SIMPLE ADDITIVE WEIGHTING TECHNIQUE- AN IMPORTANT TOOL FOR SENSITIVITY- AN ILLUSTRATION

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Abstract: Multi Criteria Decision making methods are used for taking best decisions in different situations. In this study, one of the sensitivity analysis methods –SAW(Simple Additive Weighting) method is used to obtain best decisions by changing the weight of the attributes. SAW method has been applied to select a branch of students of an engineering college to receive All Round Excellence Award for the year 2004-05 by taking subjective judgements of decision maker into consideration. This method has been applied for seven criteria which are both quantitative and qualitative in nature. It is observed that the student of ECE branch has received All Round Excellence award by applying SAW method. SAW method is not only used for ranking of alternatives but also used to find that alternative which is more sensitive to changes. In the present study this method is adopted because this is time efficient method and gives more effective and accurate solutions as compared to the solutions obtained by other sensitivity analysis methods like WSM (Weight Sum Method) and WPM (Weight Product Method).

Introduction: Decision making has acquired a greater prominence in this cut throat competitive world. A good decision at wrong time or a wrong decision at good time may result in failure. So good decision making, at good time with proper decision making approach is very important. Most of the organizations, MNCS, industries, Institutes are giving greater significance to decision making as it is the basis of the future.

Suppose a teacher has to select a student from all the branches for All Round Excellence Award .Consider five branches of an engineering college (ECE,EEE,ICE,MECH,CSE). Seven criteria are selected like academics, attendance, behaviour, etc., It is not easy to select one student out of five branches without any prejudice. In these situations, MCDM models are very useful methods to choose the best decision. But MCDM models give vague results for vague preferences of the decision makers as consider many criteria at same time. There exist many methods and in this study SAW (simple additive weighting) method is applied to the problem with seven criteria and their sub criteria corresponding to five alternatives. SAW method is multi objective decision making method. The main advantage of SAW method is that it is proportional to linear transformation of the given data weights, in this view this method is also weighted linear combination method.

There are many MCDM methods which are applied to this present problem. In this study AHP and FAHP are applied to the problem to find consistency ratio. Construction of hierarchies, multiple objectives and fuzzy sets were explained by Thomas.L.Satty[1,2]. Now in this study SAW method is applied to verify the decisions obtained by using AHP and FAHP to choose a student from some branches for All round excellence award.

The matrix of pair wise comparisons of the criteria as given by the decision maker is shown in Table 3, along with the resulting vector of priorities. The vector of priorities is the principal Eigen vector of the matrix .It gives the relative priority of the criteria measured on a ratio scale given in Table 1.Next we move to pair wise comparisons of the lower level and lastly to the pair wise comparisons of the lowest level .The elements to be compared pair wise are the engineering branches with respect to how much better one is than the other in satisfying each criterion in level 2.Thus there will be fifteen 5 x 5 matrices of judgments. Level 0 shows the overall goal of "All round Excellence award "in the zeroth level, shown in blue colour. The next level, namely level 1 shows the criteria as its elements, which are shown in orange coloured cells. Its next level namely level 2 shows the sub criteria in the Lavender coloured cells. It can be observed that not all the criteria have sub criteria. The criteria like Academics and attendance do not have any criteria as identified by the decision maker. The next level namely level 3 is the highest level given by alternatives shown in green coloured cells.

Methodology: Among all the MCDM models, the most commonly and widely used method is Analytical Hierarchy Process(AHP). This method was introduced by T.L.Satty in 1970s. The main advantage of AHP is to capture both subjective and objective evaluation measures to check the consistency. The decisions obtained by FAHP are discussed in [4]. Now we are applying SAW method to get more accurate solution and to compare decisions obtained in above mentioned methods.

