SOME VARIATIONS OF CORDIAL LABELING FOR SOME FAMOUS GRAPHS

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Abstract: J. Shiama has defined Sum Cordial labeling and some Sum Cordial graphs. A binary vertex labeling of a graph G with induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ defined by $f^*(uv) = (f(u) + f(v))\ (\text{mod}2)$ is called a sum cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Any graph which satisfies the sum cordial labeling is called a Sum Cordial graph. Here we prove the Flower graph admits Sum cordial graph. Ponraj. R, M. Sivakumar, M. Sundaram introduced a new graph labeling method called the 3-Total Product Cordial labeling, we have computed the 3-Total Product Cordial labeling on Sub-division of Flower graph. Yilmaz and Cahit introduced E-cordial labeling as a weak version of edge-graceful labeling and having bland of cordial labeling. We have obtained the E-cordial labeling for the Cartesian product of the star and ladder graphs.

Keywords: Binary labeling, Cordial labeling; Product Cordial, Sum cordial labeling, 3- Total Product cordial labeling and E –cordial labeling.

Introduction: A graph labeling is an assignment of integer to the vertices or edges or both, subject to certain conditions. Most graph labeling methods trace their origin to the graceful labeling introduced by Rosa [7] in 1967. Over the past five decades a large number of graph labeling methods have been studied. In [12] J. Shiama introduced a new graph labeling method called the Sum cordial labeling.

In [8] Ponraj. R, M. Sivakumar, M. Sundaram introduced a new graph labeling method called the 3-Total Product Cordial labeling, Using the concept of k-Total Product Cordial labeling. To understand the 3-Total Product Cordial labeling one must know some basic labelings like Binary labeling, Cordial, Product Cordial, Total Product Cordial labeling and k - Total Product Cordial labeling.

The vertex labeling f^* is said to be a **binary labeling** if $f^* : V(G) \rightarrow \{0, 1\}$ such that each edge xy is assigned the label $|f^*(x) - f^*(y)|$. The Cordial graph were introduced by Cahit(1897). A binary vertex labeling of a graph *G* is called a **Cordial labeling** if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph *G* is called Cordial if it admits cordial labeling.

A binary vertex labeling of graph *G* with induced edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ defined by $f^*(e = uv) = f(u)f(v)$ is called a Product Cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph is called Product Cordial if it admits Product Cordial labeling. The product cordial labeling were introduced by Sundaram .M *et al.* [13]. A binary vertex labeling of a graph G with induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ defined by $f^*(uv) = (f(u) + f(v))\ (\text{mod}2)$ is called a sum cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Any graph which satisfies the sum cordial labeling is called a Sum Cordial graph

A Total Product Cordial labeling of a graph G is a function $f : (V(G) \cup E(G)) \rightarrow \{0, 1\}$ such that $f(xy) = f(x) f(y)$ where $x, y \in V(G)$, $xy \in E(G)$ and the total number of 0's and 1's are balanced i.e. if $v_f(i)$ and $e_f(i)$ denote the set of vertices and edges which are labeled as i for i = 0, 1 respectively, then $|(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))| \leq 1$. If there exists a total product cordial labeling of a graph G then it is called a **Total Product Cordial graph:** Let f be a function from V(G) to $\{0, 1, \ldots k-1\}$ where k is an integer, $2 \leq k \leq |V(G)|$. For each edge uv assign the label $f(u) f(v) (\text{mod } k)$. f is called a k-Total Product Cordial labeling if $|f(i) - f(j)| \leq 1$, $i, j \in \{0, 1, \ldots k-1\}$ where $f(x)$ denotes the total number of vertices and edges labeled with x (x = 0, 1, 2… , k-1). A graph that admits a k - Total Product Cordial labeling is called **k -Total Product Cordial graph.**

In 1997, Yilmaz and Cahit introduced E-cordial labeling as follows. Let G = (V(G),E(G)) and let $f : E(G) \rightarrow \{0,1\}$. Define f^* on V(G) by $f^*(v) = \sum\{f(uv)/uv\ E(G)\}(\text{mod}2)$. The function f is called an **E-cordial labeling** of G if the number of vertices labeled 0 and the number of vertices labeled 1 differs by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 differs by at most 1. A graph that admits E-cordial labeling is called E-cordial graph.

