PROPERTIES OF NULL-ADDITIVE FUZZY MEASURE ON LOCALLY COMPACT HAUSDORFF SPACE

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Abstract: In this paper, properties of fuzzy measure on locally compact hausdorff space under the null-additivity condition, and the properties of the inner/outer regularity and the regularity of fuzzy measure are investigated.

Keywords: Fuzzy measure, null-additivity, regularity Properties, null additive fuzzy measure.

Introduction: The well-known Lusin's theorem in classical measure theory is very important and useful for discussing the continuity and the approximation of measurable function on metric spaces are investigated in [1,2]. Li [1] investigated the regularity of null-additive fuzzy measure on metric spaces and showed Lusin's theorem on fuzzy measure space under the null-additivity condition. we assume that X is a locally compact hausdorff space.and that V,F,K are classes of all open ,closed and compact set in X respectively and μ is a positive fuzzy measure. B denotes borel σ - algebra on X, it is the smallest σ algebra containing V. In this paper, properties of null-additive fuzzy measure locally compact hausdorff space under the null-additivity condition, some properties of the inner/outer regularity and the regularity of fuzzy measure are investigated.

Definition: 1.1: A **fuzzy measure** μ on (X,B) is an extended real valued set function $\mu : F \to [0,\infty]$ satisfying the following conditions.

i) $\mu(\phi)=0$

ii) $\mu(A)\leq\mu(B)$ whenever $A\subset B$ and $A,B\in F$

Definition: 1.2: A fuzzy measure μ is called **null-additive**,if $\mu(E\cup F)=\mu(E)$ whenever $E,F\in B$ and $\mu(F)=0$.

Definition:1.3: A fuzzy measure μ is called **outer regular** if $\mu(E)=\inf\{\mu(V)\backslash V\supset E, V \text{ open.}\}$

Definition:1.4: A fuzzy measure μ is called **inner regular**.if $\mu(E)=\sup\{\mu(F)\backslash F\subset E, F \text{ closed}\}$
$=\sup\{\mu(K)\backslash K\subset E, K \text{ compact}\}$

Definition: 1.5: X is **locally compact** if every point of x has a neighbourhood whose closure is compact.

Definition: 1.6: A set E in X is called **σ-compact** if E is a countable union of compact sets.

2. Main Results:

Theorem:2.1: Let X is a locally compact ,σ-compact, hausdorff spaces, μ is describe as in the statement of Definition: 1.3 & 1.4, then the following statements of the set $E \in B$ are equivalent.

a) If $E\in B$ and $\varepsilon > 0$ there is a closed set F and an open set V such that $F\subset E\subset V$ and $\mu(V-F)<\varepsilon$.

b) μ is regular on X.

c) If $E\in B$, there are sets A and B such that A is an F_σ,B is an G_δ $A\subset E\subset B$ and $\mu(B-A)=0$.

Proof: To Prove: (b)=>(a)

Since X is σ-compact,we have $X=K_1\cup K_2\cup \ldots$where each K_n is compact.

Let $E\in B$ and $\varepsilon>0$ Since every σ-compact set has σ-finite measure,we have $E=\cup(E\cap K_n)$,

$\mu(E\cap Kn)<\infty$,for all n

Hence, there exist an open set V_n containing $E\cap K_n$ suchthat

$\mu(V_n-(E\cap K_n))<\varepsilon/2^{n+1}$,n=1,2,…

By the Definition of inner regularity,

$\mu(E)=\inf\{\mu(V),V\supset E,V \text{ Open}\}$ we have,

$\mu(V)<\mu(E)+\varepsilon$

$\Rightarrow$ $\mu(V)-\mu(E)<\varepsilon$

Let $V=\cup Vn$ is an open set.

Then $V-E\subset\cup(V_n-(E\cap K_n))$

Hence, $\mu(V-E)\leq\sum\mu(V_n-(E\cap K_n)) <\sum \varepsilon/2^{n+1}=\varepsilon/2$

Thus $\mu(V-E)<\varepsilon/2$……………………….(1)

Take E^c,and apply the above argument to E^c in place of E,there is an open set $W\supset E^c$ Suchthat

$\mu(W-E^c)<\varepsilon/2$

But $W-E^c=E-W^c$ And $W^c\subset E$

Take $F=W^c$ then $F\subset E$ and $E-F= W-E^c$

Hence $\mu(E-F)<\varepsilon/2$,

$V-F\subset(V-E)\cup(E-F)$

$\mu(V-F)<\mu(V-E)+\mu(E-F) <\varepsilon/2+\varepsilon/2$ $\mu(V-F)<\varepsilon$

To Prove: (a)=>(b):

If F is a closed in X,then $F=\cup(K_n\cap F)$,Each $K_n\cap F$ is compact $\cup(K_n\cap F)=\cup K_n\cap F=X\cap F=F$

Hence, $\mu((K_1\cup K_2\cup\ldots\cup K_n)\cap F)\to\mu(F)$ as $n\to\infty$ (ie) $\mu(F)$ is the sup of $\mu(E)$ $\mu(E)=\sup\{\mu(F),F\subset E,F \text{ is closed}\}$

$\Rightarrow$ μ is regular

To prove: (a)=> (c): For any positive integer j,there exist a closed set F_j and open set V_j such that

$F_j\subset E\subset V_j$ and $\mu(V_j-F_j)<1/\varepsilon_j$ $A=\cup F_j$ and $B=\cap V_j$

A is an F_σ set,B is G_δ set and $A\subset E\subset B$,$B\subset V_j$ and $F_j\supset A$ for all j

Therefore $B-A\subset Vj-Fj\mu(B-A)\leq\mu(V_j-F_j)<1//\varepsilon_j$ for allj , as $j\to\infty\mu(B-A)\to 0$ Hence the proof.

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