Saw Method: In this study SAW method applied to the problem of seven criteria with their sub criteria's compared with five alternatives, for which the decisions already obtained by AHP, FAHP, WSM[4]. In this paper SAW method is used to find raking also to find the rate of change of rank of alternative with corresponding change in the weight of attribute. This method consists of five steps

Step-1: construct a normalized decision matrix of order for positive criteria as

$$n_{ij}= \frac{r_{ij}}{r_j^{max}} \quad i=1,2.....m \quad \text{and} \quad j=1,2,.....n,$$

$$\text{for negative criteria } n_{ij}= \frac{r_j^{min}}{r_{ij}} \quad i=1,2,.....m \quad \text{and} \quad j=1,2,.....n$$

Step-2: evaluate the rank of each alternative A_i by using following formula $A_i=\sum W_j .X_{ij}$
Where W_j is the weight vector of criteria and
X_{ij} is score of the i^{th} alternative with respect to j^{th} criteria
Step-3: to check the effect of change in the weight of one attribute on the weight of other attributes. Assume weight of p^{th} criteria changes from w_p to w_p^I as $W_p^I = W_p + \Delta_p$

Step 4: then the weight of other attributes changes as w_j^I

$$= \frac{1-w_p^I}{1-w_p}.w_j \text{ ;j=1,2...k ,j}\neq p$$

Step-5: repeat Step-2 with new weights
Step-5 gives the priority vector of alternatives from which we can obtain the desired accuracy.

Problem

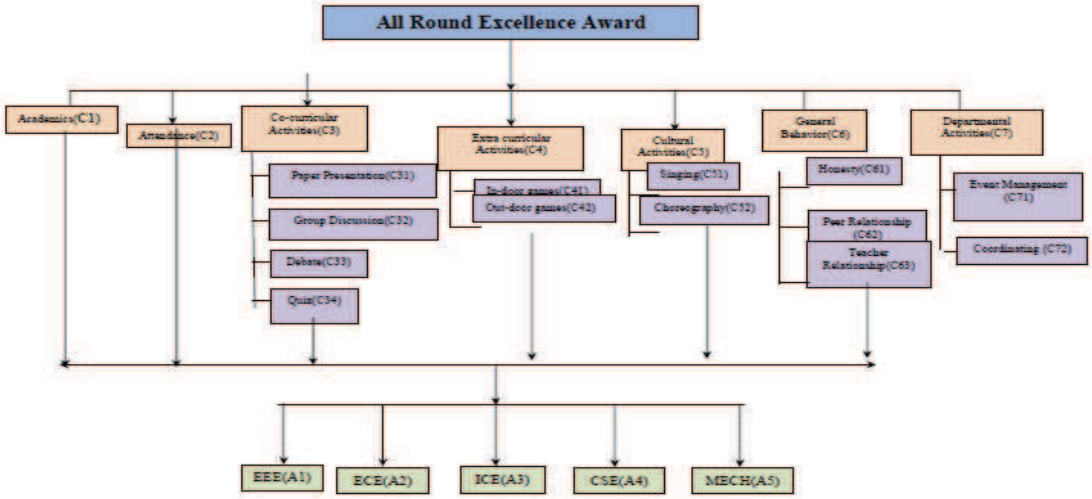

Figure: 1 Hierarchical decomposition of criteria, sub criteria and alternatives

Calculations: Step 1: Normalised decision matrix

Weight	0.316	0.04	0.2	0.2	0.2	0.2	0.5	0.5	0.5	0.5	0.33	0.33	0.33	0.5	0.5
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
A1	0.633	0.196	0.511	1	0.413	0.473	0.341	0.642	1	0.641	1	1	1	0.821	1
A2	1	1	1	1	1	1	0.219	0.285	0.411	0.641	1	1	1	1	0.655
A3	0.142	0.0196	0.111	0.329	0.13	0.236	0.268	0.821	0.264	0.265	0.329	1	0.333	0.285	0.344
A4	0.339	0.039	0.422	0.801	0.413	0.578	0.634	0.821	1	1	1	1	1	0.821	0.793
A5	0.14	0.019	0.177	0.411	0.195	0.282	1	1	1	1	0.329	1	0.333	0.642	1