In this paper, we compute the Sum Cordial labeling on flower graph, 3-Total Product Cordial labeling on Sub-division of Flower and We obtain the E-cordial labeling for the Cartesian product of the star and ladder graph.

We prove that F_n is sum cordial graph, $S(F_n)$ is 3- Total product cordial and $S_n \times L_2$ is E –Cordial We prove these results in the following theorems.

Main Theorems:

Theorem 1: The flower graph F_n is sum cordial labeling.

Proof: A **Wheel graph** W_n with n vertices is defined to be the join of $C_{n-1} + K_1$ of a isolated vertex with the cycle of length n. The **Helm graph** H_p is the graph obtained from a wheel graph by attaching a pendant edge at each vertex of the n-cycle. The **Flower graph** F_n is the graph obtained from the helm by attaching each pendent edge vertex to the centre vertex of the Wheel (W_n).

Let $F_n$be the flower graph with n vertices. Let u be the central vertex of F_n. The vertex u is called the hub vertex of the flower graph. Let $u_1, u_2, u_3, \ldots, u_{n-1}, u_n$be the vertices in the cycle of the flower. Let $v_1, v_2, v_3, \ldots, v_{n-1}, v_n$ be the end vertices of flower.

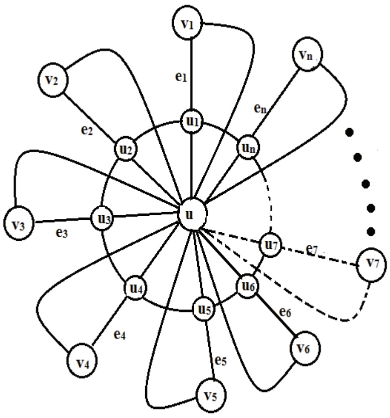


Figure. 1. Sum cordial labeling of flower F_n

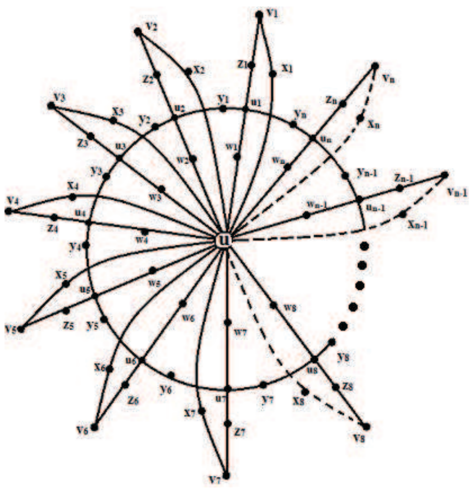
Let G = (V,G) and G be the Flower F_n that is, G = F_n. Then │V(G)│= 2n=1 and │E(G)│=4n.

Define f(u) =1, f(u_i)=1, f(v_i)=0,

Theorem 2: The graph S (F_n) is 3 - Total Product cordial labeling.

Proof: Let the Sub - division of flower S (F_n) is the graph obtained from the helm by attaching each pendant edge vertex to the center vertex of the Wheel(W_n) and Sub - dividing each edge by a vertex. u is the center vertex, w_i $(1 \le i \le n)$ is the vertex which sub - divides the uu_i edges, $u_i(1 \le i \le n)$ is the vertex of the cycle; y_i $(1 \le i \le n)$ is the vertex which sub - divides $u_1u_2, u_2u_3, ..., u_{n-1}u_n$ edges, $v_i(1 \le i \le n)$ is the end vertex of the S(F_n) graph and x_i $(1 \le i \le n)$ is the vertex which sub – divides uv_i edges. Finally z_i $(1 \le i \le n)$ is the vertex which sub - divides the v_iu_i $(1 \le i \le n)$ edges.

