A GENERALIZATION OF FUZZY SEMI CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

DR. K. BAGEERATHI

Abstract: The second and third authors have recently introduced the concepts of fuzzy $\mathscr{C}$-closed sets and fuzzy $\mathscr{C}$-closure operator in a fuzzy topological space (X,τ) where $\mathscr{C}$: [0, 1]→ [0, 1] is an arbitrary complement function. The purpose of this paper is to introduce the concept of $\mathscr{C}$- fuzzy semi closed set and investigate some of its properties in a fuzzy topological space.

Key words: $\mathscr{C}$- fuzzy semi closed sets, $\mathscr{C}$- fuzzy semi- closure operator and fuzzy topology.

Introduction: In 1968, Chang introduced the concept of fuzzy topological space [5] which is a natural generalization of topological spaces. Various results in ordinary topological spaces have been put in the fuzzy setting, and also various departures have been observed. Our notation and terminology follow that of Chang[5].

For a fuzzy set λ in a fuzzy topological space X, we denote the fuzzy closure of λ by *cl* λ that equals the infimum of fuzzy closed sets containing λ and the fuzzy interior of λ by *int* λ that equals the supremum of fuzzy open sets contained in λ. The connection between the fuzzy open sets and the fuzzy closed sets is given by the standard complement $\mathscr{C}\lambda$ of λ with membership function $\mathscr{C}\lambda(x) = 1-\lambda(x)$. Several fuzzy topologists used this type of complement while extending the concepts in general topological spaces to fuzzy topological spaces. But there are other complements in the fuzzy literature. The standard complement is obtained by using the function $\mathscr{C}$:[0, 1]→[0, 1] defined by $\mathscr{C}(x) = 1-x$ for all $x \in [0, 1]$. In a fuzzy topological space, a fuzzy subset λ is fuzzy closed if the standard complement of λ is fuzzy open. Such a fuzzy closed set can be thought of a fuzzy closed set with respect to the standard complement $\mathscr{C}\lambda$ of λ with membership function $\mathscr{C}\lambda(x) = 1-\lambda(x)$.

The notions of fuzzy closed sets and fuzzy closure with respect to the standard complement are extended to the analog concepts with respect to an arbitrary complement function $\mathscr{C}$:[0, 1] → [0, 1]. Bageerathi &Thangavelu [2] introduced the concept of fuzzy $\mathscr{C}$- closed sets and fuzzy $\mathscr{C}$- closure operator in a fuzzy topological space (X,τ) where $\mathscr{C}$: [0, 1]→ [0, 1] is an arbitrary complement function in the sense of George J.Klir and Bo Yuan [6]. Using fuzzy $\mathscr{C}$- closure operator the same authors studied fuzzy $\mathscr{C}$- semi open and fuzzy $\mathscr{C}$- semi closed sets. The purpose of this paper is to introduce the notion of $\mathscr{C}$- fuzzy semi closed sets and investigate their properties.

According to Bageerathi &Thangavelu [ ] a fuzzy subset λ of a fuzzy topological space (X,τ) is fuzzy $\mathscr{C}$ closed if $\mathscr{C}\lambda$ is fuzzy open in (X,τ) that is $\mathscr{C}\lambda \in \tau$. The sense technique may be adopted to define a $\mathscr{C}$- fuzzy semi closed set. That is a fuzzy subset λ of a fuzzy topological space (X,τ) is fuzzy $\mathscr{C}$ fuzzy semi closed if $\mathscr{C}\lambda$ is fuzzy semi open that is $\mathscr{C}\lambda \in FSO(X,\tau)$.

The concepts that are needed in this paper are discussed in the second section. The concepts of $\mathscr{C}$- fuzzy semi closed set and $\mathscr{C}$ -fuzzy semi closure operator are introduced in the third and fourth section.

2. Preliminaries: Throughout this paper (X,τ) denotes a fuzzy topological space in the sense of Chang. Let $\mathscr{C}$: [0, 1]→[0, 1] be a complement function. If λ is a fuzzy subset of (X,τ) then the complement $\mathscr{C}\lambda$ of a fuzzy subset λ is defined by $\mathscr{C}\lambda(x) = \mathscr{C}(\lambda(x))$ for all $x \in X$. A complement function $\mathscr{C}$ is said to satisfy

(i) the boundary condition if $\mathscr{C}(0) = 1$ and $\mathscr{C}(1) = 0$,

(ii) monotonic condition if $x \le y \Rightarrow \mathscr{C}(x) \ge \mathscr{C}(y)$, for all $x, y \in [0, 1]$,

(iii) involutive condition if $\mathscr{C}(\mathscr{C}(x)) = x$, for all $x \in [0, 1]$.

The properties of fuzzy complement function $\mathscr{C}$ and $\mathscr{C}\lambda$ are given in George Klir[8] and Bageerathi et al[3]. The following lemma will be useful in sequel.

