in I} A_i$ is a $\Lambda_{\mu\text{-}\alpha}$-set.

Definition: 3.7: A subset A of a GTS (X, μ), is called a (Λ, μ-α)-closed set if A = T ∩ C, where T is a $\Lambda_{\mu\text{-}\alpha}$-set and C is a μ-α-closed set. The complement of a (Λ, μ-α)-closed set is called (Λ, μ-α)-open set.

We shall denote the collection of all (Λ, μ-α)-open sets (resp. (Λ, μ-α)-closed sets) by $\Lambda_{\mu\text{-}\alpha}$-O(X) (resp. $\Lambda_{\mu\text{-}\alpha}$-C(X)).

Theorem: 3.8: Let A be a (Λ, μ-α)-closed subset of a GTS (X, μ). Then, we have,
(i). A is a (Λ, μ-α)-closed set;
(ii). A = T ∩ $\alpha c_\mu(A)$, where T is a $\Lambda_{\mu\text{-}\alpha}$-set;
(iii). A = $\Lambda_{\mu\text{-}\alpha}(A) \cap \alpha c_\mu(A)$.

Proof:
(i) ⟹ (ii): Let A=T ∩ F, where T is a $\Lambda_{\mu\text{-}\alpha}$-set and F is a μ-α-closed set in (X, μ). Since A ⊂ F, $\alpha c_\mu(A) \subset \alpha c_\mu(F)$ = F. Thus A = T ∩ F ⊇ T ∩ $\alpha c_\mu(A)$ = A. Therefore we have A = T ∩ $\alpha c_\mu(A)$.
(ii) ⟹ (iii): Let A = T ∩ $\alpha c_\mu(A)$, where T is a $\Lambda_{\mu\text{-}\alpha}$-set. Since A ⊂ T, then we have $\Lambda_{\mu\text{-}\alpha}(A) \subset \Lambda_{\mu\text{-}\alpha}(T)$ = T and hence A ⊂ $\Lambda_{\mu\text{-}\alpha}(A) \cap \alpha c_\mu(A) \subset T \cap \alpha c_\mu(A)$ =A. Therefore A = $\Lambda_{\mu\text{-}\alpha}(A) \cap \alpha c_\mu(A)$.
(iii) ⟹ (i); by observation 3.7 (i), $\Lambda_{\mu\text{-}\alpha}(A)$ is a $\Lambda_{\mu\text{-}\alpha}$- set and $\alpha c_\mu(A)$ is μ-α-closed. By (iii),
A = $\Lambda_{\mu\text{-}\alpha}(A) \cap \alpha c_\mu(A)$ and hence by definition 3.8, A is a (Λ, μ-α)-closed set.

Observation 3.9: For a GTS (X, μ)
(i). Every $\Lambda_{\mu\text{-}\alpha}$-set (every μ-α-closed set) is a (Λ, μ-α)-closed set.
(ii). $\Lambda_{\mu\text{-}\alpha}$-O(X) (resp. $\Lambda_{\mu\text{-}\alpha}$-C(X)) is closed under arbitrary union (resp. intersection).

Example: 3.10: (i). Let X = {a, c, b, c, d} and μ = {φ, {a}, {a, b}, {a, b, c}}. Then (X, μ) is a GTS. It is easy to see that {b, c} is a (Λ, μ-α)-closed set but neither μ-α-closed set nor $\Lambda_{\mu\text{-}\alpha}$-set.
(ii). Let X = {a, b, c} and μ = {φ, {a}, {a, c}}. Then (X, μ) is a GTS. In view of theorem 3.9, it is easy to see that A = {a} and B = {b} are two (Λ, μ-α)-closed subsets of (X, μ) but A ∪ B = {a, b} is not a (Λ, μ-α)-closed set in (X, μ).

