

SOFT INTERVAL VALUED INTUITIONISTIC FUZZY SEMI-PRE GENERALIZED HOMEOMORPHISM

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Abstract: This paper introduces some new notions of generalized homeomorphism of soft interval valued intuitionistic fuzzy topology along with its characterization.

Keywords: Soft interval valued intuitionistic fuzzy open set (semi-open set, pre-open set, semi-pre open set, α - open set) soft interval valued intuitionistic semi –pre generalized homeomorphisms.

Introduction: The year 1999, gave a new start to a theory which was very convenient and easily applicable to the real life problems. This theory named Soft Set Theory was developed by D.Molodtsov [16]. He paved a way for attaining solution to complicated decision making problems that are complicated due to some uncertainty and incompleteness of information.

In 2003, Maji et al [14] developed several basic notions for this theory. In 2011, Muhammad Shabir and Munazza Naz[17] developed the soft topology.

The soft set theory was pooled with interval valued intuitionistic fuzzy set by Y. Jiang et.al [20] and Jinyan Wang et.al [11] studied interval valued intuitionistic fuzzy soft sets . In 2013, Anjan Mukherjee et. al [4] developed interval valued intuitionistic fuzzy soft topological spaces. This paper aims at generalizing the soft interval valued intuitionistic fuzzy semi-pre homeomorphism along with its characterization.

1. Preliminaries

Definition:1.1[17] A pair (F,E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

Definition: 1.2[4] Let (ξ_A, E) be an element of interval valued intuitionistic fuzzy set (IVIFS) (U;E), $P(\xi_A, E)$ be the collection of all IVIFS subsets of (ξ_A, E) . A sub family τ of $P(\xi_A, E)$ is called an interval valued intuitionistic fuzzy soft topology on (ξ_A, E) , if the following axioms are satisfied:

$$[O_1] (\phi_{\xi_A}, E), (\xi_A, E) \in \tau$$

$$[O_2] \{(f_A^k, E)/k \in K\} \subseteq \tau \Rightarrow \bigcup_{k \in K} (f_A^k, E) \in \tau$$

$$[O_3]$$

If $(f_A, E), (g_A, E) \in \tau$, then $(f_A, E) \cap (g_A, E) \in \tau$

Then the pair (ξ_A, E, τ) is called an interval valued intuitionistic fuzzy soft topological space. The members of τ are called τ – open sets, where $\phi_{\xi_A} : A \rightarrow \text{IVIFS}(U)$ is defined as $\phi_{\xi_A}(e) = \{ \langle x, [0,0], [1,1] \rangle : x \in U \}, \forall e \in A$

Definition: 1.3[4] Let (ξ_A, E, τ) be an interval valued intuitionistic fuzzy soft topological space on (ξ_A, E) and (f_A, E) be an IVIFS in $P(\xi_A, E)$. Then the union of all open IVIFS set of (f_A, E) is called the interior of (f_A, E) and is denoted by $\text{int}(f_A, E)$ and defined by $\text{int}(f_A, E) = \bigcup \{(g_A, E)/(f_A, E)$ is a neighborhood of

$(g_A, E)\}$.

Definition: 1.4[4] Let (ξ_A, E, τ) be an interval valued intuitionistic fuzzy soft topological space on (ξ_A, E) and (f_A, E) be an IVIFS in $P(\xi_A, E)$. Then the intersection of all closed IVIFS set containing (f_A, E) is called the closure of (f_A, E) and is denoted by $\text{cl}(f_A, E)$ and defined by $\text{cl}(f_A, E) = \bigcap \{(g_A, E)/(g_A, E)$ is a IVIFS-closed set containing $(f_A, E)\}$.

Definition: 1.5[21] Let U be an initial universe and E be the set of parameters . Suppose that $A, B \subseteq E, \langle F, A \rangle$ and $\langle G, B \rangle$ are two interval-valued intuitionistic fuzzy soft sets, we say that $\langle F, A \rangle$ is an interval-valued intuitionistic fuzzy soft subset of $\langle G, B \rangle$ if and only if (i) $A \subseteq B$, (ii) $\forall e \in A, F(e)$ is an interval-valued intuitionistic fuzzy soft subset of $G(e)$, that is for all $x \in U, e \in A$,

$$\underline{\mu}_{F(e)}(x) \leq \underline{\mu}_{G(e)}(x),$$

$$\overline{\mu}_{F(e)}(x) \leq \overline{\mu}_{G(e)}(x)$$

$$\underline{\gamma}_{F(e)}(x) \geq \underline{\gamma}_{G(e)}(x)$$

$$\overline{\gamma}_{F(e)}(x) \geq \overline{\gamma}_{G(e)}(x)$$

Definition: 1.6[21] An interval valued intuitionistic fuzzy soft set $\langle F, A \rangle$ over U is said to be a null interval valued intuitionistic fuzzy soft set denoted by Φ , if $\forall e \in A, \mu_{F(e)}(x) = [0,0], \nu_{F(e)}(x) = [1,1] \quad x \in U$.

