# PROPERTIES OF $S_g^*$-FUNCTIONS IN TOPOLOGICAL SPACES

**S. PIOUS MISSIER , J.ARUL JESTI**

**Abstract:** The determination of this paper is to introduce the new function, namely contra-$S_g^*$-continuous function. Additionally some properties of $S_g^*$-open map, $S_g^*$-closed map, strongly $S_g^*$-continuous function and perfectly $S_g^*$-continuous functions are investigated.

**Keywords:** contra-$S_g^*$continuous function, $S_g^*$-open map, $S_g^*$-closed map, strongly $S_g^*$-continuous function, perfectly $S_g^*$-continuous function .

**Introduction:** In 1963, Norman Levine[5] introduced the concept of semi-open sets and semi-continuity in topological spaces. After the works of Levine , in the year 1987, P.Battacharyya and B.K.Lahiri[1] introduced the concept of semi-generalized closed sets with the help of semi-openness. Continuing this work, P.Sundaram [10] and et al introduced semi-generalised continuous functions and semi-generalised irresolute functions in 1991 . Also in1993, R.Devi and et al introduced semi-generalized closed maps. Recently, S.Pious Missier and J.Arul Jesti have introduced the concept of $S_g^*$-open sets[8], $S_g^*$-continuous function[9], $S_g^*$-irresolute function, $S_g^*$-open map, $S_g^*$-closed map, strongly $S_g^*$-continuous function and perfectly $S_g^*$-continuous function and studied their properties.

In this direction, we shall introduce a new functions called contra-$S_g^*$-continuous function, In addition to this, we discussed some properties of strongly $S_g^*$-continuous function and perfectly $S_g^*$-continuous function and some of its properties are discussed.

**2. Preliminaries:** Throughout this paper (X, τ), (Y, σ ) and (Z, $\eta$ ) (or X,Y and Z) represent non-empty topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ), $S_g^*Cl$(A) and $S_g^*Int$(A) denote the $S_g^*$-closure and the $S_g^*$-interior of A respectively.

**Definition 2.1:** A function $f$: (X, τ) ⟶ (Y, σ) is called a ***contra– continuous*** [3] if $f^{-1}$ (O) is closed in (X, τ) for every open subset O of (Y, σ ).

**Definition 2.2:** A function $f$: (X, τ) ⟶ (Y, σ) is called a ***contra semi– continuous*** [4] if $f^{-1}$ (O) is semi-closed in (X, τ) for every open subset O of (Y, σ ).

**Definition 2.3:** A function $f$: (X, τ) ⟶ (Y, σ) is called a ***contra semi generalised–continuous*** [4] if $f^{-1}$ (O) is semi generalized-closed in (X, τ) for every open subset O of (Y, σ ).

**Definition 2.4:** A subset A of a topological space (X,$\tau$) is called a ***$S_g^*$-open set*** [8]if there is an open set U in X such that U⊆A⊆ $sCl^*$(U). The collection of all $S_g^*$-open sets in (X, τ) is denoted by $S_g^*$O(X,$\tau$).

**Theorem 2.5** [8]**:**

(i) Every open set is $S_g^*$-open and every closed set is $S_g^*$-closed set

(ii) Every $S_g^*$-open set is semi-open and every $S_g^*$-closed set is semi-closed.

(iii) Every $S_g^*$-open set is semi-generalised open and every $S_g^*$-closed set is semi-generalised closed.

**Definition 2.6:** A topological space $(X,\tau)$ is said to be ***$S_g^*$-$T_{1/2}$ space*** if every $S_g^*$-open set of X is open in X.

**Definition 2.7:** A mapping $f: X \to Y$ is said to be ***$S_g^*$ −continuous***[9] if the inverse image of every open set in Y is $S_g^*$ −open in X.

**Defintion 2.8:** A map $f: X \to Y$ is said to be ***$S_g^*$ −irresolute***[9]if the inverse image of every $S_g^*$ −open set in Y is $S_g^*$ −open in X.

**Definition 2.9:** A mapping $f: X \to Y$ is said to be ***strongly-continuous***[6] if the inverse image of every subset in Y is both open and closed in X.

**Definition 2.10:** A mapping $f: X \to Y$ is said to be ***strongly $S_g^*$-continuous***[9] if the inverse image of every $S_g^*$-open set in Y is open in X.

