EXPECTED TIME TO RECRUITMENT FOR A SINGLE GRADE MANPOWER SYSTEM WITH DIFFERENT EPOCHS FOR DECISIONS AND EXITS HAVING INTER-DECISION TIMES AS AN ORDER STATISTICS

A.DEVI, A. SRINIVASAN

Abstract: In this paper, the problem of time to recruitment is studied using a univariate policy of recruitment for a single grade manpower system in which attrition takes places due to policy decisions. Assuming that the policy decisions and exits occur at different epochs, a stochastic model is constructed and the variance of the time to recruitment is obtained when the inter-policy decision times and inter-exit times form an order statistics and an ordinary renewal process respectively. The effect of nodal parameters on the performance measures is studied.

Keywords: Decision and exit epochs, order statistics, ordinary renewal process, single grade manpower system, univariate policy of recruitment, variance of time to recruitment.

Introduction: Attrition is a common phenomenon in many marketing organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the breakdown threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. Many researchers have studied several problems in manpower planning using different methods. In [1] and [2] the authors have discussed some manpower planning models for a single and multi-grade manpower system using Markovian and renewal theoretic approach. In [13], the author has analyzed the problem of time to recruitment for a single grade manpower system which is subjected to attrition, using a univariate policy of recruitment based on shock model approach for replacement of systems in reliability theory. In [10], the author has considered a single grade manpower system and obtained system characteristics when the loss of manpower process and inter-decision time process form a correlated pair of renewal sequence by employing a univariate cum policy of recruitment. The optimum cost of recruitment for a single grade manpower system is obtained in [15] by using several univariate and bivariate policies of recruitment. For the study of this problem corresponding to correlated inter-decision times under different policies of recruitment, one can refer to [9] and [17]. In [7], the author has initiated the study of the problem of time to recruitment for a single grade manpower system by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory thresholds for the cumulative loss of manpower in this manpower system. The author [11] has considered the problem of time to recruitment for a single grade manpower system when inter-decision times form an order statistics. Variance of the time to recruitment for a single grade manpower system with optional and mandatory thresholds is obtained in [16] when inter-decision times form an order statistics. In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decisions points. This aspect is taken into account for the first time in [3] and the variance of the time to recruitment is obtained when the inter-decision times and exit times are independent and non-identically distributed exponential random variables, using a univariate policy for recruitment and Laplace transform in the analysis. This problem is studied in [4] by using a different probabilistic analysis and in [5] and [6] the authors have extended the work in [3] and [4] for correlated inter-policy decision times. The present paper extends the research work in [6], when the inter-decision times form an order statistics.

Model Description: Consider an organization taking policy decisions at random epochs in $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…, let X_i be independent and identically distributed exponential random variables representing the amount of depletion of manpower (loss of man hours) due to i^{th} exit point with probability distribution function M(.), density function m(.) and mean $\frac{1}{\alpha}$ (α>0). It is assumed that A_i , the time between $(i-1)^{th}$ and i^{th} policy decisions,are independent and identidically distributed random variables with distribution F(.) and probability density function f(.). Let $F_{a(j)}(.)$ and $f_{a(j)}(.)$ be the distribution and the probability density function of the j^{th} order statistics(j=1,2,…,r) selected from the sample of size r from the population $\{A_i\}_{i=1}^{\infty}$. Let B_i, be the time between the $(i-1)^{th}$ and i^{th} exit times. It is assumed that $B_i's$ be independent and identically distributed random variables with probability distribution function G(.), density function g(.) and mean $\frac{1}{\delta}$ (δ>0). Let D_{i+1} be the waiting time upto (i+1) exits. Let Y be the independent breakdown threshold level for the cumulative depletion of manpower in the organization with probability distribution function H(.) and density function h(.). Let q be the probability that every policy decision has exit of personnel. It is assumed that $q \neq 0$, since q=0 corresponds to the case where exits are impossible. Let $\chi(I)$ be the indicator function of the event I. Let T be the random variable denoting the time to recruitment with mean E(T) and variance V(T). The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the breakdown threshold.