Lemma 2.2 [3]: Let $\mathscr{C}$: [0, 1] → [0, 1] be a complement function that satisfies the monotonic and involutive conditions. Then for any family $\{\lambda_\alpha : \alpha \in \Delta\}$ of fuzzy subsets of X, we have

(i) $\mathscr{C}(\sup\{\lambda_\alpha(x): \alpha\in\Delta\}) = \inf\{\mathscr{C}(\lambda_\alpha(x)): \alpha\in\Delta\} = \inf\{(\mathscr{C}\lambda_\alpha(x)): \alpha\in\Delta\}$ and

(ii) $\mathscr{C}(\inf\{\lambda_\alpha(x): \alpha\in\Delta\}) = \sup\{\mathscr{C}(\lambda_\alpha(x)): \alpha\in\Delta\} = \sup\{(\mathscr{C}\lambda_\alpha(x)): \alpha\in\Delta\}$ for $x \in X$.

Lemma 2.3 [1] If $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are the fuzzy subsets of X then

$(\lambda_1 \wedge \lambda_2) \times (\lambda_3 \wedge \lambda_4) = (\lambda_1 \times \lambda_4) \wedge (\lambda_2 \times \lambda_3)$.

Lemma 2.4 [Lemma 5.1, [2]] Suppose f is a function from X to Y. Then $f^{-1}(\mathscr{C}\mu) = \mathscr{C}(f^{-1}(\mu))$ for any fuzzy subset μ of Y.

Definition 2.5 [7] If λ is a fuzzy subset of X and μ is a fuzzy subset of Y, then $\lambda \times \mu$ is a fuzzy subset of $X \times Y$, defined by $(\lambda \times \mu)(x, y) = \min\{\lambda(x), \mu(y)\}$ for each $(x, y) \in X \times Y$.

Lemma 2.6 [Lemma 2.1, [1]] Let f : X → Y be a function. If $\{\lambda_\alpha\}$ a family of fuzzy subsets of Y, then (i) $f^{-1}(\vee \lambda_\alpha) = \vee f^{-1}(\lambda_\alpha)$ and (ii) $f^{-1}(\wedge \lambda_\alpha) = \wedge f^{-1}(\lambda_\alpha)$.

Lemma 2.7 [Lemma 2.2, [1]] If λ is a fuzzy subset of X and μ is a fuzzy subset of Y, then $\mathscr{C}(\lambda\times\mu) = \mathscr{C}\lambda \times 1 \vee 1 \times \mathscr{C}\mu$.

Lemma 2.8 [2] Let (X,τ) be a fuzzy topological space. Let $\mathscr{C}$ be a complement function that satisfies the boundary, monotonic and involutive conditions. Then for any family $\{\lambda_\alpha : \alpha\in \Delta\}$ of fuzzy subsets of X. we have $\mathscr{C}(\vee\{\lambda_\alpha: \alpha\in\Delta\})=\wedge\{\mathscr{C}\lambda_\alpha: \alpha\in\Delta\}$ and $\mathscr{C}(\wedge\{\lambda_\alpha: \alpha\in\Delta\})=\vee\{\mathscr{C}\lambda_\alpha: \alpha\in\Delta\}$.

Definition 2.9 [2]A fuzzy subset λ of X is fuzzy closed in (X,τ) if λ^c is fuzzy open in (X,τ). The fuzzy closure of λ is defined as the intersection of all fuzzy closed sets μ containing λ. The fuzzy closure of λ is denoted by $Cl\,\lambda$ that is equal to $\wedge\{\mu: \mu \geq \lambda, \mu^c \in \tau\}$.

Lemma 2.10 [2] For any fuzzy subset λ of X,

(i) $Int\,\lambda^c = Cl(\lambda^c)$ and $(Cl\lambda^c) = Int(\lambda^c)$.

(ii) $\lambda \leq Cl\,\lambda$,

(iii) λ is fuzzy closed $\Leftrightarrow$ $Cl\,\lambda = \lambda$,

(iv) $Cl\,(Cl\,\lambda) = Cl\,\lambda$,

(v) If $\lambda \leq \mu$ then $Cl\,\lambda \leq Cl\,\mu$,

(vi) $Cl\,(\lambda\vee\mu) = Cl\,\lambda \vee Cl\,\mu$,

(vii) $Cl\,(\lambda\wedge\mu) \leq Cl\,\lambda \wedge Cl\,\mu$.

(viii) For any family $\{\lambda_\alpha\}$ of fuzzy sub sets of a fuzzy topological space we have $\vee\, Cl\,\lambda_\alpha \leq Cl\,(\vee\lambda_\alpha)$ and $Cl\,(\wedge\lambda_\alpha) \leq \wedge\, Cl\,\lambda_\alpha$.

