STATUS SEQUENCE AND DETOUR STATUS SEQUENCE OF CONNECTED GRAPHS

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Abstract: Let G=(V,E) be a simple undirected connected graph. The status of a vertex $v\in V$ of a graph G is the sum of shortest distance from v to the other vertices of G. The status sequence of G is the list of its status value in non-decreasing order. The detour status of a vertex $v\in V$ of a graph G is the sum of longest distance from v to the other vertices of G. The detour status sequence of G is the list of its detour status values in non-decreasing order. In this paper a status sequence (SS) and detour status sequence (DSS) of connected graphs are investigated.

Keywords: status sequence, detour status sequence, status values, detour status values.

Introduction: Let G=(V,E) be a simple undirected connected graph. For vertices u and v in a connected graph G, the distance d(u,v) is the length of a shortest u-v path in G. A u-v path of length d(u,v) is called u-v geodesic. The status of a vertex of a graph G is the sum of the shortest distances from v to the other vertices of G. The status sequence(SS) of G is the list of its status value in non-decreasing order. The length of a longest u-v path between two vertices u and v in G is called a detour distance D(u,v) between u and v. A u-v path of length D(u,v) is a u-v detour. The detour status of a vertex v□V of a graph G is the sum of longest distance from v to the other vertices of G. The detour status sequence(DSS) of G is the list of its detour status values in non-decreasing order. In this paper, status sequence (SS) and detour status sequences (DSS) of connected graph are investigated with some examples.

2. Main Results:

Theorem:2.1: The status sequence of a crown graph $SS(C_nK_1)=\{SS(u_i) = (n+1(n-\lfloor n/2\rfloor)-1)$, $SS(v_j) = (n+1(n-\lfloor n/2\rfloor) +2n-3)\}$,when n is odd, for $n\geq3$ and $SS(C_nK_1) =\{SS(u_i) = n/2(n+2),SS(v_j)=(n/2(n+2) +2n-2)\}$,when n is even, for $n\geq4$and Detour status sequence of $DSS(C_nK_1)= \{ SS(u_i) = n(n+\lfloor n/2\rfloor) - (n-\lfloor n/2\rfloor)$, $SS(v_j) = n(n+\lfloor n/2\rfloor) - (n-\lfloor n/2\rfloor) +2n-2 \}$,when n is odd, for $n\geq3$ and $DSS(C_nK_1) =\{SS(u_i) = n/2(3n-2)$, $SS(v_j) = (n/2(3n-2) +2n-2)\}$,when n is even, for $n\geq4$

Proof: Let $V(C_nK_1) =\{u_i$ and v_i ;i=0 to n-1} be the vertex set of C_nK_1 . and Let $E(C_nK_1) = \{u_0u_{i+1} \cup u_i v_i$; i=0 to n-1 and $u_0= u_n$ }, be the edge set of C_nK_1.

Case(i) n is odd: If n=3, the SS & DSS of $C_3 K_1$ is

$SS(C_3 K_1)=\{7,7,7,11,11,11\}$

and $DSS(C_3 K_1)= \{11,11,11,15,15,15\}$

The result is true for n=3.

By mathematical induction,

Let us assume that the result is true for k= n & $n \geq3$ and n is odd.

Clearly, this result is true for k=n+2.

Therefore the SS & DSS of (C_nK_1) given by

$SS(C_nK_1) =\{SS(u_i) = n/2(n+2)$,

$SS(v_j) = (n/2(n+2) +2n-2)\}$,when n is even, for $n\geq4$

$SS (C_n K_1)= \{SS (u_i) = (n+1(n-\lfloor n/2\rfloor) -1)$,

$SS(v_j) = (n+1(n-\lfloor n/2\rfloor) +2n-3)\}$ and

$SS(u_i) = n(n+\lfloor n/2\rfloor) - (n-\lfloor n/2\rfloor)$,

$SS(v_j) = n(n+\lfloor n/2\rfloor) - (n-\lfloor n/2\rfloor) +2n-2 \}$,

when n is odd, for $n\geq3$}.

Case(ii) n is even: If n=4, the SS & DSS of $C_4 K_1$ is

$SS(C_4 K_1)=\{12,12,12,12,18,18,18,18\}$

and $DSS(C_4K_1)= \{20,20,20,20,26,26,26,26\}$

The result is true for n=4

By mathematical induction,

Let us assume that the result is true for k= n & $n\geq4$ and n is even

Clearly, this result is true for k=n+2 and n is even.

Therefore the SS & DSS of (C_nK_1) given by

$SS(C_nK_1) =\{SS(u_i) = n/2(n+2)$,

$SS(v_j) = (n/2(n+2) +2n-2)\}$

$DSS(C_nK_1) =\{SS(u_i) = n/2(3n-2)$,

$SS(v_j) = (n/2(3n-2) +2n-2)\}$,when n is even, for $n\geq4$

Theorem:2.2: The status sequence of a helm graph $SS(W_n K_1)=\{u_0=3n,\ u_j=5n-7,\ u_j=7n-8$;i=1 to n and j=n+1 to 2n }& Detour status sequence of a helm graph $DSS((W_n K_1))=\{ u_0=n(2n+1),\ u_j=2n^2,\ u_j=2n^2+2n-1$;i=1 to n and j=n+1 to 2n}

Proof: Let $V(W_n K_1) = \{u_0 ,u_i$; i=1 to 2n} be the vertex set of $(W_n K_1)$.

and Let $E(W_n K_1) = \{u_0u_i,\ u_iu_{i+1}\ ,\ u_iu_{i+3}$ for i, =1 to n and $u_1=u_{n+1}\}$, be the edge set of $(W_n K_1)$

If n=3, the SS & DSS of $(W_3 K_1)$.

$SS(W_3 K_1)=\{8,8,8,9,13,13,13 \}$

$DSS((W_3 K_1)= \{18,18,18, 21,23,23,23\}$

This result is true for n=3.

By mathematical induction,

Let as assume that the result is true for k= n & $n \geq3$.

Clearly , this result is true for all k=n+1

Therefore the SS & DSS of $(W_n K_1)$ is given by

$SS(W_n K_1)=\{u_0=3n,\ u_j=5n-7,\ u_j=7n-8$;i=1 to n and j=n+1 to 2n }$SS(w_{n+1} k_1)=\{ u_0=3n,\ u_j=5n-7,\ u_j=7n-8$;i=1 to n and j=n+1 to 2n }and

$DSS((W_n K_1))=\{ u_0=n(2n+1),\ u_j=2n^2,\ u_j=2n^2+2n-1$;i=1 to n and j=n+1 to 2n}

Conclusion: Motivated by the definition of Status sequence and detour status sequence of a graph, we have discussed the SS and DSS of connected graph. The Problem of SS and DSS of connected graphs are under investigation.

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