Main Result: In this section the distribution and the variance of time to recruitment are obtained. By the recruitment policy, recruitment is done whenever the cumulative loss of manpower exceeds the threshold Y. When the first decision is taken, recruitment would not have been done for B_1 units of time. If the loss of manpower $X_1(=S_1)$ due to the first policy of decision is greater than Y, then the recruitment is done and in this case $T=B_1=D_1$. However, if $S_1 \leq Y$, the non-recruitment period will continue till the next policy decision is taken. If the cumulative sum S_2 of the loss of manpower the first two decision exceeds Y, then recruitment is done and $T=B_1+B_2=D_2$. If $S_2 \leq Y$, then the non-recruitment period will continue till the next policy decision is taken and depending on $S_3>Y$ or $S_3 \leq Y$, recruitment is done or the non-recruitment period continues and so on. Hence

$$T = \sum_{i=0}^{\infty} D_{i+1}\, \chi\,(S_i \leq Y < S_{i+1}) \qquad (1)$$

From (1) and from the definition of $D_{i+1,}$ we get

$$E(T) = \sum_{i=0}^{\infty} (i+1)\, E(B)\, P\,(S_i \leq Y < S_{i+1}) \quad (2)$$

It can be shown that the distribution function G(.) of the inter-exit times satisfy the

$$\text{relation } G(x)= q \sum_{n=1}^{\infty} (1-q)^{n-1} F_n(x) \qquad (3)$$

Therefore from (2) and (3) it can be shown that

$$E(T) = \sum_{i=0}^{\infty} (i+1)\, [q \sum_{n=1}^{\infty} n(1-q)^{n-1} E(A)] \times P(S_i \leq Y < S_{i+1}) \qquad (4)$$

Again from (1) it is found that

$$E(T^2) = \sum_{i=0}^{\infty} (i+1)\, [V(B)+ (i+1)E^2(B)] \times P\,(S_i \leq Y < S_{i+1}) \qquad (5)$$

By using law of total probability we get

$P\,(S_i \leq Y < S_{i+1}) = J_1$

$$\text{where } J_1 = \int_0^{\infty}\int_0^{t} \overline{M}\,(t-x)\, m_i(x)\, h(t)\, dx\, dt \quad (6)$$

Equations (4), (5) and (6) determine E(T) and V(T) in the more general setting.

We know obtain explicit analytical expression for E(T) and V(T) by assuming $M(x)=1-e^{-\alpha x}$ and considering several cases on different distributions for the breakdown threshold.

Case (i): $H(x) = 1-e^{-\theta x}$

In this case from (4), (5) and (6) and on simplification we get

$$P\,(S_i \leq Y < S_{i+1}) = \frac{\alpha^i \theta}{(\alpha+\theta)^{i+1}} \qquad (7)$$

Sub case (i): $f(t) = f_{a(1)}(t) = r[1-F(t)]^{r-1} f(t)$

(8)

In this sub case from (4), (5) and (7) and on simplification it is found that

$$E(T)= \left(\frac{\alpha+\theta}{\lambda r \theta q}\right) \quad (9)$$

$$E(T^2) = \frac{2(\alpha+\theta)^2}{(\lambda r \theta q)^2} \qquad (10)$$

and

$$V(T)= \frac{(\alpha+\theta)^2}{(\lambda r \theta q)^2} \qquad (11)$$

(9) and (11) give the mean and variance of the time to recruitment for sub case (i).