Lemma 2.11 [4, 5] Let (X, τ) be a fuzzy topological space. Then a fuzzy subset $\lambda$of X is called

(i)fuzzy regular open in(X, τ) if $Int\,Cl\,\lambda = \lambda$,

(ii) fuzzy semiopen in(X, τ) if $\lambda \leq Cl\,Int\,\lambda$.

3. $\mathscr{C}$-fuzzy semi closed sets: In this section, we define $\mathscr{C}$-fuzzy semi closed sets and investigate some of their basic properties.

Definition 3.1Let (X, τ) be a fuzzy topological space and $\mathscr{C}$ be a complement function. Then a fuzzy subset λ of X is called $\mathscr{C}$- fuzzy semi closed in (X, τ) if $\mathscr{C}\lambda$ is fuzzy semi open.

Example 3.2 Let X = {a, b, c} and $\tau = \{0, \{a_{.2}, b_{.3}, c_{.3}\}, \{a_{.3}, b_{.2}, c_{.4}\}, \{a_{.3}, b_{.3}, c_{.4}\}, \{a_{.2}, b_{.2}, c_{.3}\}, 1\}$. Let $\mathscr{C}(x) = \sqrt{x}$, $0 \leq x \leq 1$, be a complement function. The family of all fuzzy closed sets is given by $\tau^c = \{0, \{a_{.8}, b_{.7}, c_{.7}\}, \{a_{.7}, b_{.8}, c_{.6}\}, \{a_{.7}, b_{.7}, c_{.6}\}, \{a_{.8}, b_{.8}, c_{.7}\}, 1\}$. Let $\lambda = \{a_{.2}, b_{.2}, c_{.2}\}$ Then $\mathscr{C}\lambda = \{a_{.447}, b_{.447}, c_{.447}\}$ and $Int\,\mathscr{C}\lambda = \{a_{.3}, b_{.3}, c_{.4}\}$ and $Cl\,Int\,\mathscr{C}\lambda = \{a_{.7}, b_{.7}, c_{.6}\}$. That implies $\mathscr{C}\lambda \leq Cl\,Int\,\mathscr{C}\lambda$. This shows that λ is $\mathscr{C}$- fuzzy semi closed.

Proposition 3.3 Let (X, τ) be a fuzzy topological space and let $\mathscr{C}$ be a complement function that satisfies the involutive condition. If λ is fuzzy semi open then $\mathscr{C}\lambda$ is $\mathscr{C}$-fuzzy semi closed.

Proof. Let λ be fuzzy semi open. Then by Definition 2.5, $\lambda \leq Cl\,Int\,\lambda$. Taking complement on both sides, $\mathscr{C}\lambda \geq Int\,Cl\,(\mathscr{C}\lambda)$. Again taking complement on both sides, $\mathscr{C}(\mathscr{C}\lambda) \leq Cl\,Int\,\mathscr{C}(\mathscr{C}\lambda))$. Hence $\mathscr{C}\lambda$ is $\mathscr{C}$- fuzzy semi closed.

Conversly assume that $\mathscr{C}\lambda$ is $\mathscr{C}$-fuzzy semi closed. Then by Definition 3.1, $\mathscr{C}(\mathscr{C}\lambda) \leq Cl\,Int\,\mathscr{C}(\mathscr{C}\lambda))$. Since $\mathscr{C}$ satisfies the involutive condition, $\lambda \leq Cl\,Int\,\lambda$. This proves that λ is fuzzy semi open.

The following example shows that the converse of the above proposition is not true if $\mathscr{C}$ satisfies the involutive condition.

Example 3.4 Let X = {a, b, c} and $\tau = \{0, \{a_{.6}, b_{.3}\}, \{b_{.4}, c_{.7}\}, \{a_{.2}, c_{.5}\}, \{b_{.3}\}, \{a_{.6}, b_{.4}, c_{.7}\}, \{a_{.2}, b_{.4}, c_{.7}\}, \{c_{.5}\}, \{a_{.2}\}, \{a_{.6}, b_{.3}, c_{.5}\}, \{a_{.2}, b_{.3}\}, \{a_{.2}, b_{.3}, c_{.5}\}, \{b_{.3}, c_{.5}\}, 1\}$. Then (X, τ) is a fuzzy topological space. Let $\mathscr{C}(x) = \dfrac{1-x}{1+3x}$, $0 \leq x \leq 1$, be a complement function that satisfies the involutive condition. The family of all fuzzy closed sets $\tau^c = \{0, \{a_{.4}, b_{.7}, c_1\}, \{a_1, b_{.6}, c_{.3}\}, \{a_{.8}, b_1, c_{.5}\}, \{a_1, b_{.7}, c_1\}, \{a_{.4}, b_{.6}, c_{.3}\}, \{a_{.8}, b_{.6}, c_{.3}\}, \{a_1, b_1, c_{.5}\}, \{a_{.8}, b_1, c_1\}, \{a_{.4}, b_{.7}, c_{.5}\}, \{a_{.8}, b_{.7}, c_1\}, \{a_{.8}, b_{.7}, c_{.5}\}, \{a_1, b_{.7}, c_{.5}\}, 1\}$. Let $\lambda = \{a_{.2}\}$ it can be computed that $Int\lambda = \{a_{.2}\}$, $ClInt\lambda = \{a_{.4}, b_{.7}, c_{.5}\}$. This shows that $\lambda \leq ClInt\,\lambda$. Now $\mathscr{C}\lambda = \{a_{.5}\}$then $\mathscr{C}(\mathscr{C}\lambda) = \{a_{.2}\}$ it can be computed that $ClInt\,\mathscr{C}(\mathscr{C}\lambda) = \{a_{.4}, b_{.7}, c_{.5}\}$. Hence $\mathscr{C}(\mathscr{C}\lambda) \leq ClInt\,\mathscr{C}(\mathscr{C}\lambda)$.

