SLIGHTLY $\alpha\psi$-CONTINUOUS FUNCTION

M. PARIMALA, R. PERUMAL

Abstract: The concept of $\alpha\psi$-closed sets in a topological space are introduced by R. Devi et. al. [3]. In this paper, we introduce the notion of slightly $\alpha\psi$-continuous functions utilizing $\alpha\psi$-open sets. Furthermore, basic properties and preservation theorems of slightly $\alpha\psi$-continuous functions are investigated.

Keywords: *$\alpha\psi$-closed sets, slightly-continuous, slightly $\alpha\psi$-continuous, $\alpha\psi$-connected functions.*

Introduction: Jain [6] introduced the notion of slightly continuous functions. He defined a function $f : X \to Y$ to be slightly continuous if, for every $x \in X$ and every clopen subset V of Y containing f(x) there exists an open subset U of X with $x \in U$ and $f(U) \subseteq V$. Slight semi-continuity [12], Slight α-continuity [11], Slight pre-continuity [1], and Slight β-continuity [5] are analogously defined as weak forms of slight continuity. In this paper, the notion of slightly $\alpha\psi$-continuous functions is introduced and basic properties of slightly $\alpha\psi$-continuous functions are investigated and obtained.

All through this paper, (X, τ) and (Y, σ) (or X and Y) stand for topological spaces with no separation axioms assumed, unless otherwise stated. Let $A \subseteq X$, the closure of A and the interior of A will be denoted by cl(A) and int(A) respectively.

Definition 1.1. A subset A of a space (X, τ) is called a

1. a semi-open set [8] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$,
2. an α-open set [10] if $A \subseteq int(cl(int(A)))$ and an α-closed set if $cl(int(cl(A))) \subseteq A$ and
3. a semi-generalized closed (briefly sg-closed) set [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ). The complement of sg-closed set is called sg-open set,
4. a ψ-closed set [14] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X, τ). The complement of ψ-closed set is called ψ-open set,

Let (X, τ) be a space and let A be a subset of X. A is called $\alpha\psi$-closed set [3] if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open set of (X, τ). The complement of an $\alpha\psi$-closed set is called $\alpha\psi$-open. The collection of all $\alpha\psi$-closed (resp. $\alpha\psi$-open) subsets of X will be denoted by $\alpha\psi$-C(X)(resp. $\alpha\psi$-O(X)). The intersection of all $\alpha\psi$-closed sets of X containing A is called $\alpha\psi$-closure of A and is denoted by $\alpha\psi cl(A)$ [3]. The union of all $\alpha\psi$-open sets of X contained in A is called $\alpha\psi$-interior of A and is denoted by $\alpha\psi int(A)$ [3].

2. Slightly $\alpha\psi$-continuous functions: In this section, the notion of slightly $\alpha\psi$-continuous functions is introduced. If A is both $\alpha\psi$-open and $\alpha\psi$-closed, then it is said to be $\alpha\psi$-clopen. The family of all $\alpha\psi$-clopen (resp. clopen) sets of X is denoted by $\alpha\psi$ CO(X) (resp.CO(X)). The family of all $\alpha\psi$-clopen (resp. clopen) sets of X containing $x \in X$ is denoted by $\alpha\psi$ CO(X, x) (resp. CO(X, x)).

Definition 2.1. A function $f : X \to Y$ is said to be slightly $\alpha\psi$-continuous at a point $x \in X$ if for each clopen subset V in Y containing f(x), there exist an $\alpha\psi$-open subset U in X containing x such that $f(U) \subseteq V$. The function f is said to be slightly $\alpha\psi$-continuous if it has this property at each point of X.

Theorem 2.2. For a function $f : (X, \tau) \to (Y, \sigma)$ the following statements are equivalent:

(i) f is slightly $\alpha\psi$-continuous;
(ii) for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is $\alpha\psi$-open;
(iii) for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is $\alpha\psi$-closed;
(iv) for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is $\alpha\psi$-clopen;
(v) for every $A \subseteq X$, $f(\alpha\psi cl(A)) \subset \alpha\psi cl(f(A))$.