**Definition 2.11:** A mapping $f: X \to Y$ is said to be ***perfectly-continuous***[7] if the inverse image of every open set in Y is open and closed in X.

**Definition 2.12:** A mapping $f: X \to Y$ is said to be ***perfectly $S_g^*$-continuous***[9] if the inverse image of every $S_g^*$-open set in Y is open and closed in X.

**Definition 2.13:**A map $f: X \to Y$ is said to be ***$S_g^*$-open map***[9] if $f$(U) is $S_g^*$-open in Y for every open set U in X.

**Definition 2.14:** A map $f: X \to Y$ is said to be a ***$S_g^*$-closed map***[9] if $f$(U) is $S_g^*$-closed in Y for every closed set U in X.

## 3. CONTRA-$S_g^*$-CONTINUOUS FUNCTION

**Definition 3.1:** A function $f: X \to Y$ is said to be ***contra-$S_g^*$-continuous*** if the inverse image of every open set in Y is $S_g^*$ −closed in X.

**Theorem 3.2:** The following are equivalent for a function $f:(X,\tau) \to (Y,\sigma)$. Assume that $S_g^*$O(X)(resp. $S_g^*$C(X)) be closed under any union(resp. intersection)

1. $f$ is contra-$S_g^*$-continuous.
2. The inverse image of each closed set in Y is $S_g^*$-open in X.
3. For each $x \in X$ and each closed set V∈C(Y,$f(x)$), there exists U∈ $S_g^*$O(X , $x$) such that $f$(U)⊂V.
4. $f(s_g^*Cl(A))$ ⊂ker($f(A)$) for every subset A of X.
5. $s_g^*Cl(f^{-1}(B)) \subset f^{-1}(ker(B))$ for every subset B of Y.

**Proof:**

(1)⟹(3):Let $x \in X$ and V be a closed set in Y with $f(x)$ ∈V. By (1), it follows that $f^{-1}$(Y\V)=X\$f^{-1}$(V) is $S_g^*$-closed and so $f^{-1}$(V) is $S_g^*$-open. Take U=$f^{-1}$(V). Thus we obtain $x$ ∈U and $f$(U)⊂V.

(3)⟹(2): Let V be any closed set in Y with $x \in f^{-1}$(V). Since $f(x)$ ∈V, by (3) there exist an $S_g^*$-open set U in X containing $x$ such that $f$(U)⊂V. it follows that $x$ ∈U⊂ $f^{-1}$(V). Hence $f^{-1}$(V) is $S_g^*$-open.

(2)⟹(1): Obvious.

(2)⟹(4):Let A be any subset of X. Let y∉ker($f$(A)). Then there exists a closed set F containing y such that $f$(A)∩F=$\phi$. Hence we have A∩ $f^{-1}$(F)=$\phi$ and $s_g^*Cl$ (A)∩ $f^{-1}$(F)=$\phi$ which implies $s_g^*Cl$ ($f$(A))∩F=$\phi$ and y∉ $f(s_g^*Cl$ (A)).Hence (2).

(4)⟹(5): Let B be any subset of Y. By (4) $f(s_g^*Cl(f^{-1}$ (B))) ⊂ker(B) and $s_g^*Cl(f^{-1}$ (B)) ⊂ $f^{-1}$(ker(B)).

(5)⟹(1):Let B be any open set in Y. By (5) $s_g^*Cl(f^{-1}$ (B)) ⊂ $f^{-1}$(ker(B))= $f^{-1}$(B) and $s_g^*Cl(f^{-1}$ (B))= $f^{-1}$(B). Thus we obtain $f^{-1}$(B) is $S_g^*$-closed set in X.

**Theorem 3.3:** Every contra-continuous function is contra-$S_g^*$-continuous.

**Proof:** Let $f: X \to Y$ be a contra-continuous function and let V be any closed set in Y. Then $f^{-1}$(V) is open in X. Since every open set is $S_g^*$ −open, $f^{-1}$(V) is $S_g^*$ −open in X. Hence $f: X \to Y$ is contra- $S_g^*$-continuous.