Definition: 3.11: A GTS (X, μ) is said to be
1. μ-α-T_0 if for any pair of distinct point in X there exists a μ-α-open set containing one of the points not the other;
2. μ-α-$T_{1/2}$ if for each x ∈ X {x} is eitherμ-α-open or μ-α-closed;
3. μ-α-T_1 if for each pair of distinct points x and y of X, there exist μ-α-open set U_x containing x but not y and μ-α-open set U_y containing y but not x;
4. μ-α-R_0 if for each μ-α-open set U and each x ∈ U, $\alpha c_\mu(\{x\}) \subseteq$ U.

Theorem: 3.12: For a GTS (X, μ), the following conditions are equivalent:
(i). X is a μ-α-T_o;
(ii). Every singleton is (Λ, μ-α)-closed.

Proof: (i) ⟹ (ii): Let x ∈ X. Since X is μ-α-T_0, then for every point x ≠ y there exists a set A_y containing x and is disjoint from {y} such that A_y is either μ-α-open or μ-α-closed. Let L be intersection of all μ-α-open sets A_y and F be intersection of all μ-α-closed sets A_y. Clearly L is a $\Lambda_{\mu\text{-}\alpha}$-set and F is μ-α-closed set. Note that {x} = L ∩ F. This shows that {x} is a (Λ, μ-α)-closed set.
(ii)⟹(i): Let x and y be two different point of X. Then by (ii), {x} = L ∩ F, where L is $\Lambda_{\mu\text{-}\alpha}$-set and F is μ-α-closed set. If F does not contain y, then X – F is a μ-α-open set containing y and we are done. If F contains y, then y ∉ L and thus for some μ-α-open set U containing L, we have y ∉ U. Hence X is a μ-α-T_o.

Theorem: 3.13
For a GTS (X, μ), the following conditions are equivalent:
(i). X is a μ-α-$T_{1/2}$'
(ii). Every subset of X is (Λ, μ-α)-closed.

Proof: (i) ⟹ (ii): Let A⊂ X. Let A_1 be the set of all μ-α-open singleton of X – A and
A_2 = X – (AUA_1). Let F = $\cap_{x\in A1}$\{X – {x}} and L = $\cap_{x\in A2}$\{X – {x}}. Note that F is μ-α-closed and L is $\Lambda_{\mu\text{-}\alpha}$-set. Moreover, A = F ∩ L. Thus A is (Λ, μ-α)-closed.
(ii) ⟹ (i): Let x □ X. Assume that {x} is not μ-α-open. Then X –{x} is not μ-α-closed and since A is (Λ, μ-α)-closed A = T ∩ F, where T is a $\Lambda_{\mu\text{-}\alpha}$ - set and F is μ-α-closed set. Then the only possibility for F = X and T = X–{x}, then A is a $\Lambda_{\mu\text{-}\alpha}$-set, i.e., A = $\Lambda_{\mu\text{-}\alpha}(A)$. Since X is the only superset of A, then A is μ-α-open. Hence {x} is μ-α-closed.

Definition: 3.14: A GTS (X, μ) is said to be weak μ-α-R_0 if every (Λ, μ-α)-closed singleton is a $\Lambda_{\mu\text{-}\alpha}$-set.

Theorem: 3.15: Every μ-α-R_0 GTS (X, μ) is a weak μ-α-R_0.

Proof: Suppose that (X, μ) is a μ-α-R_0 GTS. Let x □ X with {x} = L ∩ F, where L is a $\Lambda_{\mu\text{-}\alpha}$- set and F is μ-α-closed set. Let y □ $\Lambda_{\mu\text{-}\alpha}(\{x\})$ be such that y≠ x. Then clearly y □ L. Thus y ∉ F and since X is μ-α-R_0, then $\alpha c_\mu(\{y\})$ □ X – F. This shows that x ∉ $\alpha c_\mu(\{y\})$. Thus there exist μ-α-open set containing x disjoint from y. Thus y ∉ $\Lambda_{\mu\text{-}\alpha}(\{x\})$ and this is a contradiction. Hence, (X, μ) is a weak μ-α-R_0 space.

Example: 3.16: Consider $X = \{a, b, c\}$ and $\mu = \{\varphi, \{a\}\}$. Then (X, μ) is a GTS which is weak μ-α-R_0 space but not μ-α-R_0.