Sub case (ii): $f(t) = f_{a(r)}(t) = r[F(t)]^{r-1} f(t)$ (12)

In this sub case from (4), (5) and (7) and on simplification it is found that

$$E(T)= \left(\frac{\alpha+\theta}{\lambda \theta q}\right)\left(\sum_{j=1}^{r}\frac{1}{j}\right) \quad (13)$$

$$E(T^2) = \left(\frac{\alpha+\theta}{\lambda^2 \theta q}\right)\left(\sum_{j=1}^{r}\frac{1}{j^2}\right) + J_2\left(\sum_{j=1}^{r}\frac{1}{j}\right)^2 \qquad (14)$$

$$\text{where } J_2=\left(\frac{(\alpha+\theta)(2(\alpha+\theta)-\theta q)}{(\lambda\theta q)^2}\right)$$

and

$$V(T)= \left(\frac{\alpha+\theta}{\lambda^2 \theta q}\right)\left(\sum_{j=1}^{r}\frac{1}{j^2}\right) + J_3\left(\sum_{j=1}^{r}\frac{1}{j}\right)^2 \qquad (15)$$

$$\text{Where } J_3= \left(\frac{(\alpha+\theta)((\alpha+\theta)-\theta q)}{(\lambda\theta q)^2}\right)$$

(13) and (15) give the mean and variance of the time to recruitment for sub case (ii).

Case (ii): $H(x) = [1-e^{-\theta x}]^2$ which is the extended exponential distribution [12] with scale parameter θ and shape parameter 2.

In this case from (4), (5) and (6) and on simplification we get

$$P(S_i \leq Y < S_{i+1}) \left(\frac{2\alpha^i\theta}{(\alpha+\theta)^{i+1}} - \frac{2\alpha^i\theta}{(\alpha+2\theta)^{i+1}}\right) \qquad (16)$$

Sub case (i): $f(t) = f_{a(1)}(t) = r[1-F(t)]^{r-1} f(t)$

(17)

In this sub case from (4), (5) and (7) and on simplification it is found that

$$E(T) = \left(\frac{3\alpha + 2\theta}{2\lambda r\theta q}\right) \quad (18)$$

and

$$V(T) = \frac{5\alpha^2 + 4\theta^2 + 12\alpha\theta}{4(\lambda r\theta q)^2} \quad (19)$$

(18) and (19) give the mean and variance of the time to recruitment for sub case(i).

Sub case (ii): $f(t) = f_{a(r)}(t) = r[F(t)]^{r-1} f(t)$ (20)

In this sub case from (4), (5) and (7) and on simplification it is found that

$$E(T) = \left(\frac{3\alpha + 2\theta}{2\lambda\theta q}\right)\left(\sum_{j=1}^{r}\frac{1}{j}\right) \quad (21)$$

and

$$V(T) = \left(\frac{3\alpha + 2\theta}{2\lambda^2\theta q}\right)\left(\sum_{j=1}^{r}\frac{1}{j^2}\right) + D\left(\sum_{j=1}^{r}\frac{1}{j}\right)^2 \quad (22)$$

where $D = \left(\frac{(12\alpha\theta + 4\theta^2 - 6\alpha\theta q - 4\theta^2 q + 5\alpha^2)}{4(\lambda\theta q)^2}\right)$

(21) and (22) give the mean and variance of the time to recruitment for sub case(ii).

Case(iii): $H(x) = p_1 e^{-(\theta_1+\mu)x} + p_2 e^{-\theta_2 x}$, which is the distribution function with SCBZ property [14]

where $p_1 = \frac{\theta_1 - \theta_2}{\mu + \theta_1 - \theta_2}$ *and* $p_2 = 1 - p_1$

For the present case, from (4), (5) and (6) and on simplification it is found that

$$P(S_i \le Y < S_{i+1}) = \frac{p_1\alpha^i(\theta_1 + \mu)}{(\alpha + \theta_1 + \mu)^{i+1}} + \frac{p_2\alpha^i\theta_2}{(\alpha + \theta_2)^{i+1}} \quad (23)$$

Sub case (i): $f(t) = f_{a(1)}(t) = r[1 - F(t)]^{r-1} f(t)$ (24)

