A MULTI – ITEM INVENTORY MODEL WITH VARIABLE RATE OF DETERIORATION

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Abstract: Inventory is the stock held in reserve to meet the future demand. Perishable inventory is deteriorating items like fruits, vegetables, medicines, alcohols etc. which can be stored only for a limited period of time. Mathematical models is to study the optimum time of placing the order, optimum deteriorated units and optimum order quantity by focusing the demand and the deterioration pattern. A deterministic EOQ multi-item model with exponential rate of demand and the time to deterioration of items being a variable is developed. Shortages are allowed and are fully backlogged. The optimum time of placing the order, the optimum initial inventory, the optimum order quantity and the optimum deteriorated units are derived for the i^{th} item and explained using numerical illustration. Sensitivity analysis is performed for the parameter of the model.

Keywords: Inventory, Inventory control, Inventory models, Non cost parameter, EOQ.

Introduction: Most of the existing inventory models in the literature assume that items can be stored indefinitely to meet the future demands. However certain types of commodities either deteriorate or become obsolete in the course of time and hence they are unusable. The effect of deterioration is very important in many inventory systems. Deterioration is defined as decay or damage to the item so that it cannot be used for its intended purpose. Food items, drugs, pharmaceuticals and radioactive substances are examples of items which deteriorate during the storage period. Hence this loss must be taken into consideration while dealing with the total cost in an inventory model.

Inventory problem for deteriorating items have been studied extensively by many researchers from time to time. Research in this area started with the work of Whitin, who considered fashion goods that deteriorate at the end of the prescribed storage period. Wagner and Whitin have discussed the replenishment policy of the inventory of an item having a demand pattern varying over time and they developed dynamic programming algorithm for the determination of EOQ. An exponentially decaying inventory was first developed by Ghare and Schrader. Emmons has developed a model in the case of decay of radioactive nuclear generators. Covert and Philip have obtained and EOQ model for items with a variable rate of deterioration by assuming two parameter Weibull distribution for the time to deterioration of the item. Misra has developed a deterministic model for both constant and variable rate of deterioration of items with a finite production rate. Shah has developed and order level lot size inventory model for deterioration items. Friedman and Hoch have developed a dynamic lot size model for deteriorating items.

Dave and Patel have analysed and inventory model for deteriorating items with time proportional demand. Nahmias has given a detailed survey of literature on perishable inventory models. A simple optimal solution for determining the EOQ of a system with a linear increasing demand is given by Ritchie. Mukerjee has given optimum ordering interval for time varying decay rate in inventory. Datta and Pal have developed and order level inventory system with power demand pattern for items with variable rate of deterioration. Also shortages are allowed and are fully backlogged. Bahar Kashani has considered a replenishment schedule for deteriorating items with time proportional demand. Mandal and Phaujder have considered and inventory model for deteriorating items with stock dependent consumption rate.

A complete literature survey on inventory models for deteriorating items is given in Goyal and Giri. Joanna Kezia has condidered and EOQ model for deteriorating items with shortage. A deteriorating inventory model with stock- dependent demand and partial backlogging under conditions of permissible delay in payments is given by Dye. Yang have developed a forward recursive algorithm for inventory lot-size models with power –form demand and shortages.

In the present study a multi-item inventory model is considered. The demand of an item is exponential in nature while the rate of deterioration is a variable depending on time. Shortages are allowed and are fully backlogged.

Mathematical Model: The model is developed under the following assumptions:

a. Replenishment is instantaneous and lead time is zero. b. The system operates for a prescribed period of time t. c. T is the fixed length of each production cycle. d. There are **n** non- overlapping items in the system. e. C_{1i} is the inventory holding cost per unit per unit time of the i^{th} term, i = 1,2,…,n. f. C_{2i} is the storage cost per unit per unit time of the i^{th} term, i = 1,2,…,n. g. C_{23i} is the storage cost per unit per unit time of the i^{th} term, i = 1,2,…,n. h. Shortages are allowed and fully backlogged. i. Rate of deterioration of the i^{th} term is a variable function of time which is θ_i t, $0 < \theta_i \leq 1$: i = 1,2,…,n. It can be observed that this is a special form of a two parameter Weibull distribution function. J. Demand rate is $d_i e^{t/T}$ at any time t where d_i is the demand of the i^{th} term, i = 1,2,…,n.

Numerical Illustration: Here we consider a numerical illustration to study the model. The values of the parameters of the model are taken as θ= 0.5, T = 10 days, C_{1i} = Rs. 2.5, C_{2i} = Rs. 1.5, C_{3i} = Rs. 0.5, d_i = 10 units.