Theorem: 3.17: For a GTS (X, μ), the following conditions are equivalent:

(i). X is a μ-α-T_1;

(ii). X is a μ-α-T_0 and a μ-α-R_0;

(iii). X is a μ-α-T_0 and is a weak μ-α-R_0.

Proof: (i) $\Longrightarrow$ (ii): If X is μ-α-T_1 then it is μ-α-T_0. Let U be a μ-α-open set such that $x \in U$. Let $y \notin U$. Then x and y are distinct points of X. So by (i) there exists a μ-α-open set G such that $y \in G$ but $x \notin G$ and so $y \notin \alpha c_\mu(\{x\})$. Thus (X, μ) is μ-R_0.

(ii) $\Longrightarrow$ (iii): Follows from Theorem: 3.16.

(iii) $\Longrightarrow$ (i): In view of theorem 3.13 and Definition: 3.15 it follows that every singleton subset of X is $\Lambda_{\mu\text{-}\alpha}$-set and the rest follows from Definition: 3.1.

4. $\Lambda_{\mu\text{-}\alpha}$-D SETS:

Definition: 4.1: A subset A of a GTS (X, μ) is called a $\Lambda_{\mu\text{-}\alpha}$-D set if there are two $(\Lambda, \mu\text{-}\alpha)$-open sets U and V in X such that $U \neq X$ and $A = U - V$.

Definition: 4.2: A GTS (X, μ) is called

(i). $\Lambda_{\mu\text{-}\alpha}$-$D_0$ if for any distinct pair of points x and y of X there exists a $\Lambda_{\mu\text{-}\alpha}$-D set of X containing x but not y or $\Lambda_{\mu\text{-}\alpha}$-D set of X containing y but not x.

(ii). $\Lambda_{\mu\text{-}\alpha}$-$D_1$ if for any distinct pair of points x and y of X there exist a $\Lambda_{\mu\text{-}\alpha}$-D set of X containing x but not y and a $\Lambda_{\mu\text{-}\alpha}$-D set of X containing y but not x.

(iii). $\Lambda_{\mu\text{-}\alpha}$-$D_2$ if for any distinct pair of points x and y of X there exist disjoint $\Lambda_{\mu\text{-}\alpha}$-D sets G and H of X containing x and y respectively.

Definition: 4.3: A GTS (X, μ) satisfies $(\Lambda, \mu\text{-}\alpha)$-property if for any distinct pair of points x and y of X there exist disjoint $(\Lambda, \mu\text{-}\alpha)$-open sets containing one but not the other.

Remark: 4.4: The following holds for a GTS (X, μ):

(i). If (X, μ) satisfies $(\Lambda, \mu\text{-}\alpha)$-property, then it is $\Lambda_{\mu\text{-}\alpha}$-$D_0$.

(ii). If (X, μ) is $\Lambda_{\mu\text{-}\alpha}$-$D_i$, then it is $\Lambda_{\mu\text{-}\alpha}$-$D_{i-1}$, $i = 1,2$.

Theorem: 4.5: A GTS (X, μ) is

(i). $\Lambda_{\mu\text{-}\alpha}$-$D_0$ if and only if it satisfies $(\Lambda, \mu\text{-}\alpha)$-property.

(ii). (X, μ) is $\Lambda_{\mu\text{-}\alpha}$-$D_1$ if and only if $\Lambda_{\mu\text{-}\alpha}$-$D_2$.

Proof:

(i) By remark 4.4, one part is trivial. Let (X, μ) be $\Lambda_{\mu\text{-}\alpha}$-$D_0$ .So for any pair of distinct points of x and y of X at least one belongs to a $\Lambda_{\mu\text{-}\alpha}$-D-set U. Suppose $U = P - Q$ for which $P \neq X$ and P and Q are $(\Lambda, \mu\text{-}\alpha)$-open set in X. Without loss of generality let $x \in U$ and $y \notin U$. This implies that $x \in P$. For the case $y \notin U$ we have (i) $y \notin P$ (ii) $y \in P$ and $y \in Q$. For (i), the space X satisfies $(\Lambda, \mu\text{-}\alpha)$-property since $x \in P$ and $y \notin P$. For (ii), the space X also satisfies $(\Lambda, \mu\text{-}\alpha)$-property since $y \in Q$, but $x \notin Q$.