Proof. (i) $\Rightarrow$ (ii): Let V be a clopen subset of Y and let $x \in f^{-1}(V)$. Since f is slightly $\alpha\psi$-continuous, by (i) there exists an $\alpha\psi$-open set Ux in X containing x such that $f(Ux) \subseteq V$; hence $Ux \subseteq f^{-1}(V)$. We obtain that $f^{-1}(V) = [\{Ux : x \in f^{-1}(V)\}$. Thus, $f^{-1}(V)$ is $\alpha\psi$-open.

(ii) $\Rightarrow$ (iii): Let V be a clopen subset of Y . Then $Y - V$ is clopen. By (ii) $f^{-1}(Y - V) = X - f^{-1}(V)$ is $\alpha\psi$-open. Thus $f^{-1}(V)$ is $\alpha\psi$-closed.

(iii) $\Rightarrow$ (iv): It can be shown easily.

(iv) $\Rightarrow$ (v): Let A be a subset of X. Then $\alpha\psi cl(f(A))$ is $\alpha\psi$-closed in Y , by (iv) $f^{-1}(\alpha\psi cl(f(A)))$ is $\alpha\psi$-clopen in X and $A \subset f^{-1}(f(A)) \subset f^{-1}(\alpha\psi cl(f(A)))$; that is $f^{-1}(\alpha\psi cl(f(A))) \subseteq f^{-1}(\alpha\psi cl(f(A)))$.
Hence $f(\alpha\psi cl(A)) \subseteq \alpha\psi cl(f(A))$.

(i) $\Rightarrow$ (v): It is obvious.

Corollary 2.3. Let (X, τ) and (Y, σ) be topological spaces. The following statements are equivalent for a function $f : X \to Y$:

(i) f is slightly $\alpha\psi$-continuous;
(ii) for every $x \in X$ and each clopen subset $V \subseteq Y$ containing f(x), there exist a $U \ \alpha\psi \in O(X, x)$ such that $f(U) \subseteq V$.

Theorem 2.4. If $f : X \to Y$ is slightly $\alpha\psi$-continuous and $A \in \alpha\psi O(X)$ then the restriction $f_{|A} : A \to Y$ is slightly $\alpha\psi$-continuous.

Proof. Let V be a clopen subset of Y.We have $(f_{|A})^{-1}(V) = f^{-1}(V) \cap A$. Since $f^{-1}(V)$ and A are $\alpha\psi$-open, therefore $(f_{|A})^{-1}(V)$ is $\alpha\psi$-open in the relative topology of A. Thus $f_{|A}$ is slightly $\alpha\psi$-continuous.

Definition 2.5. A function $f : X \to Y$ is said to be

(i) $\alpha\psi$-irresolute [3] if for every $\alpha\psi$-open subset U of Y , $f^{-1}(U)$ is $\alpha\psi$-open in X,

(ii) αψ-open [3] if for every αψ-open subset U of X, f(U) is αψ-open in Y ,
(iii) αψ-continuous [3] if for every open subset U of Y , $f^{-1}(U)$ is αψ-open in X,
(iv) contra αψ-continuous [4] if for every open subset U of Y , f−1(U) is αψ-closed in X.

Theorem 2.6. Let f : X → Y and g : Y → Z be two functions. Then, the following properties hold:
(i) If f is αψ-irresolute and g is slightly αψ-continuous, then g o f is slightly αψ-continuous.
(ii) If f is αψ-continuous and g is slightly continuous, then g o f is slightly αψ-continuous.

Proof. (i) Let V be any clopen set in Z. By the slight αψ-continuity of g, $g^{-1}(V)$ is αψ-open. Since f is αψ-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is αψ-open. Therefore, g o f is slightly αψ-continuous.
(ii) Let V be any clopen set in Z. By the slight continuity of g, $g^{-1}(V)$ is open. Since f is αψ-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is αψ-open. Therefore, g o f is slightly αψ-continuous.