For this sub case from (4), (5) and (7) and on simplification it is found that

$$E(T) = \left[\frac{p_1(\alpha + \theta_1 + \mu)\theta_2 + p_2(\alpha + \theta_2)(\theta_1 + \mu)}{\lambda r q(\theta_1 + \mu)\theta_2}\right]$$

(25)

and

$$V(T) = \left[\frac{(2p_1 - p_1^2)(\alpha + \theta_1 + \mu)^2(\theta_2)^2 + (2p_2 - p_2^2)(\alpha + \theta_2)^2(\theta_1 + \mu)^2 - 2p_1p_2(\alpha + \theta_1 + \mu)(\theta_1 + \mu)(\alpha + \theta_2)}{(\lambda r q)^2(\theta_1 + \mu)^2(\theta_2)^2}\right]$$

(26)

(25) and (26) give the mean and variance of the time to recruitment for sub case(i).

Sub case (ii): $f(t) = f_{a(r)}(t) = r[F(t)]^{r-1} f(t)$ (27)

For this sub case from (4), (5) and (7) and on simplification it is found that

$$E(T) = \left[\frac{p_1(\alpha + \theta_1 + \mu)\theta_2 + p_2(\alpha + \theta_2)(\theta_1 + \mu)}{\lambda q(\theta_1 + \mu)\theta_2}\right]\left(\sum_{j=1}^{r}\frac{1}{j}\right)$$

(28)

and

$$V(T) = \left[\frac{p_1(\alpha + \theta_1 + \mu)\theta_2 + p_2(\alpha + \theta_2)(\theta_1 + \mu)}{\lambda^2 q(\theta_1 + \mu)\theta_2}\right]\left(\sum_{j=1}^{r}\frac{1}{j^2}\right) + \left(\sum_{j=1}^{r}\frac{1}{j}\right)^2 \times \left[\frac{p_1(\alpha + \theta_1 + \mu)(\theta_2)^2\left[\begin{matrix}2\alpha + (\mu + \theta_1)(2 - q) - \\ p_1(\alpha + \theta_1 + \mu)\end{matrix}\right] + p_2(\alpha + \theta_2)(\theta_1 + \mu)^2[2\alpha + \theta_2(2 - q) - p_2(\alpha + \theta_2)] - 2p_1p_2(\alpha + \theta_1 + \mu)(\alpha + \theta_2)(\theta_1 + \mu)(\theta_2)}{(\lambda q)^2(\theta_1 + \mu)^2(\theta_2^2)}\right]$$

(29)

(28) and (29) give the mean and variance of the time to recruitment for sub case(ii).

Note

- ❖ When q=1, our results agree with the results in [11] for case(i).
- ❖ When r=1, our results agree with the results in [4] for all the three cases.
- ❖ When q=1 and r=1, our results for cases (ii) and (iii) are consistent with those of [12] and [14] respectively in which the authors have used Laplace transform to obtain their results.

Findings: From the above table the following observations are presented which agree with reality,

- ➢ When α increases and keeping all the other parameter fixed, the average loss of manpower increases. Therefore the mean and variance of time to recruitment increase for all the three cases.
- ➢ As λ increases, on the average, the inter-decision time decreases and consequently the mean and variance of time to recruitment decrease for all the three cases when the other parameters are fixed.
- ➢ As q, the probability that every policy decision has exit of personnel increases, the mean and variance of time to recruitment decrease for all the three cases when the other parameters are fixed.
- ➢ When r increases and keeping all the other parameter fixed, the average loss of manpower decreases. Therefore the mean and variance of time to recruitment decrease for all the three cases.

Conclusion: The model discussed in this paper is found to be more realistic and new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs and (ii) associating a probability for any decision to have exit points. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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A.Srinivasan, Associate Professor in Mathematics, Bishop Heber College,
Tiruchirappalli – 620 017, Tamil Nadu, India.mathsrinivas@yahoo.com
A.Devi, Assistant Professor in Mathematics, Bishop Heber College,
Tiruchirappalli – 620 017, Tamil Nadu, India. devi150187@gmail.com