Azad [2] established that the union of fuzzy semi closed sets is not fuzzy semi closed. Analogously the union of any two $\mathscr{C}$- fuzzy semi closed sets is need not be $\mathscr{C}$-fuzzy semi closed. The intersection of fuzzy semi closed sets is a fuzzy semi closed set. However the following example shows that the intersection of any two $\mathscr{C}$- fuzzy semi closed sets is not $\mathscr{C}$- fuzzy semi closed.

Example 3.5: Let X = {a, b, c} and $\tau = \{0, \{c_{.3}\}, \{a_{.6},\}, \{a_{.6}, c_{.3}\}, 1\}$. Let $\mathscr{C}(x) = 1-x/1-2x$, $0 \leq x \leq 1$, be a complement function. From this, we see that the complement function $\mathscr{C}$ does not satisfy the monotonic and involutive conditions. The family of all fuzzy closed sets is given by $\tau^c = \{0, \{a_1, b_1, c_{.7}\}, \{a_{.4}, b_1, c_1\}, \{a_{.4}, b_{.7}, c_1\}, 1\}$. Let $\lambda = \{a_{.6}\}$ and $\mu = \{c_{.3}\}$, then $\mathscr{C}\lambda = \{a_{.182}, b_{.1}, c_1\}$ and $\mathscr{C}\mu = \{a_{.1}, b_{.1}, c_{.438}\}$, it can be found that $Int\,\mathscr{C}\lambda = \{a_{.3}\}$ and $Cl\,Int\,\mathscr{C}\lambda = \{a_{.4}, b_{.7}, c_1\}$. That implies $\mathscr{C}\lambda \leq Cl\,Int\,\mathscr{C}\lambda$. This shows that λ is $\mathscr{C}$- fuzzy semi closed. Also $Int\,\mathscr{C}\mu = \{c_{.3}\}$ and $Cl\,Int\,\mathscr{C}\mu = \{a_{.4}, b_{.7}, c_1\}$. That implies $\mathscr{C}\mu \leq Cl\,Int\,\mathscr{C}\mu$. This shows that μ is $\mathscr{C}$- fuzzy semi closed. Now $(\lambda\vee\mu) = \{a_{.7}, b_{.6}, c_{.5}\}$ then $\mathscr{C}(\lambda\vee\mu) = \{a_{.837}, b_{.775}, c_{.707}\}$ it can be found that $Int\,\mathscr{C}(\lambda\vee\mu) = \{a_{.3}, b_{.3}, c_{.4}\}$ and $Cl\,Int\,\mathscr{C}(\lambda\vee\mu) = \{a_{.4}, b_{.7}, c_1\}$ This implies that $\mathscr{C}(\lambda\vee\mu) \nleq Cl\,Int\,\mathscr{C}(\lambda\vee\mu)$. By using Proposition 3.2, $\lambda\vee\mu$ is not $\mathscr{C}$- fuzzy semi closed.

Example 3.6 Let X = {a, b, c} and $\tau = \{0, \{a_{.2}, b_{.3}, c_{.3}\}, \{a_{.3}, b_{.2}, c_{.4}\}, \{a_{.3}, b_{.3}, c_{.4}\}, \{a_{.2}, b_{.2}, c_{.3}\}, 1\}$. Let $\mathscr{C}(x) = 1-x/1-2x$, $0 \leq x \leq 1$, be a complement function. From this, we see that the complement function $\mathscr{C}$ does not satisfy the monotonic and involutive conditions. The family of all fuzzy closed sets is given by $\tau^c = \{0, \{a_{.8}, b_{.7}, c_{.7}\}, \{a_{.7}, b_{.8}, c_{.6}\}, \{a_{.7}, b_{.7}, c_{.6}\}, \{a_{.8}, b_{.8}, c_{.3}\}\ 1\}$. Let $\lambda = \{a_{.1}, b_{.6}, c_{.1}\}$ and $\mu = \{a_{.7}, b_{.1}, c_{.5}\}$, then $\mathscr{C}\lambda = \{a_{.316}, b_{.775}, c_{.316}\}$ and $\mathscr{C}\mu = \{a_{.837}, b_{.316}, c_{.707}\}$, it can be found that $Int\,\mathscr{C}\lambda = \{a_{.2}, b_{.3}, c_{.3}\}$ and $Cl\,Int\,\mathscr{C}\lambda = \{a_{.7}, b_{.7}, c_{.6}\}$. That implies $\mathscr{C}\lambda \leq Cl\,Int\,\mathscr{C}\lambda$. This shows that λ is $\mathscr{C}$- fuzzy semi closed. Also $Int\,\mathscr{C}\mu = \{a_{.3}, b_{.3}, c_{.4}\}$ and $Cl\,Int\,\mathscr{C}\mu = \{a_{.7}, b_{.7}, c_{.6}\}$ That implies $\mathscr{C}\mu \leq Cl\,Int\,\mathscr{C}\mu$. This shows that μ