**Remark 3.4:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 3.5:** Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Therefore $\tau^c = \{\emptyset, X, \{b, c\}, \{c\}\}$and $S_g^*O(X, \tau) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$. Define $f: (X, \tau) \to (X, \tau)$ by $f(a) = c, f(b) = a$ and $f(c) = b$. Here $f^{-1}\{b, c\} = \{a, c\}$ which is $S_g^*$-open but not open. Hence $f$ is contra-$S_g^*$-continuous but not contra-continuous.

**Theorem 3.6:** Every contra-$S_g^*$-continuous function is contra-semi-continuous.

**Proof:** Let $h: A \to B$ be a contra-$S_g^*$-continuous function and let O be any closed set in B. Then $h^{-1}$(O) is $S_g^*$-open in A. Since every $S_g^*$-open set is semi-open, $h^{-1}$(O) is semi-open in X. Hence $h: A \to B$ is contra- semi-continuous.

**Remark 3.7:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 3.8:** Let $X = \{a, b, c, d\}$. $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, c, d\}, \{a, b, c\}\}$. Therefore $\tau^c = \{\emptyset, X, \{b\}, \{d\}, \{b, d\}, \{a, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$, SO$(X, \tau)$= $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $S_g^*O(X, \tau) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}\}$. Define $f: (X, \tau) \to (X, \tau)$ by $f(a) = f(d) = d, f(b) = b$ and $f(c) = a$. Here $f^{-1}\{d\} = \{a, d\}$ which is semi-open but not $S_g^*$-open. Hence $f$ is contra-semi-continuous but not contra-$S_g^*$-continuous.

**Theorem 3.9:** Every contra-$S_g^*$-continuous function is contra-semi-generalised-continuous.

**Proof:** Let $h: A \to B$ be a contra-$S_g^*$-continuous function and let O be any closed set in B. Then $h^{-1}$(O) is $S_g^*$-open in A. Since every $S_g^*$-open set is semi-generalised-open, $h^{-1}$(O) is semi-generalised-open in X. Hence $h: A \to B$ is contra- semi-generalised-continuous.

**Remark 3.10:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 3.11:** It follows from Example 3.8 that SO$(X, \tau)$= SgO$(X, \tau)$

**Definition 3.12:** A space (X,$\tau$) is called ***$S_g^*$-locally indiscrete space*** if $S_g^*$-open set is closed.

**Theorem 3.13:** If $f: X \to Y$ is contra-$S_g^*$-continuous and if X is a $S_g^*$-locally indiscrete space, then $f$ is continuous.

**Proof:** Let U be any closed set in Y. Since $f$ is contra-$S_g^*$-continuous, $f^{-1}$(U) is$S_g^*$-open in X. Also since X is $S_g^*$-locally indiscrete space, $f^{-1}$(U) is closed in X. Hence $f$ is continuous.

**Theorem 3.14:** If $f: X \to Y$ is contra-$S_g^*$-continuous and $g: Y \to Z$ is continuous, then $gof: X \to Z$ is contra-$S_g^*$-continuous.

**Proof:** Let V be any closed set in $Z$. Since $g$ is continuous, $g^{-1}$(V) is closed in $Y$. Also since $f$ is contra-$S_g^*$-continuous, $f^{-1}(g^{-1}$(V)) is $S_g^*$-open in X. Hence $gof$ is contra-$S_g^*$-continuous.

**Theorem 3.15:** Let Y be a $S_g^* - T_{1/2}$ space. If $f: X \to Y$ is contra-$S_g^*$-continuous and $g: Y \to Z$ is contra-$S_g^*$-continuous, then $gof: X \to Z$ is $S_g^*$-continuous.

**Proof:** Let V be any closed set in $Z$. Since $g$ is contra-$S_g^*$-continuous, $g^{-1}$(V) is $S_g^*$-open in $Y$. Since Y is a $S_g^* - T_{1/2}$ space, $g^{-1}$(V) is open in $Y$. Also since $f$ is contra-$S_g^*$-continuous, $f^{-1}(g^{-1}$(V)) is $S_g^*$-closed in X. Hence $gof$ is $S_g^*$-continuous.

**Theorem 3.16:** Let Y be a $S_g^* - T_{1/2}$ space. If $f: X \to Y$ is contra-$S_g^*$-continuous and $g: Y \to Z$ is $S_g^*$-continuous, then $gof: X \to Z$ is contra-$S_g^*$-continuous.