(ii) By remark 4.3, one part is trivial. Suppose X is $\Lambda_{\mu\text{-}\alpha}$-$D_1$. It follows from the definition that for any two distinct points x and y in X there exist $\Lambda_{\mu\text{-}\alpha}$-D-sets G and H such that $x \in G$ but $y \notin G$ and $y \in H$ but $x \notin H$. Let $G = U - V$ and $H = W - D$, where U, V, W and D are $(\Lambda, \mu\text{-}\alpha)$-open sets in X. By the fact that $x \notin H$, we have two case, i.e., either $x \notin W$ or both W and D contains x. If $x \notin W$, then from $y \notin G$ either (i) $y \notin U$ or (ii) $y \in U$ and $y \in V$. If (i) is the case, then it follows from $x \in U - V$ that $x \in U - V \cup W$ and also it follows from $y \in W - D$ that $y \in W - U \cup D$ are disjoint. If (ii) is the case, it follows that $x \in U - V$, $y \in V$ and $(U - V) \cap V = \varphi$. If $x \in W$ and $x \in D$ we have $y \in W - D$, $x \in D$ and $(W - D) \cap D = \varphi$. Thus X is $\Lambda_{\mu\text{-}\alpha}$-$D_2$.

Definition: 4.6: Let (X, μ) be a GTS. A point $x \in X$ is said to be a $\Lambda_{\mu\text{-}\alpha}$-neat point if there does not exist any $(\Lambda, \mu\text{-}\alpha)$-open set containing x other than X.

Theorem: 4.7: For a GTS (X, μ) that satisfies $(\Lambda, \mu\text{-}\alpha)$- property the following is equivalent:

(i). (X, μ) is $\Lambda_{\mu\text{-}\alpha}$-$D_1$;

(ii). (X, μ) has no $\Lambda_{\mu\text{-}\alpha}$-neat point.

Proof: (i) $\Longrightarrow$ (ii) (X, μ) is $\Lambda_{\mu\text{-}\alpha}$-$D_1$, so each point x of X is contained in a $\Lambda_{\mu\text{-}\alpha}$-D set $O = U - V$ and thus $x \in U$. By definition $U \neq X$ and U is $(\Lambda, \mu\text{-}\alpha)$ - open. This implies that x is not a $\Lambda_{\mu\text{-}\alpha}$ -neat point.

(ii) $\Rightarrow$ (i) Since (X, μ) that satisfies $(\Lambda, \mu\text{-}\alpha)$ property, then for each distinct pair of points x, $y \in X$, at least one of them, choose x for example is contained in a $(\Lambda, \mu\text{-}\alpha)$ -open set U not containing the point y. Thus U is a $\Lambda_{\mu\text{-}\alpha}$-D set different from X (as every $(\Lambda, \mu\text{-}\alpha)$ -open set is a $\Lambda_{\mu\text{-}\alpha}$-D set). Since by (ii) (X, μ) has no $\Lambda_{\mu\text{-}\alpha}$ -neat point. So there exist a $(\Lambda, \mu\text{-}\alpha)$ -open set V $(\neq X)$ containing y. Thus $y \in V - U$ and $x \notin V - U$ is a $\Lambda_{\mu\text{-}\alpha}$-D set. Hence X is $\Lambda_{\mu\text{-}\alpha}$-$D_1$.

Definition: 4.8: Let (X, μ) be a GTS. A point $x \in X$ is called $(\Lambda, \mu\text{-}\alpha)$ -cluster point of A if for every $(\Lambda, \mu\text{-}\alpha)$ - open set U of X containing x we have $A \cap U \neq \emptyset$. The set of all $(\Lambda, \mu\text{-}\alpha)$ -closure of A and is denoted by $A^{(\Lambda, \mu\text{-}\alpha)}$.