Corollary 2.7. Let f : X → Y and g : Y → Z be two functions. Then, the following properties hold:
(i) If f is αψ-irresolute and g is slightly continuous, then g o f is slightly αψ-continuous.
(ii) If f is αψ-continuous and g is slightly continuous, then g o f is slightly αψ-continuous.

Corollary 2.8. Let f : X → Y be an αψ-irresolute, αψ-open surjection and g : Y → Z be a function. Then g is slightly αψ-continuous if and only if g o f is slightly αψ-continuous.

Proof. If g be slightly αψ-continuous. Then by Theorem 2.6., g o f is slightly αψ-continuous. Conversely, let g o f be slightly αψ-continuous and V be clopen set in Z. Then $(g \circ f)^{-1}(V)$ is αψ-open. Since f is αψ-open surjection, then $f((g \circ f)^{-1}(V)) = g^{-1}(V)$ is αψ -open in Y . This shows that g is slightly αψ-continuous.

A space X is called αψ-connected provided that X is not the union of two disjoint non-empty αψ-open sets. Now we have following Theorem.

Theorem 2.9. If f : X → Y is a slightly αψ-continuous surjection and X is αψ-connected, then Y is connected.

Proof. Suppose that Y is a disconnected space. Then there exist non-empty disjoint open sets U and V such that Y = U ∪ V . Therefore, U and V are clopen sets in Y . Since f is slightly αψ-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are αψ-open in X. Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint and $X = f^{-1}(U) \cup f^{-1}(V)$. Since f is surjective, $f^{-1}(U)$ and $f^{-1}(V)$ are non-empty. Therefore, X is not αψ-connected. This is a contradiction and hence Y is connected.

Corollary 2.10. The inverse image of a disconnected space under a surjection slightly αψ-continuous is αψ-disconnected.

It is easy to show that slight continuity, contra αψ-continuity, αψ-continuity imply slight αψ-continuity. None of these implications is reversible.

αψ -continuity
⇓
slight continuity ⇒ slight αψ-continuity ⇐ contra αψ-continuity

Example 2.11. Let X = R with the usual topology τ and let Y = {a, b, c} with the topology τ = {Y, ϕ, {a}, {b}, {a, b}}. Then the function f : (X, τ) → (Y, σ) defined by $f(x)=\begin{cases} a, \ if \ x \in Q \\ b, \ if \ x \notin Q \end{cases}$ is slightly αψ-continuous but not αψ-continuous.

Example 2.12. Let X = R with the usual topology τ and Y = {a, b, c, d} with the topology τ = {Y, ϕ, {a}, {b}, {a, b}, {a, b, c}}. Then the function f : (X,τ) → (Y, σ) defined by $f(x)=\begin{cases} c, \ if \ x \in Q \\ a, \ if \ x \notin Q \end{cases}$ is slightly αψ-continuous but not contra αψ-continuous, since $f^{-1}(\{b, c, d\})$ is not αψ-open set.

Example 2.13. Let X = {a, b, c}, τ = {X, ϕ, {a}, {b, c}} and σ = {X, ϕ, {c}}. Let f : (X,τ) → (Y, σ) be the identity function. Then f is slightly αψ-continuous. But it is not slightly continuous since $f^{-1}(\{a\}) = \{a\}$ is not open in X.

Corollary 2.14. If f : (X,τ) → (Y, σ) is αψ-continuous, contra αψ-continuous or slightly continuous surjection and X is αψ-connected, then Y is connected.

Recall that a space X is said to be
(1) extremally disconnected if the closure of every open set of X is open,
(2) locally indiscrete [9] if every open set of X is closed in X,
(3) 0-dimensional if its topology has a base consisting of clopen sets.

Theorem 2.15. If f : X→ Y is slightly αψ-continuous and Y is extremally disconnected, then f is αψ-continuous.

Proof. Let x ∈ X and let V be an open subset of Y containing f(x). Then cl(V) is open and hence clopen. Therefore there exists an αψ-open set U ⊆ X with x ∈ U and f(U) ⊆ cl(V). Thus f is αψ-continuous.

Theorem 2.16. If f : X → Y is slightly αψ-continuous and Y is locally indiscrete, then f is αψ-continuous and contra αψ-continuous.