is $\mathscr{C}$- fuzzy semi closed. Now $(\lambda\wedge\mu) = \{a_{.7}, b_{.6}, c_{.5}\}$ then $\mathscr{C}(\lambda\wedge\mu) = \{a_{.837}, b_{.775}, c_{.707}\}$ it can be found that $Int\,\mathscr{C}(\lambda\vee\mu) = \{a_{.3}, b_{.3}, c_{.4}\}$ and $Int\ Cl\ \mathscr{C}(\lambda\wedge\mu) = \{a_{.7}, b_{.7}, c_{.6}\}$.This implies that $\mathscr{C}(\lambda\wedge\mu) \not\leq IntCl\ \mathscr{C}(\lambda\wedge\mu)$. By using Proposition 2.2, $\lambda\vee\mu$ is not $\mathscr{C}$- fuzzy semi closed.

Remark 3.7: Further, the Example 3.6 shows that the intersection of any two $\mathscr{C}$- fuzzy semi closed sets is not $\mathscr{C}$ - fuzzy semi closed, eventhough the complement function $\mathscr{C}$ satisfies the monotonic and involutive conditions. If the complement function $\mathscr{C}$ satisfies the monotonic and involutive conditions then the arbitrary intersection of any two $\mathscr{C}$- fuzzy semi closed sets is $\mathscr{C}$- fuzzy semi closed as shown in the following Proposition.

Proposition 3.8: Let (X, τ) be a fuzzy topological space and let $\mathscr{C}$ be a complement function that satisfies the monotonic and involutive conditions.Then the arbitrary intersection of any two $\mathscr{C}$- fuzzy semi closed sets is $\mathscr{C}$- fuzzy semi closed.

Proof. Let $\{\lambda_\alpha\}$ be a collection of all $\mathscr{C}$- fuzzy semi closed sets of a fuzzy topological space X. Then by Definition 3.1, $\{\mathscr{C}\lambda_\alpha\}$ be a collection of all fuzzy semi open sets. Then for each α , there exists a fuzzy semi open set $\mathscr{C}\mu_\alpha$ such that $\mathscr{C}\mu_\alpha \leq Int\ cl\ \mu_\alpha$. By using Lemma 3.3in [4], $\vee\{\mathscr{C}\mu_\alpha\}$ is fuzzy semi open. By using Lemma 2.4, $\mathscr{C}\{\wedge\mu_\alpha\}$ is fuzzy semi open. By Definition 3.1, $\wedge\mu_\alpha$ is $\mathscr{C}$- fuzzy semi closed. This implies that $\wedge\lambda_\alpha$ is $\mathscr{C}$- fuzzy semi closed.

It is clear that every fuzzy regular open is $\mathscr{C}$- fuzzy semi closed set. But the converse is not true as shown by the following example.

Example 3.9 From Example 3.8, let X = {a, b, c} and $\tau = \{0, \{a_{.2}, b_{.3}, c_{.3}\}, \{a_{.3}, b_{.2}, c_{.4}\}, \{a_{.3}, b_{.3}, c_{.4}\}, \{a_{.2}, b_{.2}, c_{.3}\}, 1\}$. Let $\lambda = \{a_{.1}, b_{.2}, c_{.1}\}$ Then $\mathscr{C}\lambda = \{a_{.316}, b_{.447}, c_{.316}\}$ $Int\ \mathscr{C}\lambda = \{a_{.2}, b_{.3}, c_{.4}\}$ and $Cl\ Int\ \mathscr{C}\lambda = \{a_{.7}, b_{.7}, c_{.6}\}$. That implies $\mathscr{C}\lambda \leq Cl\ Int\ \mathscr{C}\lambda$. This shows that λ is $\mathscr{C}$- fuzzy semi closed. Also λ is not fuzzy regular open.

4. $\mathscr{C}$-fuzzy semi closure: In this section,we introduce the concepts of $\mathscr{C}$- fuzzy semi-closure operators and investigate some of their basic properties.

