b-CHROMATIC NUMBER OF A TRIANGULAR BELT

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Abstract: Let G=(V,E) be a simple undirected connected graph. A proper k coloring c of a graph G is a *b-coloring* if for every color class c_i, there is a vertex with color i which has at least one neighbor in every other color classes. Such vertex is called a *b-vertex*. The *b-chromatic number* of a graph G, denoted by φ(G), is the largest integer k for which G admits a b-coloring for k colors and G is called *b-colorable graph*. The concept of b-coloring was introduced by Irving and Manlove [2]. In this paper, b-chromatic number of a triangular belt is investigated.

Keywords: b-coloring, b-chromatic number, b-vertex, b-colorable graph.

Introduction: Let G=(V,E) be a simple undirected connected graph. A proper k coloring is a function c:V (G)→{1, 2, ..., k}such thatc(u)≠c(v) for all uv ∈E(G). The color class c_i is the subset of vertices of G that are assigned to color i. The chromatic number χ(G) is the minimum integer k for which G admits proper k-coloring and Δ(G) is the maximum degree of G. A proper k coloring c of a graph G is a b-coloring.if for every color class c_i, there is a vertex with color i which has at least one neighbor in every other color classes. Such vertex is called a b-vertex. The b-chromatic numberof a graph G, denoted by φ(G), is the largest integer k for which G admits a b-coloring for k colors and G is called b-colorable graph. The concept of b-coloring was introduced by Irving and Manlove [2]. They investigated that if G admits a b-coloring with m colors, G must have at least m vertices with at least m-1. In this paper, b-chromatic number of a triangular belt is investigated.

2. Main Results:

Theorem:2.1: For any triangular belt Tb_n , the b-Chromatic number is $\varphi(Tb_n) \leq \Delta(G) +1$ for all n≥4.

Proof: Let Tbn be the graph with vertex set

$V(Tb_n) =\{u_i , v_i / i =1 \text{ to } n \}$and

$E(Tb_n) = \{u_iu_{i+1}, v_iv_{i+1}, u_{i+1}v_i, u_jv_j / i=1 \text{ to } n-1 \text{ and } j=1 \text{ to } n\}$, the edge set of Tb_n.

$| V(Tb_n) | =2n$ and $| E(Tb_n) | = 4n-3$

Here, 2 vertices u_1, v_n are of degree 2, 2 vertices u_n, v_1 are of degree 3, 2n-4 vertices u_i , v_i (for i=2 to n-1) are of degree 4. ∴ $\Delta(Tb_n) = 4$.

If n= 4 Let Tb_4 be the graph with vertex set

$V(Tb_4) =\{u_i , v_i / i =1 \text{ to } 4 \}$and

$E(Tb_4) = \{u_iv_i, u_ju_{j+1} / i=1 \text{ to } 4 \text{ and } j=1 \text{ to } 3\}$,

the edge set of Tb_4.$| V(Tb_4) | =8$ and $| E(Tb_4) | = 13$

Here, 2 vertices u_1, v_4 are of degree 2, 2 vertices u_4, v_1 are of degree 3 and 4 vertices u_i,v_i (for i=2 ,3) are of degree 4.

∴ $\Delta(Tb_4) = 3$.

Consider the color class C = $\{c_1,c_2,c_3,c_4, c_5\}$. Assign a proper coloring to the vertices as follows.

Give the color c_1 to the vertices u_4 and v_1, the color c_2 to the vertex v_2 , the color c_3 to the vertices u_1 and v_3. the color c_4 to the vertices u_2 and v_4 and the color c_5 to the vertices u_3.

$\varphi(Tb_4) = 5$ $\varphi(Tb_4) = 5 \leq \Delta(Tb_n) +1$.

∴ $\varphi(Tb_4) \leq \Delta(Tb_n) +1$.

Hence the result is true for all n≥4. Consider the color class C = $\{c_1,c_2,c_3,c_4, c_5\}$. Assign a proper coloring to the vertices as follows.

Hence, we color the vertices as follows:

$c(u_k) = c_1 = c(u_i)$ for k≡0(mod 4) and

i≡1(mod 5) for i,k=1 to n

$c(u_i) = c_3 = c(u_k)$for i≡1(mod 4) and

k≡3(mod 5) for i,k=1 to n

$c(u_j) = c_4 = c(u_k)$for j≡2(mod 4) and

k≡4(mod 5) for j,k=1 to n

$c(u_l) = c_5 = c(u_i)$for l≡3(mod 4) and

i≡0(mod 5) for i,l=1 to n

$c(v_k) = c_2$ for k≡2(mod 5) for k=1 to n,

which is b-coloring with the b-vertices v_i (for i= 1 to n)for the color classes c_1, c_2, c_3, c_4, c_5 respectively.

$\varphi(Tb_n) = 5 \leq \Delta(Tb_n) +1$.

∴ $\varphi(Tb_n) \leq \Delta(Tb_n) +1$.

Conclusion: Motivated by the definition of b-Chromatic number of a graph, we have discussed the b-Chromatic number of a triangular belt. The Problem of b-Chromatic number of some cycle related graphs are under investigation.

References:

1. M. Blidia, F. Maffray And Z. Zemir, "On b-Colorings In Regular Graphs", *Discrete Applied Mathematics*, Vol.157, Pp. 1787-1793, 2009.
2. R. W. Irving And D. F. Manlove, "The b-Chromatic Number Of A Graph", *Discrete Applied Mathematics*, Vol.91, Pp. 127-141, 1999.
3. B. Effantin And H. Kheddouci, "The b-Chromatic Number Of Some Power Graphs", *Discrete Mathematics And Theoretical Computer Science*, Vol.6, Pp. 45-54, 2003.
4. F. Havet, C. L. Sales And L. Sampaio, "b-Coloring Of Tight Graphs", *Discrete Applied Mathematics*, Vol. 160, Pp. 2709-2715, 2012.
5. M. Kouider, M. Mah΄Eo,"Some Bounds For The b-Chromatic Number Of A Graph", *Discrete Mathematics*, Vol. 256, Pp. 267-277, 2002.
6. C. L. Sales And L. Sampaio, "b-Coloring Of M-Tight Graphs", *Electronic Notes In Discrete Mathematics*,

Vol. 35, Pp. 209-214, 2009.
7. S. K. Vaidya And Rakhimol V. Isaac, The b-Chromatic Number Of Some Path Related Graphs, International Journal Of Mathematics And Scientific Computing (ISSN: 2231-5330), Vol. 4, No. 1, 2014
8. D. Vijayalakshmi, K. Thilagavathi and N. Roopesh, "b-chromatic number of $M(C_n)$, $M(P_n)$, $M(F_1,n)$ and $M(W_n)$", *Open Journal of Discrete Mathematics*, vol. 1, pp. 85-88, 2011.
9. D. B. West, *Introduction To Graph Theory*, Prentice-Hall Of India, New Delhi, 2003.

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