Figure 2 : 3 - Total Product cordial on S (F_n)



Let S (V (F_n)) = { u, w_i, u_i, z_i, v_i, y_i, x_i ; $1 \le i \le n$ }

S (E (F_n) ={ uw_i, w_iu_i ; $1 \le i \le n$ }∪{u_iz_i, z_iv_i; $1 \le i \le n$}∪{ u_1y_1, y_1u_2, .., y_nu_1}∪{ux_i, x_iv_i; $1 \le i \le n$ }

f (u) = 0; f (w_i) = 0; $1 \le i \le n$

f (u_i) = f (y_i) = 2; $1 \le i \le n$

Case i: $n \equiv 0 (\text{mod } 3)$ Let n = 3t

Special case :

$$\begin{cases} \text{If } t = 1;\ f(v_1) = f(z_1) = 2 \\ \text{Otherwise};\ f(v_1) = f(z_1) = 0 \end{cases}$$

Sub case i:

If t is odd

f (x_1) = 0

If $1 \le t \le 7$; f (x_i) = 0; $1 \le i \le \left\lceil \frac{t}{2} \right\rceil$

$f(x_{\lceil \frac{t}{2} \rceil + i}) = 2 ;\ 1 \le i \le n - \left\lceil \frac{t}{2} \right\rceil$

$$\text{If } t > 11 \begin{cases} f(x_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} - m \right\rceil \\ f(x_{\lceil \frac{t}{2} - m \rceil + i}) = 2 ;\ 1 \le i \le n \left\lceil \frac{t}{2} - m \right\rceil \end{cases}$$

$$\text{when } t \equiv \begin{cases} 0 \ (\text{mod } 3)\ ;\ t \ge 9 \\ 2 \ (\text{mod } 3)\ ;\ t \ge 11 \\ 1 \ (\text{mod } 3)\ ;\ t \ge 13 \end{cases}$$

Then the value of 'a' can be computed as follows;

$$a = \begin{cases} \left\lceil \frac{t}{3} \right\rceil;\ t \ge 9 \\ \left\lceil \frac{t-2}{3} \right\rceil;\ t \ge 11 \\ \left\lceil \frac{t-4}{3} \right\rceil;\ t \ge 13 \\ \text{Then } m = \lceil a - 1 \rceil \end{cases}$$

$f(z_i) = 0 ;\ 1 \le i \le \left\lceil \frac{t}{3} \right\rceil;\ f(z_{\lceil \frac{t}{3} \rceil + i}) = 2;$

$1 \le i \le n - \left\lceil \frac{t}{3} \right\rceil$

$$\text{If } 5 \le t \le 11 \begin{cases} f(v_i) = 0 ;\ 1 \le i \le \left\lceil \frac{t}{2} - 1 \right\rceil \\ f(v_{\lceil \frac{t}{2} - 1 \rceil + i}) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{2} - 1 \right\rceil \end{cases}$$

$$\text{For } t > 11 \begin{cases} f(v_i) = 0 ;\ 1 \le i \le \left\lceil \frac{t}{2} - m \right\rceil \\ f(v_{\lceil \frac{t}{2} - m \rceil + i}) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{2} - m \right\rceil \end{cases}$$

$$\text{when } t \equiv \begin{cases} 1 \ (\text{mod } 3)\ ;\ t \ge 13 \\ 0 \ (\text{mod } 3)\ ;\ t \ge 15 \\ 2 \ (\text{mod } 3)\ ;\ t \ge 1 \end{cases}$$

Then the value of 'a' can be computed as follows;

$$a = \begin{cases} \left\lceil \frac{t-1}{3} \right\rceil;\ t \ge 13 \\ \left\lceil \frac{t-3}{3} \right\rceil;\ t \ge 15 \\ \left\lceil \frac{t-5}{3} \right\rceil;\ t \ge 17 \end{cases}$$

Then $m = \left\lceil \frac{a}{2} \right\rceil$

Here f (0) = 14t + 1; f (1) = f (2) = 14t

Hence f is 3 - Total Produce Cordial labeling.

Sub case ii:

If t is even

Special case : {

If $t = 2$; $f(x_1) = f(z_1) = 2$

Otherwise; $f(x_1) = f(z_1) = 0$

If $t = 4$ $\begin{cases} f(x_1) = 0 \\ f(x_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} - 1 \right\rceil \\ f\left(x_{\left\lceil \frac{t}{2} - 1 \right\rceil + i}\right) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{2} - 1 \right\rceil \end{cases}$

For $t > 4$ $\begin{cases} f(x_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} - m \right\rceil \\ f\left(x_{\left\lceil \frac{t}{2} - m \right\rceil + i}\right) = 2, 1 \le i \le n - \left\lceil \frac{t}{2} - m \right\rceil \end{cases}$

when $t \equiv \begin{cases} 0 \pmod 3;\ t \ge 6 \\ 2 \pmod 3;\ t \ge 8 \\ 1 \pmod 3;\ t \ge 10 \end{cases}$