Step-2: Ranks of alternatives

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	SUM	Rank
A1	0.200028	0.00784	0.1022	0.2	0.0826	0.0946	0.1705	0.321	0.5	0.3205	0.33	0.33	0.33	0.4105	0.5	3.899768	2
A2	0.316	0.04	0.2	0.2	0.2	0.2	0.1095	0.1425	0.2055	0.3205	0.33	0.33	0.33	0.5	0.3275	3.7515	3
A3	0.044872	0.000784	0.0222	0.0658	0.026	0.0472	0.134	0.4105	0.132	0.1325	0.10857	0.33	0.10989	0.1425	0.172	1.878816	5
A4	0.107124	0.00156	0.0844	0.1602	0.0826	0.1156	0.317	0.4105	0.5	0.5	0.33	0.33	0.33	0.4105	0.3965	4.075984	1
A5	0.04424	0.00076	0.0354	0.0822	0.039	0.0564	0.5	0.5	0.5	0.5	0.10857	0.33	0.10989	0.321	0.5	3.62746	4

The ranking of alternatives obtained as $A_4 > A_1 > A_2 > A_5 > A_3$

Step 3:Change of weight of alternative A_3

$$W_3^I = W_3 + \Delta_3 = 0.2 + 0.2 = 0.4$$

Step 4:The remaining weights of alternatives are obtained by

Weight	0.079	0.01	0.4	0.05	0.05	0.05	0.125	0.125	0.125	0.125	0.0825	0.0825	0.0825	0.125	0.125
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
A1	0.633	0.196	0.511	1	0.413	0.473	0.341	0.642	1	0.641	1	1	1	0.821	1
A2	1	1	1	1	1	1	0.219	0.285	0.411	0.641	1	1	1	1	0.655
A3	0.142	0.0196	0.111	0.329	0.13	0.236	0.268	0.821	0.264	0.265	0.329	1	0.333	0.285	0.344
A4	0.339	0.039	0.422	0.801	0.413	0.578	0.634	0.821	1	1	1	1	1	0.821	0.793
A5	0.14	0.019	0.177	0.411	0.195	0.282	1	1	1	1	0.329	1	0.333	0.642	1

Step 5: By repeating step 2 we get

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	SUM
A1	0.050007	0.00196	0.2044	0.05	0.02065	0.02365	0.042625	0.08025	0.125	0.080125	0.0825	0.0825	0.0825	0.102625	0.125	1.153792
A2	0.079	0.01	0.4	0.05	0.05	0.05	0.027375	0.035625	0.051375	0.080125	0.0825	0.0825	0.0825	0.125	0.081875	1.287875
A3	0.011218	0.000196	0.0444	0.01645	0.0065	0.0118	0.0335	0.102625	0.033	0.033125	0.0271425	0.0825	0.0274725	0.035625	0.043	0.508554
A4	0.026781	0.00039	0.1688	0.04005	0.02065	0.0289	0.07925	0.102625	0.125	0.125	0.0825	0.0825	0.0825	0.102625	0.099125	1.166696
A5	0.01106	0.00019	0.0708	0.02055	0.00975	0.0141	0.125	0.125	0.125	0.125	0.0271425	0.0825	0.0274725	0.08025	0.125	0.968815

Weights and Ranks of alternatives

S.NO	Alternatives	Weights of alternatives	Rank of alternatives
1	EEE	1.153792	3
2	ECE	1.287875	1
3	ICE	0.508554	5
4	CSE	1.166696	2
5	MECH	0.968815	4

Conclusion: The above illustration of selecting a branch of student for all round excellence award, the decisions are obtained by the methods AHP, FAHP and WSM are taken into considerations. In this study SAW method is applied but this method is applied to a decision matrix of positive criteria then maximum weight is taken into account. Here the alternative A2 is with maximum weight, then A2 is more sensitive alternative. Finally student of ECE branch received all round excellence award.

Hence the decisions obtained by all the methods that is AHP, FAHP, WSM &SAW Method are similar. But there is a change in the weights of alternatives from method to method.

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