Proof. Let V be any open set of Y . Since Y is locally indiscrete, V is clopen and hence $f^{-1}(V)$ is αψ-open and αψ-closed in X. Therefore f is αψ-continuous and contra αψ-continuous.

Theorem 2.17. If f : X → Y is slightly αψ-continuous and Y is 0-dimensional, then f is αψ-continuous.

Proof. Let x ∈ X and V ⊆ Y be any open set containing f(x). Since Y is 0-dimensional, there exists a clopen set U containing f(x) such that U ⊆ V. But f is slightly αψ-continuous and there exists G ∈ αψO(X, x) such that f(x)

$\in f(G) \subseteq U \subseteq V$. Hence f is αψ-continuous.

3. Separation axioms related to αψ-open sets:

Theorem 3.1. Let $f : X \to Y$ be a function and $g : X \to X \times Y$ the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. Then g is slightly αψ-continuous if and only if f is slightly αψ-continuous.

Proof. Let $V \in CO(Y)$, then $X \times V$ is clopen in $X \times Y$. Since g is slightly αψ -continuous, then $f^{-1}(V) = g^{-1}(X \times V) \in \alpha\psi O(X)$. Thus, f is slightly αψ-continuous. Conversely, let $x \in X$ and F be a clopen subset of $X \times Y$ containing g(x). Then $F \cap (\{x\} \times Y)$ is clopen in $\{x\} \times Y$ containing g(x). Also $\{x\} \times Y$ is homeomorphic to Y. Hence $\{y \in Y : (x, y) \in F\}$ is a clopen subset of Y . Since f is slightly αψ-continuous, $[\{f^{-1}(y) : (x, y) \in F\}$ is an αψ-open set of X. Further $x \in \cup\{f^{-1}(y) : (x, y) \in F\} \subseteq g^{-1}(F)$. Hence $g^{-1}(F)$ is αψ-open. Then g is slightly αψ-continuous.

A subset A of a topological space X is said to be *αψ-closed if for each $x \in X - A$ there exists a αψ-clopen set U containing x such that $U \cap A = \phi$. A topological space X is said to be ultra Hausdorff [13] if every two distinct points of X can be separated by disjoint clopen sets.

Theorem 3.2. If $f : X \to Y$ is slightly αψ-continuous and Y is ultra Hausdorff and the times of two αψ-open sets is αψ-open, then

(i) The graph G(f) of f is *αψ-closed in the times space $X \times Y$,

(ii) The set $\{(x_1, x_2) : f(x_1) = f(x_2)\}$ is *αψ-closed in the times space $X \times X$.

Proof. (i) Let $(x, y) \in (X \times Y) - G(f)$. Then $f(x) = y$. Since Y is ultra Hausdorff, there exists clopen sets V and W such that $y \in V$ and $f(x) \in W$ and $U \cap W = \phi$. Since f is slightly αψ-continuous, there exists an αψ-clopen set U containing x such that $f(U) \subseteq W$. Therefore, we obtain $V \cap f(U) = \phi$ and hence $(U \times V) \cap G(f) = \phi$ and $U \times V$ is an αψ -clopen set of $X \times Y$. This shows that G(f) is *αψ - closed in $X \times Y$.

(ii) Set $A = \{(x_1, x_2) : f(x_1) = f(x_2)\}$. Let $(x_1, x_2) \notin A$ then $f(x_1) \neq f(x_2)$. Since Y is ultra Hausdorff, there exist V_1, $V_2 \in CO(Y)$ containing $f(x_1)$, $f(x_2)$ respectively, such that $V_1 \cap V_2 = \phi$. Since f slightly αψ-continuous, there exists αψ-clopen sets U_1,U_2 of X such that $x_1 \in U_1$, $f(U_1) \subseteq V_1$ and $x_2 \in U_2$, $f(U_2) \subseteq V_2$ and hence $f(U_1) \cap f(U_2) = \phi$. Thus $(x_1, x_2) \in U_1 \times U_2$ and $(U_1 \times U_2) \cap A = \phi$. Moreover $U_1 \times U_2$ is αψ-clopen in $X \times X$ and A is *αψ-closed in the product space $X \times X$.