Then the value of 'a' can be computed as follows;

$$a = \begin{cases} \left\lceil \frac{t}{3} \right\rceil; & t \ge 6 \\ \left\lceil \frac{t-2}{3} \right\rceil; & t \ge 8 \\ \left\lceil \frac{t-4}{3} \right\rceil; & t \ge 10 \end{cases}$$

Then $m = \left\lceil \frac{a+2}{2} \right\rceil$

For $t \le 4$; $f(z_n) = 2$

If $t \equiv 1 \pmod 3$; $f(z_i) = 0$; $1 \le i \le \left\lceil \frac{t}{3} - 1 \right\rceil$

When $t \ge 6$; $\begin{cases} f\left(z_{\left\lceil \frac{t}{3} - 1 \right\rceil + i}\right) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{3} - 1 \right\rceil \\ \text{otherwise; } f(z_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{3} \right\rceil; \\ f\left(z_{\left\lceil \frac{t}{3} \right\rceil + i}\right) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{3} \right\rceil \end{cases}$

If $t = 4$ $\begin{cases} f(v_1) = 0 \\ f(v_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} \right\rceil \\ f\left(v_{\left\lceil \frac{t}{2} \right\rceil + i}\right) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{2} \right\rceil \end{cases}$

For $t > 4$ $\begin{cases} f(v_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} - m \right\rceil \\ f\left(v_{\left\lceil \frac{t}{2} - m \right\rceil + i}\right) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{2} - m \right\rceil \end{cases}$

when $t \equiv \begin{cases} 0 \pmod 3;\ t \ge 6 \\ 2 \pmod 3;\ t \ge 8 \\ 1 \pmod 3;\ t \ge 10 \end{cases}$

Then the value of 'a' can be computed as follows;

$$a = \begin{cases} \left\lceil \frac{t}{3} \right\rceil; & t \ge 6 \\ \left\lceil \frac{t-2}{3} \right\rceil; & t \ge 8 \\ \left\lceil \frac{t-4}{3} \right\rceil; & t \ge 10 \end{cases}$$

Then $m = \left\lceil \frac{a}{2} \right\rceil$

Here $f(0) = f(1) = 14t$; $f(2) = 14t + 1$

Hence f is 3-Total Produce Cordial labeling.

Case ii: $n \equiv 1 \pmod 3$

Let $n = 3t + 1$

Special case :

If $t = 1$; $\begin{cases} f(v_1) = f(z_1) = 2 \\ \text{Otherwise; } f(v_1) = f(z_1) = 0 \end{cases}$

Sub case i: If t is odd

Assign the label to the vertex x_i, z_i $(1 \le i \le n)$ as in case(i).

Then $m = \left\lceil \frac{a}{2} \right\rceil$

If $3 \le t \le 5$ $\begin{cases} f(v_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} \right\rceil \\ f\left(v_{\left\lceil \frac{t}{2} \right\rceil + i}\right) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{2} \right\rceil \end{cases}$

For $t > 5$ $\begin{cases} f(v_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} - m \right\rceil \\ f\left(v_{\left\lceil \frac{t}{2} - m \right\rceil + i}\right) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{2} - m \right\rceil \end{cases}$

when $t \equiv \begin{cases} 1 \pmod 3;\ t \ge 7 \\ 0 \pmod 3;\ t \ge 9 \\ 2 \pmod 3;\ t \ge 11 \end{cases}$

Then the value of 'a' can be computed as follows;

$$a = \begin{cases} \left\lceil \frac{t+2}{3} \right\rceil; & t \ge 7 \\ \left\lceil \frac{t}{3} \right\rceil; & t \ge 9 \\ \left\lceil \frac{t-2}{3} \right\rceil; & t \ge 11 \end{cases}$$

Then $m = \left\lceil \frac{a}{2} \right\rceil$

Here f (0) = f (1) = f (2) = 14t + 5
Hence f is 3-Total Produce Cordial labeling.