Definition 4.1 Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset λ of X, the $\mathscr{C}$- fuzzy semi closure of λ (briefly $\mathscr{C}$-*scl* λ), is the intersection of all $\mathscr{C}$- fuzzy semi closed sets containing λ. That is $\mathscr{C}\text{-}scl\ \lambda = \wedge\{\mu : \mu \geq \lambda, \mu \text{ is } \mathscr{C}\text{- fuzzy semi closed}\}$.

The concepts of "fuzzy $\mathscr{C}$- semi closure" and "$\mathscr{C}$- fuzzy semi closure" are identical if $\mathscr{C}$ is the standard complement function.

Proposition 4.2 If the complement functions $\mathscr{C}$ satisfies the monotonic and involutive conditions. Then for any fuzzy subset λ of X, (i) $\mathscr{C}(SInt\lambda) = \mathscr{C}\text{-}scl(\mathscr{C}\lambda)$ and (ii) $\mathscr{C}(\mathscr{C}\text{-}scl\ \lambda) = SInt(\mathscr{C}\lambda)$, where $SInt\lambda$ is the union of all fuzzy semi open sets contained in λ.

Proof. By using Definition , $SInt\lambda = \vee\{\mu : \mu \leq \lambda, \mu$ is fuzzy semi open$\}$. Taking complement on both sides, we get $\mathscr{C}(SInt(\lambda)(x)) = \mathscr{C}(\sup\{\mu(x): \mu(x) \leq -\lambda(x), \mu$ is fuzzy semi open$\})$. Since $\mathscr{C}$ satisfies the monotonic and involutive conditions, by using Lemma 1.38, $\mathscr{C}(SInt(\lambda)(x)) = \inf\{\mathscr{C}(\mu(x)): \mu(x) \leq \lambda(x), \mu$ is fuzzy semi open$\}$. This implies that $\mathscr{C}(SInt(\lambda)(x)) = \inf\{\mathscr{C}\mu(x): \mathscr{C}\mu(x) \geq \mathscr{C}\lambda(x), \mu$ is fuzzy semi open$\}$. By using Proposition 2.2, $\mathscr{C}\mu$ is $\mathscr{C}$- fuzzy semi closed, by replacing $\mathscr{C}\mu$ by η, we see that $\mathscr{C}(SInt(\lambda)(x)) = \inf\{\eta(x): \eta(x) \geq \mathscr{C}\lambda(x), \mathscr{C}\eta$ is fuzzy semi open$\}$. By Definition 2.10, $\mathscr{C}(SInt(\lambda)(x)) = \mathscr{C}\text{-}scl(\mathscr{C}\lambda)(x)$. This proves that $\mathscr{C}(SInt\lambda) = \mathscr{C}\text{-}scl(\mathscr{C}\lambda)$. By using Definition 2.10, $\mathscr{C}\text{-}scl\ \lambda = \wedge\{\mu: \lambda \leq \mu, \mu \text{ is } \mathscr{C}\text{- fuzzy semi closed}\}$. Taking complement on both sides, we get $\mathscr{C}(\mathscr{C}\text{-}scl\ \lambda(x)) = \mathscr{C}(\inf\{\mu(x): \mu(x) \geq \lambda(x), \mu \text{ is } \mathscr{C}\text{- fuzzy semi closed}\})$. Since $\mathscr{C}$ satisfies the monotonic and involutive conditions, by using Lemma 1.38, $\mathscr{C}(\mathscr{C}\text{-}scl\ \lambda(x)) = \sup\{\mathscr{C}(\mu(x)): \mu(x) \geq \lambda(x), \mu \text{ is } \mathscr{C}\text{-fuzzy semi closed}\}$. That implies $\mathscr{C}(\mathscr{C}\text{-}scl\ \lambda(x)) = \sup\{\mathscr{C}\mu(x): \mathscr{C}\mu(x)) \leq \mathscr{C}\lambda(x), \mu \text{ is } \mathscr{C}\text{- fuzzy semi closed}\}$. By using Proposition 2.2, $\mathscr{C}\mu$ is fuzzy semi open, by replacing $\mathscr{C}\mu$ by η, we see that $(\mathscr{C}\text{-}scl\ \lambda(x)) = \sup\{\eta(x): \eta(x) \leq \mathscr{C}\lambda(x), \eta$ is fuzzy semi open$\}$. By using Definition 2.10, $\mathscr{C}(\mathscr{C}\text{-}scl\ \lambda(x)) = SInt(\mathscr{C}\lambda)(x)$. This proves $\mathscr{C}(\mathscr{C}\text{-}scl(\lambda)) = SInt(\mathscr{C}\lambda)$.