Definition: 1.7[21]

An interval valued intuitionistic fuzzy soft set $\langle F, A \rangle$ over U is said to be an absolute interval valued intuitionistic fuzzy soft set denoted by Σ , if $\forall e \in A, \mu_{F(e)}(x) = [0,0], \nu_{F(e)}(x) = [1,1] \quad x \in U$.

2. Soft Interval Valued Intuitionistic Fuzzy Semi-Pre Generalized:

Homeomorphism: In this section we have introduced soft interval valued intuitionistic fuzzy semi-pre generalized homeomorphism and has listed their characterizations.

Definition:2.1:A mapping $\phi_{\psi} : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is called a soft interval valued intuitionistic fuzzy semi pre generalized continuous (SIVIFSPG continuous)

mapping if $\varphi_{\psi}^{-1}(V, K)$ is a soft interval valued intuitionistic fuzzy semi pre generalized closed set in (X, τ, E) for every soft interval valued intuitionistic fuzzy closed set (V, K) of (Y, σ, K) .

Definition:2.2:A mapping $\varphi_{\psi}: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is called a soft interval valued intuitionistic fuzzy semi pre generalized open (SIVIFSPGO) mapping if $\varphi_{\psi}(F, E)$ is a soft interval valued intuitionistic fuzzy semi pre generalized open in Y for each soft interval valued intuitionistic fuzzy open set (F, E) of X .

Definition: 2.3 Let $\varphi_{\psi}: (X, \tau, E) \rightarrow (Y, \sigma, K)$ be a soft bijective mapping. Then φ_{ψ} is said to be a soft interval valued intuitionistic fuzzy semi pre generalized homeomorphism (SIVIFSPGHM) if φ_{ψ} is both a SIVIFSPG continuous mapping and a SIVIFSPGO mapping.

Theorem: 2.4 Let $\varphi_{\psi}: (X, \tau, E) \rightarrow (Y, \sigma, K)$ be a soft bijective mapping. If φ_{ψ} is a SIVIFSPG continuous mapping, then the following statements are equivalent:

- (i) φ_{ψ} is a SIVIFSPGO mapping.
- (ii) φ_{ψ} is a SIVIFSPGHM.
- (iii) φ_{ψ} is a SIVIFSPGC mapping.

Proof: Straight forward from the definition.
References

Definition:2.5:A mapping $\varphi_{\psi}: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is called a soft interval valued intuitionistic fuzzy M- semi pre generalized open (SIVIFMSPGO) mapping if $\varphi_{\psi}(F, E)$ is a soft interval valued intuitionistic fuzzy semi pre generalized open in Y for each soft interval valued intuitionistic fuzzy semi pre generalized (SIVIFSPG)open set (F, E) of X .

Definition: 2.6 A mapping $\varphi_{\psi}: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is called a soft interval valued intuitionistic fuzzy semi pre generalized irresolute (SIVIFSPG) mapping if $\varphi_{\psi}^{-1}(V, K)$ is a soft interval valued intuitionistic fuzzy semi pre generalized closed in Y for every SIVIFSPGCS (V, K) of Y .

Definition: 2.7 Let $\varphi_{\psi}: (X, \tau, E) \rightarrow (Y, \sigma, K)$ be a soft bijective mapping. Then φ_{ψ} is said to be a soft interval valued intuitionistic fuzzy M-semi pre

generalized homeomorphism (SIVIFMSPGHM) if φ_{ψ} is both a SIVIFSPG irresolute mapping and a SIVIFMSPGO mapping.

Theorem: 2.8 Every SIVIFMSPGHM is a SIVIFSPGHM but not conversely.

Proof: Assume that $\varphi_{\psi}: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is SIVIFMSPGHM. Let $(V, K) \cong (Y, K)$ be a SIVIFCS. Then (V, K) is a SIVIFSPGCS in Y . By hypothesis $\varphi_{\psi}^{-1}(V, K)$ is a SIVIFSPGCS in X . Hence φ_{ψ} is a SIVIFSPG continuous mapping. Let $(F, E) \cong X$ be a SIVIFOS. Then (F, E) is a SIVIFSPGOS in X . By hypothesis, $\varphi_{\psi}(F, E)$ is a SIVIFSPGOS in Y .

Hence φ_{ψ} is a SIVIFSPGO mapping.Thus φ_{ψ} SIVIFSPGHM.

Remark: 2.9 The composition of two SIVIFMSPGHM is a SIVIFMSPGHM.

Theorem: 2.10 Let $\varphi_{\psi}: (X, \tau, E) \rightarrow (Y, \sigma, K)$ be a soft bijective mapping. If φ_{ψ} is a SIVIFSPG irresolute mapping, then the following statements are equivalent:

- (i) φ_{ψ} is a SIVIFMSPGO mapping.
- (ii) φ_{ψ} is a SIVIFMSPGHM.
- (iii) φ_{ψ} is a SIVIFMSPGCM.