Sub case ii: If t is even

Special case :

$f(x_1) = 0$

If $t = 4$ $\begin{cases} f(x_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} \right\rceil \\ f(x_{\left\lceil \frac{t}{2} \right\rceil + i}) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{2} \right\rceil \end{cases}$

For $t > 4$ $\begin{cases} f(x_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} - m \right\rceil \\ f(x_{\left\lceil \frac{t}{2} - m \right\rceil + i}) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{2} - m \right\rceil \end{cases}$

when $t \equiv \begin{cases} 0 \pmod 3;\ t \ge 6 \\ 2 \pmod 3;\ t \ge 8 \\ 1 \pmod 3;\ t \ge 10 \end{cases}$

Then the value of ‘a’ can be computed as follows;

$$a = \begin{cases} \left\lceil \frac{t}{3} \right\rceil;\ t \ge 6 \\ \left\lceil \frac{t-2}{3} \right\rceil;\ t \ge 8 \\ \left\lceil \frac{t-4}{3} \right\rceil;\ t \ge 10 \end{cases}$$

Then $m = \left\lceil \frac{a}{2} \right\rceil$

$f(z_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{3} \right\rceil;\ f(z_{\left\lceil \frac{t}{3} \right\rceil + i}) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{3} \right\rceil$

If $t \equiv 2 \pmod 3$ When $t \ge 8$ $\begin{cases} f(v_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{3} + 1 \right\rceil; \\ f(v_{\left\lceil \frac{t}{3} + 1 \right\rceil + i}) = 2; 1 \le i \le n - \left\lceil \frac{t}{3} + 1 \right\rceil \end{cases}$

Otherwise; $f(v_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{3} \right\rceil;\ f(v_{\left\lceil \frac{t}{3} \right\rceil + i}) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{3} \right\rceil$

Here f (0) = f (1) = f (2) = 14t + 5
Hence f is 3-Total Produce Cordial labeling.

Case iii:

$n \equiv 2 \pmod 3$

Let $n = 3t + 2$

Special case :

If $t = 1$; $\begin{cases} f(v_1) = f(z_1) = 2 \\ \text{Otherwise};\ f(v_1) = f(z_1) = 0 \end{cases}$

Sub case i:

If t is odd

Assign the label to the vertices x_i, z_i ($1 \le i \le n$) as in case (i). Then $m = \left\lceil \frac{a}{2} \right\rceil$

If $t \le 5$ $\begin{cases} f(v_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} \right\rceil \\ f(v_{\left\lceil \frac{t}{2} \right\rceil + i}) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{2} \right\rceil \end{cases}$

For $t > 5$ $\begin{cases} f(v_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} - m \right\rceil \\ f(v_{\left\lceil \frac{t}{2} - m \right\rceil + i}) = 2; 1 \le i \le n - \left\lceil \frac{t}{2} - m \right\rceil \end{cases}$

when $t \equiv \begin{cases} 1 \pmod 3;\ t \ge 7 \\ 0 \pmod 3;\ t \ge 9 \\ 2 \pmod 3;\ t \ge 11 \end{cases}$

Then the value of ‘a’ can be computed as follows;

$$a = \begin{cases} \left\lceil \frac{t+2}{3} \right\rceil;\ t \ge 7 \\ \left\lceil \frac{t}{3} \right\rceil;\ t \ge 9 \\ \left\lceil \frac{t-2}{3} \right\rceil;\ t \ge 11 \end{cases}$$

Then $m = \left\lceil \frac{a}{2} \right\rceil$

Here f (0) = 14t + 9; f (1) = f (2) = 14t +10
Hence f is 3-Total Produce Cordial labeling.

Sub case ii:

If t is even

Special case :

If $\begin{cases} t = 1;\ f(z_1) = 2 \\ \text{Otherwise};\ f(z_1) = 0 \end{cases}$

$f(x_1) = 0$

Assign the label to the vertices x_i, z_i ($1 \le i \le n$) as in case(i).