Substituting the above values for the parameters in equation $L = \theta_i /3T\, C_{1i}$, $M = \theta_i /2T\, C_{3i}$, $N = C_{1i} + C_{2i} / T$ and $P = - C_{2i}$ and solving Newton Raphson method, the optimum time of placing the order is found to be $t_1^* =$ 2.5days. The optimum initial inventory using equations $S_i^* = d_i T[e^{t1*/T} -1 + \theta_i /2t_1^{*2} e^{t1*/T} - \theta_i T t_1^* e^{t1*/T} + \theta_i T^2 e^{t1*/T} - \theta_i T^2]$ is $S_i^* = 44.20$, the optimum deteriorated units using equation is $Q_i^* = d_i + D_i^*$, $D_i^* = 15.77$ and the optimum order quantity using is $Q_i^* = 25.77$.

Sensitivity Analysis: Sensitivity Analysis is performed to study the optimum time of placing the order, the optimum initial inventory, the optimum order quantity and optimum deteriorated units by varying the parameters of the model.

θ_i	t^*_1	S^*_i	Q^*_i	D^*_i
0.3	2.504249	37.943001	19.485889	9.485888
0.4	2.503025	41.069584	22.628197	12.628198
0.5	2.502271	44.201588	25.769886	15.769887
0.6	2.501752	47.335922	28.910885	18.910885
0.7	2.501368	50.472725	32.052620	22.052622

T	t^*_1	S^*_i	Q^*_i	D^*_i
9	2.253279	37.120663	21.516258	11.516257
10	2.502271	44.201588	25.769886	15.769887
11	2.751583	52.230740	30.967617	20.967617
12	3.001091	61.300297	37.203236	27.203236

d_i	t^*_1	S^*_i	Q^*_i	D^*_i
8	2.502271	35.361271	20.615910	12.615910
9	2.502271	39.781429	23.192898	14.192898
10	2.502271	44.201588	25.769886	15.769887
11	2.502271	48.621746	28.346876	17.346876
12	2.502271	53.041908	30.923864	18.923864

From Table 1 it is evident that when rate of deterioration θ_i increases, the optimum time of placing the order t^*_1 decreases, marginally whereas the optimum initial inventory S^*_i, the optimum order quantity Q^*_i and the deteriorated unit D^*_i increase. When the order cycle T increases, the optimum time of placing the order t^*_1, the optimum initial inventory S^*_i, the optimum order quantity Q^*_i and the deteriorated unit D^*_i increase. When the demand di increases, the optimum time of placing the order t^*_i remains constant, whereas the optimum initial inventory S^*_i, optimum order quantity Q^*_i and the deteriorated unit D^*_i increase.

The details of sensitivity analysis for the cost parameters of the model are shown in Table 2.

Table 2

C_{1i}	t^*_1	S^*_i	Q^*_i	D^*_i
2.3	2.633749	48.704231	28.572784	18.572784
2.4	2.566323	46.351875	27.094898	17.094898
2.5	2.502271	44.201588	25.769886	15.769887
2.6	2.441345	42.229664	24.578056	14.578056
2.7	2.383323	40.416672	23.503588	13.503588

C_{2i}	t^*_1	S^*_i	Q^*_i	D^*_i
1.3	2.283098	37.428230	21.780775	11.780775
1.4	2.395490	40.791798	23.724209	13.724208
1.5	2.502271	44.201588	25.769886	15.769887
1.6	2.603851	47.650154	27.907190	17.907190
1.7	2.700601	51.130226	30.125914	20.125914

C_{3i}	t^*_1	S^*_i	Q^*_i	D^*_i
0.3	2.502466	44.208202	25.773987	15.773988
0.4	2.502368	44.204678	25.771723	15.771723
0.5	2.502271	44.201588	25.769886	15.769887
0.6	2.502173	44.198666	25.768219	15.768218
0.7	2.502075	44.194966	25.765785	15.765786

When the holding cost C_{1i} increases, the optimum time of placing the order t^*_1, the optimum initial inventory S^*_i, the optimum order quantity Q^*_i and the deteriorated unit D^*_i decrease. When the shortage cost C_{2i} increases, the optimum time of placing the order t^*_1, the optimum initial inventory S^*_i, the optimum order quantity Q^*_i and the deteriorated unit D^*_i increase. When the cost of each deteriorated unit C_{3i} increases, there is a marginal decrease in the optimum time of placing the order t^*_1, the optimum initial inventory S^*_i, the optimum order quantity Q^*_i and the optimum deteriorated units D^*_i.

Conclusion: Thus a deterministic EOQ multi-item model with exponential rate of demand and the time to deterioration of items being a variable is discussed. Shortages are allowed and are backlogged.

The optimum time of placing the order, the optimum initial inventory, the optimum order quantity and the optimum deteriorated units are derived for the i^{th} item.

The working of the model is explained through a numerical illustration. Sensitivity analysis is carried out for the parameters to study the behaviour of the model is evaluating the optimum time of placing the order, optimum initial inventory optimum order quantity and optimum deteriorated units.

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