Proposition 4.3 Let (X, τ) be a fuzzy topological space and let $\mathscr{C}$ be a complement function that satisfies the monotonic and involutive conditions. Then for the fuzzy subsets λ and μ of a fuzzy topological space X, we have

(i) $\lambda \leq \mathscr{C}\text{-}scl\ \lambda$,

(ii) λ is $\mathscr{C}$- fuzzy semi closed $\Leftrightarrow \mathscr{C}\text{-}scl\ \lambda = \lambda$,

(iii) $\mathscr{C}\text{-}scl(\mathscr{C}\text{-}scl\ \lambda) = \mathscr{C}\text{-}scl\ \lambda$,

(iv) If $\lambda \leq \mu$ then $\mathscr{C}\text{-}scl\ \lambda \leq \mathscr{C}\text{-}scl\ \mu$.

Proof. The proof for (i) follows from $\mathscr{C}\text{-}scl\ \lambda = \inf\{\mu: \mu \geq \lambda, \mu \text{ is } \mathscr{C}\text{- fuzzy semi closed}\}$.

Let λ be $\mathscr{C}$- fuzzy semi closed. Since $\mathscr{C}$ satisfies the monotonic and involutive conditions, by using Proposition 3.2, $\mathscr{C}\lambda$ is fuzzy semi open. By using Proposition 3.2 in [2], $SInt(\mathscr{C}\lambda) = \mathscr{C}\lambda$. By using Proposition 3.11, $\mathscr{C}(\mathscr{C}\text{-}scl\ \lambda) = \mathscr{C}\lambda$. Taking complement on both sides, we get $\mathscr{C}(\mathscr{C}(\mathscr{C}\text{-}scl\ \lambda)) = \mathscr{C}(\mathscr{C}\lambda)$. Since the complement function $\mathscr{C}$ satisfies the involutive condition, $\mathscr{C}\text{-}scl\ \lambda = \lambda$. Conversely, we assume that $\mathscr{C}\text{-}scl\ \lambda = \lambda$. Taking complement on both sides, we get $\mathscr{C}(\mathscr{C}\text{-}scl\ \lambda) = \mathscr{C}\lambda$. By using Proposition 2.11, $SInt\,\mathscr{C}\lambda = \mathscr{C}\lambda$. By using Proposition 3.2 in [2], $\mathscr{C}\lambda$ is fuzzy semi open. Again by using Proposition 3.2, λ is $\mathscr{C}$ - fuzzy semi closed. Thus (ii) proved.

By using Proposition 3.11, $\mathscr{C}(\mathscr{C}\text{-}scl\ \lambda) = SInt(\mathscr{C}\lambda)$. This implies that $\mathscr{C}(\mathscr{C}\text{-}scl\ \lambda)$ is fuzzy semi open. By using Proposition 3.2, $\mathscr{C}\text{-}scl\ \lambda$ is $\mathscr{C}$- fuzzy semi closed. By applying (ii), we have $\mathscr{C}\text{-}scl(\mathscr{C}\text{-}scl\ \lambda) = \mathscr{C}\text{-}scl\ \lambda$. This proves (iii).

Suppose $\lambda \leq \mu$. Since $\mathscr{C}$ satisfies the monotonic condition, $\mathscr{C}\lambda \geq \mathscr{C}\mu$, that implies $SInt\,\mathscr{C}\lambda \geq SInt\,\mathscr{C}\mu$. Taking complement on both sides, $\mathscr{C}(SInt\,\mathscr{C}\lambda) \leq \mathscr{C}(SInt\,\mathscr{C}\mu)$. Then by using Proposition 4.2, $\mathscr{C}\text{-}scl\ \lambda \leq \mathscr{C}$-

scl μ.This proves (iv).

Proposition 4.4 Let (X, τ) be a fuzzy topological space and let $\mathscr{C}$ be a complement function that satisfies the monotonic and involutive conditions. Then for any two fuzzy subsets λ and μ of a fuzzy topological space, we have (i) $\mathscr{C}$-*scl* $(\lambda\vee\mu)$ = $\mathscr{C}$-*scl* $\lambda\vee$ $\mathscr{C}$-*scl* μ and (ii) $\mathscr{C}$-*scl* $(\lambda \wedge \mu) \leq$ $\mathscr{C}$-*scl* $\lambda \wedge$ $\mathscr{C}$-*scl* μ.