Proof: Straight forward from the definition.

Theorem: 2.11 Let $\varphi_{\psi}: (X, \tau, E) \rightarrow (Y, \sigma, K)$ be a SIVIFMSPGHM, then

$spgcl(\varphi_{\psi}^{-1}(V, K)) \cong \varphi_{\psi}^{-1}(spgcl(V, K))$ for every SIVIFS (V, K) in .

Proof: Let $(V, K) \cong Y$. Then $spgcl(V, K)$ is a SIVIFSPGCS in Y . Since φ_{ψ} is a SIVIFSPG irresolute mapping, $\varphi_{\psi}^{-1}(spgcl(V, K))$ is a SIVIFSPGCS in X . This implies

$$spgcl(\varphi_{\psi}^{-1}(V, K)) = \varphi_{\psi}^{-1}(V, K).$$

Hence

$$spgcl(\varphi_{\psi}^{-1}(V, K)) \cong spgcl(\varphi_{\psi}^{-1}(spgcl(V, K))) = \varphi_{\psi}^{-1}(spgcl(V, K)).$$

Conclusion : The present work is devoted to study the behavior of soft interval valued intuitionistic fuzzy semi-pre generalized homeomorphism and has established results connected with it.

References:

1. M.E Abd El.Monsef, S.N.El.Deeb and R.A Mahmoud, "g open sets and g - continuous mapping",Bull.Fac.Sci., Assiut- University, 1st ed. vol. 12, 1983, pp. 77-90.
2. Abdulkadir Aygunoglu, Halis Aygun , Some notes on soft topological space , Neural Computers and Applications , 1st ed.vol. 21,2012, 113-119 .
3. D. Andrijevic , Semi-pre- open sets , Mat.Vesnik, 38, (1986) , 24-32.
4. Anjan Mukherjee , Ajoy Kanti Das, Abhijit Saha, Interval valued intuitionistic fuzzy soft topological spaces, Annals of Fuzzy Mathematics and Informatics,6(3),(2013),689-703.
5. I.Arockiarani, A. Arokialancy , Generalized soft g g-closed sets and soft g g-closed sets in soft topological spaces,International Journal Of

- Mathematical Archive, 4(2), (2013) , 1-7.
6. Atanssov. K, Intuitionistic fuzzy sets , Fuzzy Sets and Systems, 20(1986), 87- 96.
 7. Atanssov. K, Gargov. G, Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, 31(3) , (1986),343- 349.
 8. Chang. C, Fuzzy Topological Spaces, J.Math Anal.Appl, 24(1968),182-190.
 9. Coker. D, An introduction to fuzzy subspaces in intuitionistic fuzzy topological spaces, Journal Of Fuzzy Mathematics, 4(1996), 749-764.
 10. Gunduz. C and Bayramov.S , Intuitionistic fuzzy soft modules , Comput. Math.Appl.62(2011), 2480-2486.
 11. jinyan Wang, Minghao Yin, Wenxiang Gu, Notes On: interval-valued intuitionistic Fuzzy Soft Sets and their properties, Comput. Math.Appl, 60(2010), 906-918.
 12. K.Kannan ,Soft generalized closed sets in soft topological spaces,Journal of Theoretical and Applied Information Technology, 37(1) , (2012) , 17-20.
 13. N .Levine,Semi-open sets and semi-continuity in topological spaces, American Math Monthly, 70, (1963),36-41.
 14. P.K Maji, A.R. Roy and R. Biswas, Fuzzy soft sets , Computers And Mathematics With Applications, 9(3), (2001),589-602.
 15. P.K Maji, R. Biswas and A.R. Roy , Soft set theory, Computers And Mathematics With Applications, 45(4-5), (2003),555-562.
 16. P.K Maji, A.R. Roy and R. Biswas, On intuitionistic fuzzy soft sets , Journal Of Fuzzy Mathematics ,12(3), (2004),669-683.
 17. D.Molodtsov, Soft set theory – First results, Computers And mathematics with applications,37(4-5), (1999), 19-31.
 18. Muhammad Shabir, Munazza Naz , On soft topological spaces, Computers And Mathematics With Applications,61(7), (2011), 1786-1799.
 19. Tanay.B,Kandemir. M.B, Topological structure of fuzzy soft sets, Computers And Mathematics With applications,61, (2011), 2952-2957.
 20. Y.Yumak and A.K .Kaymakci, Soft β -open sets and their Applications, arXiv: 1312.6964v1, (2013), [math.GN].
 21. Yuncheng Jiang, Yong Tang, Qimai Chen, hai Liu, Jianchao Tang, Interval valued intuitionistic fuzzy soft sets and their properties, Computers And Mathematics With Applications,60, (2010), 906-918.
 22. L.A Zadeh, Fuzzy sets, Information And Control, 8(1965),338-353.
 23. Zhaowen Li, Rongchen Cui On the topological structure of intuitionistic fuzzy soft sets, Annals Of Fuzzy Mathematics And Informatics, 5(1),(2013),229-239.

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