Definition 3.3. A space X is said to be

(i) αψ-T_0 (resp. αψ-T_1) if for each $x, y \in X$ such that $x = y$, there exists an αψ-open set containing x but not y, or (resp. and) an αψ-open set containing y but not x;

(ii) αψ-T_2 if for each $x, y \in X$ such that $x = y$, there exist disjoint αψ-open sets U and V such that $x \in U$ and $y \in V$;

(iii) αψ-regular if for each closed set F of X and each point $x \notin F$, there exists disjoint αψ-open sets U and V such that $F \subseteq U$ and $x \in V$;

(iv) αψ-normal if for every pair of disjoint closed sets F_1 and F_2 of X there exist disjoint αψ-open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

Theorem 3.4. Let Y be a 0-dimensional space and $f : X \to Y$ be a slightly αψ-continuous injection. Then the following properties hold:

(i) If Y is T_1(resp. T_2), then X is αψ-T_1 (resp. αψ-T_2).

(ii) If f is either open or closed, then X is αψ-regular.

(iii) If f is closed and Y is normal, then X is αψ-normal.

Proof. (i) We prove only the second statement, the proof of the first being analogous. Let Y be T_2. Since f is injective, for any pair of distinct points $x, y \in X$, $f(x) = f(y)$. Since Y is T_2, there exists open sets V_1, V_2 in Y such that $f(x) \in V_1$, $f(y) \in V_2$ and $V_1 \cap V_2 = \phi$. Since Y is 0-dimensional space, there exists U_1,$U_2 \in CO(Y)$ such that $f(x) \in U_1 \subseteq V_1$ and $f(y) \in U_2 \subseteq V_2$. Consequently $x \in f^{-1}(U_1) \subseteq f^{-1}(V_1)$, $y \in f^{-1}(U_2) \subseteq f^{-1}(V_2)$, and $f^{-1}(U_1) \cap f^{-1}(U_2) = \phi$. Since f is slightly αψ-continuous, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are αψ-open sets and this implies that X is αψ-T_2.

(ii) First suppose f is open. Let $x \in X$ and U be an open set containing x. Then, $f(x) \in f(U)$ which is open in Y because of the openness of f. On the other hand, 0-dimensionality of Y gives the existence of a $V \in CO(Y)$ such that $f(x) \in V \subseteq f(U)$. So, $x \in f^{-1}(V) \subseteq U$ (sinse f is injective). Again f is slightly αψ-continuous and $f^{-1}(V)$ is an αψ-clopen set in X by Theorem 2.2. and hence $x \in f^{-1}(V) = Cl(f^{-1}(V)) \subseteq U$. This implies that X is αψ-regular.

Now suppose f is closed. Let $x \in X$ and F be a closed set of X such that $x \notin F$. Then, $f(x) \notin f(F)$ and $f(x) \in Y - f(F)$ which is an open set in Y since f is closed. But Y is 0-dimensional and there exists a clopen set V in Y such that $f(x) \in V \subseteq Y - f(F)$. Since f is slightly αψ-continuous, we have $x \in f^{-1}(V) \in \alpha\psi CO(X)$ and $F \subseteq X - f^{-1}(V) \in \alpha\psi CO(X)$. Therefore X is αψ-regular.

(iii) Let F_1 and F_2 be any two closed sets in X such that $F_1 \cap F_2 = \phi$. Since f is closed and injective, we have $f(F_1)$ and $f(F_2)$ are two closed sets in Y with $f(F_1) \cap f(F_2) = \phi$. By normality of Y , there exist two open sets U and V in Y such that $f(F_1) \subseteq U$, $f(F_2) \subseteq V$ and $U \cap V = \phi$. Let $y \in f(F_1)$, then $y \in U$. Since Y is 0-dimensional and U is open in Y , there exists a clopen set Uy such that $y \in Uy \subseteq U$. Then $f(F_1) \subseteq \cup\{Uy : Uy \in CO(Y), y \in f(F_1)\} \subseteq U$, and thus $F_1 \subseteq \cup\{f^{-1}(Uy) : Uy \in CO(Y), y \in f(F_1)\} \subseteq f^{-1}(U)$. Since f is slightly αψ-continuous, $f^{-1}(Uy)$ is αψ-open for each $Uy \in CO(Y)$ so that $G = \cup\{f^{-1}(Uy) : y \in f(F_1)\}$ is αψ-open in X and $F_1 \subseteq G \subseteq f^{-1}(U)$. Similarly, there exists an αψ-open set H in X such that $F_2 \subseteq H \subseteq f^{-1}(V)$ and $G \cap H \subseteq f^{-1}(U \cap V) = \phi$. This shows that X is αψ-normal.