Definition: 4.9: A GTS (X, μ) is called $(\Lambda, \mu\text{-}\alpha)$ – symmetric if for every x and y in X, $x \in \{y\}^{(\Lambda, \mu\text{-}\alpha)}$ implies $y \in \{x\}^{(\Lambda, \mu\text{-}\alpha)}$.

Lemma: 4. 10: For a GTS (X, μ), the following conditions are equivalent:

(i). $A \subseteq A^{(\Lambda, \mu\text{-}\alpha)}$;

(ii). $A^{(\Lambda, \mu\text{-}\alpha)} = \bigcap\{F : A \subseteq F$ and F is $(\Lambda, \mu\text{-}\alpha)$ -closed set$\}$;

(iii). $A \subseteq B \Longrightarrow A^{(\Lambda, \mu\text{-}\alpha)} \subseteq B^{(\Lambda, \mu\text{-}\alpha)}$;

(iv). A is $(\Lambda, \mu\text{-}\alpha)$ -closed iff $A = A^{(\Lambda, \mu\text{-}\alpha)}$;

(v). $A^{(\Lambda, \mu\text{-}\alpha)}$ is $(\Lambda, \mu\text{-}\alpha)$ -closed.

Theorem: 4. 11: A GTS (X, μ) is $(\Lambda, \mu\text{-}\alpha)$ $(\Lambda, \mu\text{-}\alpha)$ – symmetric iff for $x \in X$, $\{x\}^{(\Lambda, \mu\text{-}\alpha)} \subseteq E$ and E is $(\Lambda, \mu\text{-}\alpha)$ -closed in (X, μ).

Proof: Assume that $x \in \{y\}^{(\Lambda, \mu\text{-}\alpha)}$ but $y \notin \{x\}^{(\Lambda, \mu\text{-}\alpha)}$. This means that the complement of $\{x\}^{(\Lambda, \mu\text{-}\alpha)}$ contains y. Now $\{y\}$ is a subset of the complement of $\{x\}^{(\Lambda, \mu\text{-}\alpha)}$. This implies that $\{y\}^{(\Lambda, \mu\text{-}\alpha)}$ is a subset of the complement of $\{x\}^{(\Lambda, \mu\text{-}\alpha)}$. Now the complement of $\{x\}^{(\Lambda, \mu\text{-}\alpha)}$ contains x which is a contradiction.

Conversely suppose that $\{x\} \subseteq E$ and E is (Λ, μ-α) -open in (X, μ) but $\{x\}^{(\Lambda, \mu\text{-}\alpha)} \nsubseteq E$. Then $\{x\}^{(\Lambda, \mu\text{-}\alpha)}$ intersects the complement of E. Let y be a member of this intersection. Now we have $x \in \{y\}^{(\Lambda, \mu\text{-}\alpha)}$ which is the subset of the complement of E and hence $x \notin E$. But this is a contradiction.

Theorem 4. 12: For a (Λ, μ-α) -symmetric GTS (X, μ) the following are equivalent:

(i). (X, μ) satisfies the (Λ, μ-α) property;

(ii). (X, μ) is $\Lambda_{\mu\text{-}\alpha}$-$D_0$;

(iii). (X, μ) is $\Lambda_{\mu\text{-}\alpha}$-$D_1$;

Proof: (i) ⟺ (ii): Follows from the Theorem 3.4.

(iii) ⟹ (ii): Follows from the remark 3.3.

(i) ⟹ (iii): Let $x \neq y$ and by (i), we may assume that $x \in E \subseteq X - \{y\}$ for some (Λ, μ-α) -open set E in (X, μ). Then $x \notin \{y\}^{(\Lambda, \mu\text{-}\alpha)}$ and $y \notin \{x\}^{(\Lambda, \mu\text{-}\alpha)}$. Hence there exists (Λ, μ-α) -open sets G and H such that $y \in G \subseteq X - \{x\}$ and $x \in H \subseteq X - \{y\}$. Since every (Λ, μ-α) -open set is a $\Lambda_{\mu\text{-}\alpha}$-D set, we have that (X, μ) is $\Lambda_{\mu\text{-}\alpha}$-$D_1$ space.

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