**Proof:** Let V be any closed set in $Z$. Since $g$ is $S_g^*$-continuous, $g^{-1}$(V) is $S_g^*$-closed in $Y$.So $[g^{-1}(V)]^c$ is $S_g^*$-open in Y. Since Y is a $S_g^* - T_{1/2}$ space,$[g^{-1}(V)]^c$ is open in $Y$ which implies $g^{-1}$(V) is closed in Y. Also since $f$ is contra-$S_g^*$-continuous, $f^{-1}(g^{-1}$(V)) is $S_g^*$-open in X. Hence $gof$ is contra-$S_g^*$-continuous.

**Theorem 3.17:** If $f: X \to Y$ is $S_g^*$-irresolute and $g: Y \to Z$ is contra-$S_g^*$-continuous, then $gof: X \to Z$ is contra-$S_g^*$-continuous.

**Proof:** Let V be any closed set in $Z$. Since $g$ is contra-$S_g^*$-continuous, $g^{-1}$(V) is $S_g^*$-open in $Y$. Also since $f$ is $S_g^*$-irresolute, $f^{-1}(g^{-1}$(V)) is $S_g^*$-open in X. Hence $gof$ is contra-$S_g^*$-continuous.

**Theorem 3.18:** If $f: X \to Y$ is contra-$S_g^*$-continuous and $g: Y \to Z$ is contra-continuous, then $gof: X \to Z$ is $S_g^*$-

continuous.

**Proof:** Let O be any open set in $Z$. Since $g$ is contra-continuous, $g^{-1}$(O) is closed in $Y$. Since $f$ is contra-$S_g^*$-continuous, $f^{-1}(g^{-1}$(O)) is $S_g^*$-open in X. Hence $gof$ is $S_g^*$-continuous.

**Remark 3.19:** The composition of two contra-$S_g^*$-continuous functions need not be contra-$S_g^*$-continuous as shown in the following example.

**Example 3.20:** Let $X = Y = Z = \{a, b, c\}$. $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$ and $\eta = \{\emptyset, Z, \{a\}\}$.

$S_g^*$O(X , $\tau$) = $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$. $S_g^*$O(Y , $\sigma$) = $\{\emptyset, Y, \{a\}, \{a, b\}, \{a, c\}\}$ Define $f: (X, \tau) \to$ (Y , $\sigma$) by $f(a) = c, f(b) = a$ and $f(c) = b$ and define $g$: (Y , $\sigma$) $\to$ $(Z , \eta)$ by $g(a) = b, g(b) = c$ and $g(c) = a$. Here $f$ and $g$ are contra-$S_g^*$-continuous. But $(gof)^{-1}\{b, c\}$ = $f^{-1}(g^{-1}\{b, c\}\} = f^{-1}\{a, b\}$=$\{b, c\}$ which is not $S_g^*$-open in (X , $\tau$) Hence $gof$: (X , $\tau$) $\to$ $(Z , \eta)$ is not contra-$S_g^*$-continuous.

**4.On Strongly $S_g^*$-continuous:**

**Theorem 4.1:** Every strongly-$S_g^*$-continuous function is $S_g^*$-continuous function.

**Proof:** Let $f: X \to Y$ be strongly-$S_g^*$-continuous. Let O be any open set in Y. Since every open set is $S_g^*$-open, O is $S_g^*$-open in Y. Therefore $f^{-1}$(O) is open in X which implies $f^{-1}$(O) is $S_g^*$-open in X. Hence $f$ is $S_g^*$-continuous.

**Remark 4.2:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 4.3:** Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma$=$\{\emptyset, Y, \{a\}, \{a, b\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $(a) = a$ , $f(b) = c$ and $f(c) = b$. Here $\tau = S_g^*$O(X , $\tau$) and $S_g^*$O(Y , $\sigma$)=$\{\emptyset, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Here $\{a, c\}$ is $S_g^*$-open in Y, but $f^{-1}\{a, c\} = \{a, b\}$ is not open in X. So $f$ is not strongly-$S_g^*$-continuous.