Proof. Since $\mathscr{C}$ satisfies the involutive condition, $\mathscr{C}$-*scl* $(\lambda\vee\mu)$ = $\mathscr{C}$-*scl* $(\mathscr{C}(\mathscr{C}(\lambda\vee\mu)))$. Since $\mathscr{C}$ satisfies the monotonic and involutive conditions, by using Proposition 4.2, $\mathscr{C}$-*scl* $(\lambda\vee\mu) = \mathscr{C}$S*Int*$(\mathscr{C}(\lambda\vee\mu)))$. Using Lemma 2.4, $\mathscr{C}$-*scl* $(\lambda\vee\mu) = \mathscr{C}$(S*Int* $(\mathscr{C}\lambda\wedge\mathscr{C}\mu))$. Again by Proposition3.11, $\mathscr{C}$-*scl* $(\lambda\vee\mu)\leq$((S*Int* $\mathscr{C}\lambda)\wedge$(S*Int* $\mathscr{C}\mu$))= $\mathscr{C}$(S*Int* $\mathscr{C}$ $\lambda)\vee\mathscr{C}$(S*Int* $_{\mathscr{C}}\mathscr{C}\mu$). By using Proposition 3.11, $\mathscr{C}$-*scl* $(\lambda\vee\mu) \leq$ $\mathscr{C}$-*scl* $\lambda \vee$ $\mathscr{C}$-*scl* μ. Also $\mathscr{C}$-*scl* $(\lambda)\leq$ $\mathscr{C}$-*scl* $(\lambda\vee\mu)$ and $\mathscr{C}$-*scl* $(\mu) \leq$ $\mathscr{C}$-*scl* $(\lambda\vee\mu)$ that implies $\mathscr{C}$-*scl* $_{\mathscr{C}}(\lambda) \vee$ $\mathscr{C}$-*scl* $(\mu) \leq$ $\mathscr{C}$-*scl* $(\lambda\vee\mu)$. It follows that $\mathscr{C}$-*scl* $(\lambda\vee\mu)$ = $\mathscr{C}$-*scl* $\lambda \vee$ $\mathscr{C}$-*scl* μ. Since $\mathscr{C}$-*scl* $(\lambda\wedge\mu) \leq$ $\mathscr{C}$-*scl* λ and $\mathscr{C}$-*scl* $(\lambda\wedge\mu) \leq$ $\mathscr{C}$-*scl* μ, it follows that $\mathscr{C}$-*scl* $(\lambda\wedge\mu) \leq$ $\mathscr{C}$-*scl* $\lambda \wedge$ $\mathscr{C}$-*scl* μ.

Proposition 4.5Let $\mathscr{C}$ be a complement function that satisfies the monotonic and involutive conditions. Then for any family $\{\lambda_\alpha\}$ of fuzzy subsets of a fuzzy topological space, we have (i) $\vee$ ($\mathscr{C}$-*scl* λ_α) $\leq$ $\mathscr{C}$-*scl* ($\vee\lambda_\alpha$) and (ii) $\mathscr{C}$-*scl* $(\wedge\lambda_\alpha) \leq \wedge($ $\mathscr{C}$-*scl* λ_α)

Proof. For every β, $\lambda_\beta \leq \vee\lambda_\alpha \leq$ $\mathscr{C}$-*scl* $(\vee\lambda_\alpha)$. By using Proposition 3.12, $\mathscr{C}$-*scl* $\lambda_\beta \leq$ $\mathscr{C}$-*scl* $(\vee\lambda_\alpha)$ for every β. This implies that $\vee$ $\mathscr{C}$-*scl* $\lambda_\beta \leq$ $\mathscr{C}$-*scl* $(\vee\lambda_\alpha)$. This proves (i). Now $\wedge\lambda_\alpha \leq \lambda_\beta$ for every β. Again using Proposition 3.12, we get $\mathscr{C}$-*scl* $(\wedge\lambda_\alpha) \leq$ $\mathscr{C}$-*scl* λ_β. This implies that $\mathscr{C}$-*scl* $(\wedge\lambda_\alpha) \leq \wedge$ $\mathscr{C}$-*scl* λ_α. This proves (ii).

References:

1. K.K.Azad, On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity, *J.Math.Anal.Appl*. 82(1)(1981), 14-32.
2. Bageerathi, K.,Sutha,G. and Thangavelu, P.(2011).A generalization of fuzzy closed sets, *International Journal of fuzzy systems and rough systems*, 4(1): 1-5.
3. K.Bageerathi, P.Thangavelu, On generalization of nearly fuzzy open sets, Proceedings of the International Seminar On New Trends in Applications of Mathematics (ISNTAM 2011) Bharata Mata College, Kerala, Jan 31, Feb 1-2(2011). **(To appear).**
4. C.L.Chang, Fuzzy topological spaces, *J.Math.Anal.Appl*. 24 (1968), 182-190.
5. George J.Klir and Bo Yuan, Fuzzy Sets and Fuzzy Logic Theory and Applications,*Prentice – Hall, Inc*., 2005.
6. T.H.Yalvac, Semi-interior and semi-closure of fuzzy set, *J.Math.Anal.Appl.132* (2)(1988), 356-364.
7. L.A. Zadeh, Fuzzy sets*, Inform. and Control* 8(1965), 338-353.

***

Dr. K. Bageerathi/Assistant Professor of Mathematics/Aditanar College of Arts and Science/Tiruchendur/
Tamil Nadu-628216/India/ mrcsm5678@rediffmail.com