Then $m = \left\lceil \frac{a}{2} \right\rceil$

$f(v_1) = 0$

If $t \ge 4$ $\begin{cases} f(v_i) = 0;\ 1 \le i \le \left\lceil \frac{t}{2} - m \right\rceil \\ f(v_{\left\lceil \frac{t}{2} - m \right\rceil + i}) = 2;\ 1 \le i \le n - \left\lceil \frac{t}{2} - m \right\rceil \end{cases}$

when $t \equiv \begin{cases} 1 \pmod 3;\ t \ge 4 \\ 0 \pmod 3;\ t \ge 6 \\ 2 \pmod 3;\ t \ge 8 \end{cases}$

Then the value of ‘a’ can be computed as follows

$$a = \begin{cases} \left\lceil \frac{t+2}{3} \right\rceil;\ t \ge 4 \\ \left\lceil \frac{t}{3} \right\rceil;\ t \ge 6\ \left\lceil \frac{t-2}{3} \right\rceil;\ t \ge 8 \end{cases}$$

Then $m = \left\lceil \frac{a}{2} \right\rceil$

Here f (0) = 14t + 9; f (1) = f (2) = 14t + 10
Hence f is 3-Total Produce Cordial labeling. The illustrations for the various cases in the proof of the above theorem is given in the appendix.

Theorem 3: The graph obtained by Cartesian product of star and ladder graph i.e., $S_n \times L_2$ admits E-cordial labeling.

Proof : Let G be the graph obtained by Cartesian product of star S_n and ladder graph L_2 i.e., $S_n \times L_2$ and V(G) = { V_{ij} / i = 1,2,…,n and j = 1,2 }, where i represents the vertices of the star graph S_n and j represents the vertices of ladder graph L_2.

Note that $|V(G)| = 4(n+1)$ and $|E(G)| = 4(2n+1)$ as $|V(S_n)|$ $= n$ and $|E(S_n)| = n$.

Define

$(v_{i1}, v_{ij}) = 0, j = 2,3;$ $\quad i \leq n+1$

$(v_{ij}, v_{i4}) = 1, j = 2,3;$ $\quad i \leq n+1 \quad (1)$

$(v_{ij}, v_{n+1j}) = 0, j = 3,4;$ $\quad i \leq n$

$(v_{ij}, v_{4j}) = 1, j = 3,4;$ $\quad i \leq n, \ (2)$

In view of the above defined labeling pattern f satisfies the conditions for E-cordial labeling as shown in Table 1

Table 1

Integers	Vertex condition	Edge condition
n	$v_f(0) = v_f(1) = n$	$e_f(0) = e_f(1) = n$

Appendix Theorem 1

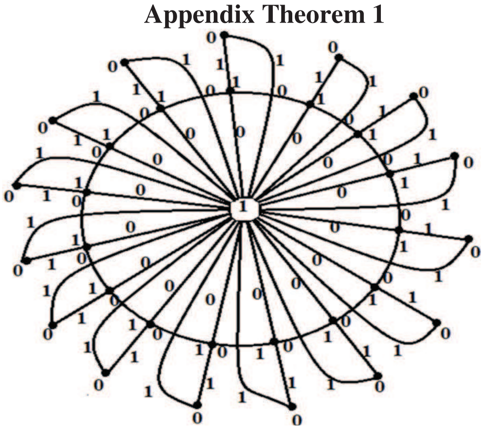


Fig. 3 Sum cordial graph on Flower graph F_{15}

The flower graph F_{15} is the sum cordial labeling.

Let u, $u_1, u_2, u_3, \ldots, u_{15}$, $v_1, v_2, v_3, \ldots v_{15}$ be the vertices of the flower graph F_{15}.

Then $|V(G)| = 2n+1 = 31$ and $|E(G)| = 4n = 60$

$f(u) = 1$, $f(ui) = 1$, $1 \leq i \leq n = 15$; $f(u_1) = f(u_2) = \ldots = f(u_{15}) = 1$