**Theorem 4.4:** Every strongly-continuous function is strongly-$S_g^*$-continuous function.

**Proof:** Let $f: X \to Y$ be strongly-continuous. Let O be any $S_g^*$-open set in Y. Since $f$ is strongly-continuous, $f^{-1}$(O) is open in X. Hence $f$ is $S_g^*$-continuous.

**Remark 4.5:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 4.6:** Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma$=$\{\emptyset, Y, \{a, b\}\}$. The identity map $f: (X, \tau) \to (Y, \sigma)$ is strongly-$S_g^*$-continuous but $f$ is not strongly continuous. For the subset $\{a, b\}$ of $(Y, \sigma)$, $f^{-1}\{a, b\} = \{a, b\}$ is open in $(X, \tau)$ but not closed in $(X, \tau)$.

**Theorem 4.7:** If $f: X \to Y$ is strongly-$S_g^*$-continuous and $g: Y \to Z$ is strongly-$S_g^*$-continuous, then $gof: X \to Z$ is strongly-$S_g^*$-continuous.

**Proof:** Let O be any $S_g^*$-open set in $Z$. Since $g$ is strongly-$S_g^*$-continuous, $g^{-1}$(O) is open in $Y$. By theorem 2.5(i), $g^{-1}$(O) is $S_g^*$-open in $Y$, Since $f$ is strongly-$S_g^*$-continuous, $f^{-1}(g^{-1}$(O)) is open in X. Hence $gof$ is strongly-$S_g^*$-continuous.

**Theorem 4.8:** If $f: X \to Y$ is strongly-$S_g^*$-continuous and $g: Y \to Z$ is contra-$S_g^*$-continuous, then $gof: X \to Z$ is contra-continuous.

**Proof:** Let O be any closed set in $Z$. Since $g$ is contra-$S_g^*$-continuous, $g^{-1}$(O) is $S_g^*$-open in $Y$. Also since $f$ is strongly-$S_g^*$-continuous, $f^{-1}(g^{-1}$(O)) is open in X. Hence $gof$ is contra-continuous.

**Theorem 4.9:** If $f: X \to Y$ is continuous and $g: Y \to Z$ is strongly-$S_g^*$-continuous, then $gof: X \to Z$ is strongly-$S_g^*$-continuous.

**Proof:** Let O be any $S_g^*$-open set in $Z$. Since $g$ is strongly-$S_g^*$-continuous, $g^{-1}$(O) is open in $Y$. Also since $f$ is continuous, $f^{-1}(g^{-1}$(O)) is open in X. Hence $gof$ is strongly-$S_g^*$-continuous.

**Theorem 4.10:**Let X be any topological spaces and Y be a $S_g^*$-$T_{1/2}$ space and $f: X \to Y$ be a map. Then the following are equivalent:

(i) $f$ is strongly-$S_g^*$-continuous

(ii) $f$ is continuous.

**Proof:** (i)⟹(ii) Let U be any open set in Y. By theorem 2.5, U is $S_g^*$-open in Y. Then by (i), $f^{-1}$(U) is open in X. Hence $f$ is continuous.

(ii)⟹(i) Let O be any $S_g^*$open set in Y. Since Y is $S_g^*$-$T_{1/2}$ space, O is open in Y. Then by (ii), $f^{-1}$(U) is open in X. Hence $f$ is strongly-$S_g^*$-continuous.

**Theorem 4.11:**Let X be any topological spaces and Y be a $S_g^*$-$T_{1/2}$ space and $f: X \to Y$ be a map. Then the following are equivalent:

(i) $f$ is $S_g^*$-irresolute

(ii) $f$ is strongly-$S_g^*$-continuous.

(iii) $f$ is continuous

(iv) $f$ is $S_g^*$-continuous.

**Proof:** The proof is straight forward.

**5. On perfectly $S_g^*$-continuous:**

**Theorem 5.1:** Every perfectly $S_g^*$-continuous function is perfectly continuous.

**Proof:** Let $f: X \to Y$ be perfectly $S_g^*$-continuous and O be any open set in Y. Since every open set is $S_g^*$-open, O is $S_g^*$-open in Y. Therefore $f^{-1}$(O) is both open and closed in X. Hence $f$ is perfectly continuous.