Next we use the concept of an αψ-open set to define an analogue of the notion of a set connected function.

Definition 3.5. A space X is said to be αψ-connected between subsets A and B provided there is no αψ-clopen set F for which $A \subseteq F$ and $F \cap B = \phi$.

Definition 3.6. A function f : X → Y is said to be set αψ-connected if whenever X is αψ-connected between A and B, then f(X) is connected between f(A) and f(B) with respect to the relative topology on f(X).

The next result is analogous to the characterization of set connected functions obtained by
Kwak [7].

Theorem 3.7. A function f : X → Y is set αψ-connected if and only if $f^{-1}(F)$ is αψ-clopen for every clopen subset F of f(X) (with respect to the relative topology on f(X)).

Proof. Necessity. Assume that F is a clopen subset of f(X) with respect to the relative topology on f(X). Suppose that $f^{-1}(F)$ is not αψ-closed in X. Then there exists $x \in X - f^{-1}(F)$ such that for every αψ-open set U with $x \in U$ and $U \cap f^{-1}(F) = \phi$. We claim that the space X is set αψ-connected between x and $f^{-1}(F)$. Suppose there exists an αψ-clopen set A such that $f^{-1}(F) \subseteq A$ and $x \notin A$. Then $x \in X - A \subseteq X - f^{-1}(F)$ and evidently X – A is an αψ -open set containing x and disjoint from $f^{-1}(F)$ this contradiction implies that X is set αψ-connected between x and $f^{-1}(F)$. Since f is set αψ-connected, f(X) is connected between f(x) and $f(f^{-1}(F))$. But $f(f^{-1}(F)) \subseteq F$ which is clopen in f(X) and $f(x) \notin F$, which is a contradiction. Therefore $f^{-1}(F)$ is αψ-closed in X and an argument using complements will show that $f^{-1}(F)$ is also αψ-open.

Sufficiency. Suppose X is αψ-connected between A and B and also f(X) is not connected between f(A) and f(B) (in the relative topology on f(X)). Thus there is a set $F \subseteq f(X)$ that is clopen in the relative topology on f(X) such that $f(A) \subseteq F$ and $F \cap f(B) = \phi$. Then $A \subseteq f^{-1}(F)$, $B \cap f^{-1}(F) = \phi$ and $f^{-1}(F)$ is αψ-clopen, which implies that X is not αψ-connected between A and B. It follows that f is set αψ-connected.

Corollary 3.8. Every slightly αψ-continuous surjection is set αψ-connected.

Theorem 3.9. Every set αψ-connected function is slightly αψ-continuous.

Proof. Assume f : X → Y is set αψ-connected. Let F be a clopen subset of Y. Then $F \cap f(X)$ is clopen in the relative topology on f(X). Since f is set αψ -connected, by Theorem 3.7., $f^{-1}(F) = f^{-1}(F \cap f(X))$ is αψ-clopen in X.

Corollary 3.10. A surjective function is slightly αψ-continuous if and only if it is set αψ-connected.

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M. Parimala//Assistant Professor/ Department of Mathematics/Bannari Amman Institute of Technology
Sathyamangalam-638401/Tamil Nadu/ India/ rishwanthpari@gmail.com
R. Perumal /Assistant Professor/ Department of Mathematics/Kumaraguru College of Technology
Coimbatore-641049/Tamil Nadu, India/ perumalkctmaths@gmail.com