$f(vi) = 0$, $1 \leq i \leq n = 30$; $f(v_1) = f(v_2) = \ldots = f(v_{15}) = 0$

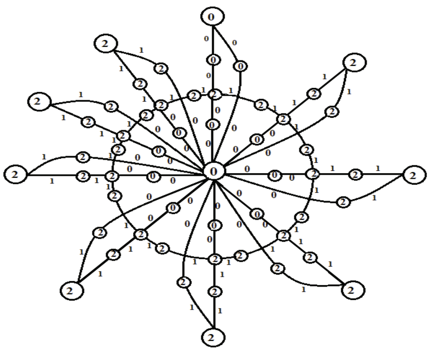
Theorem : 2

Case i: $n \equiv 0 \pmod 3$

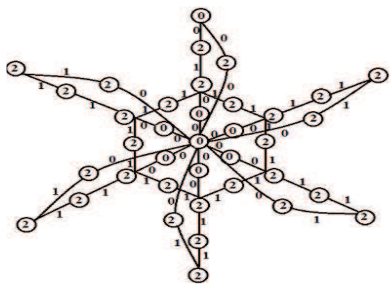
Let n = 3t If t is odd; t = 3

Here f (0) = 14t + 1 = 43; f (1) = f (2) = 14t = 42

f is 3-Total product cordial labeling



4(a) $S(F_9)$



4 (b) $S(F_6)$

Figure 4 : 3 -Total Product Cordial on Sub - division of Flower

If t is even; t = 2

Here f (0) = f (1) = 14t = 28 ; f (2) = 14t + 1 = 29

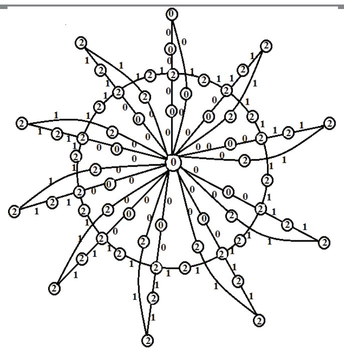
Hence f is 3-Total Produce Cordial labeling.

Case ii : $n \equiv 1 \pmod 3$

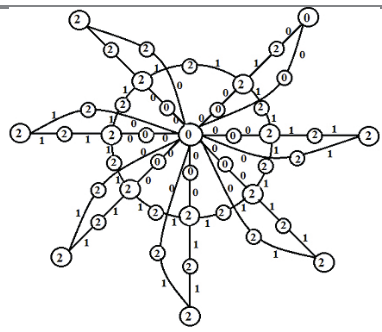
Let n = 3t + 1

If t is odd; if t = 3

Here f (0) = f (1) = f (2) = 14t + 5 = 47. Hence f is 3-Total Product cordial labeling



5(a) $S(F_{10})$



5(b) $S(F_7)$

Figure 5: 3-Total Product Cordial on Sub - division of Flower

If t is even; t = 2

Here f (0) = f (1) = f (2) = 14t + 5 = 33

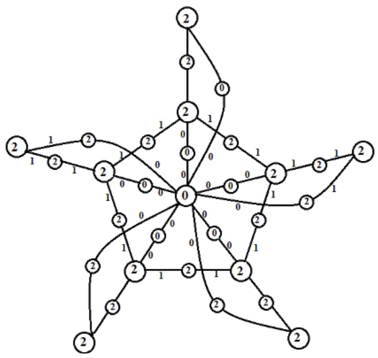
Hence f is 3-Total Produce Cordial labeling

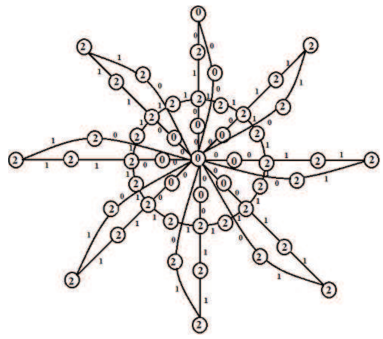
Case iii: n ≡ 2(mod 3)

Let n = 3t + 2 If t is odd; if t = 1

Here f (0) = 14t + 9 = 23; f (1) = f (2) = 14t +10 = 24

Hence f is 3-Total Produce Cordial labeling





6(b) $S(F_8)$

Figure 6: 3-Total Product Cordial on Sub - division of Flower

If t is even; if t = 2

Here f (0) = 14t + 9 = 37; f (1) = f (2) = 14t + 10 = 38

Hence f is 3-Total Produce Cordial labeling.

Theorem 3:

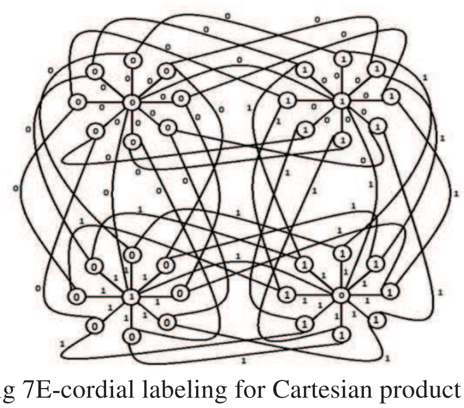


Fig 7E-cordial labeling for Cartesian product of

$S_8 \times L_2$

6(a)S(F_5)

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