**Remark 5.2:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 5.3:** Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b, c\}\} = \tau^c$ and $Y = \{d, e, f\}$ with $\sigma$=$\{\emptyset, Y, \{d\}, \{d, e\}, \{d, f\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $(a) = d$ , $f(b) = f$ and $f(c) = e$. Here $\tau = S_g^*$O(X , $\tau$) and $S_g^*$O(Y , $\sigma$)=$\{\emptyset, Y, \{d\}, \{d, e\}, \{d, f\}\}$. Here $\{d, e\}$ and $\{d, f\}$ are $S_g^*$-open in Y, but $f^{-1}\{d, e\} = \{a, c\}$ and $f^{-1}\{d, f\} = \{a, b\}$ are not open as well as not closed in X. So $f$ is not perfectly-$S_g^*$-continuous.

**Theorem 5.4:** Let $f: (X, \tau) \to (Y, \sigma)$ be strongly-$S_g^*$-continuous. Then $f$ is perfectly-$S_g^*$-continuous if $(X, \tau)$ is a discrete topology.

**Proof:** Let U be any $S_g^*$-open set in $(Y,\sigma)$. By hypothesis, $f^{-1}$(U) is open in$(X,\tau)$. Since $(X,\tau)$ is a discrete topology, $f^{-1}$(U) is closed in$(X,\tau)$. That is $f^{-1}$(U) is both open and closed in$(X,\tau)$. Hence $f$ is perfectly-$S_g^*$-continuous.

**Theorem 5.5:** If $f: X \to Y$ is perfectly-$S_g^*$-continuous and $g: Y \to Z$ is perfectly-$S_g^*$-continuous, then $gof: X \to Z$ is perfectly-$S_g^*$-continuous.

**Proof:** Let O be any $S_g^*$-open set in $Z$. Since $g$ is perfectly-$S_g^*$-continuous, $g^{-1}$(O) is both open and closed in $Y$. By theorem 2.5(i), $g^{-1}$(O) is both $S_g^*$-open and $S_g^*$-closed in $Y$, Since $f$ is perfectly-$S_g^*$-continuous, $f^{-1}(g^{-1}$(O)) is open and closed in X. Hence $gof$ is perfectly-$S_g^*$-continuous.

**Theorem 5.6:** If $f: X \to Y$ is continuous and $g: Y \to Z$ is perfectly-$S_g^*$-continuous, then $gof: X \to Z$ is perfectly-$S_g^*$-continuous.

**Proof:** Let O be any $S_g^*$-open set in $Z$. Since $g$ is perfectly-$S_g^*$-continuous, $g^{-1}$(O) is both open and closed in $Y$. Since $f$ is continuous, $f^{-1}(g^{-1}$(O)) is open and closed in X. Hence $gof$ is perfectly-$S_g^*$-continuous.

**Theorem 5.7:** If $f: X \to Y$ is perfectly-$S_g^*$-continuous and $g: Y \to Z$ is $S_g^*$-irresolute, then $gof: X \to Z$ is perfectly-$S_g^*$-continuous.

**Proof:** Let O be any $S_g^*$-open set in $Z$. Since $g$ is $S_g^*$-irresolute, $g^{-1}$(O) is $S_g^*$-open in Y. Since $f$ is perfectly-$S_g^*$-continuous, $f^{-1}(g^{-1}$(O)) is both open and closed in X. Hence $gof$ is perfectly-$S_g^*$-continuous.

**Theorem 5.8:** If $f: X \to Y$ is contra-continuous and $g: Y \to Z$ is perfectly-$S_g^*$-continuous, then $gof: X \to Z$ is perfectly-$S_g^*$-continuous.

**Proof:** Let O be any $S_g^*$-open set in $Z$. Since $g$ is perfectly-$S_g^*$-continuous, $g^{-1}$(O) is both open and closed in $Y$. Since $f$ is contra-continuous, $f^{-1}(g^{-1}$(O)) is closed and open in X. Hence $gof$ is perfectly-$S_g^*$-continuous.

**Theorem 5.9:** If $f: X \to Y$ is perfectly-$S_g^*$-continuous and $g: Y \to Z$ is $S_g^*$-continuous, then $gof: X \to Z$ is perfectly-continuous.

**Proof:** Let O be any open set in $Z$. Since $g$ is $S_g^*$-continuous, $g^{-1}$(O) is $S_g^*$-open in Y. Since $f$ is perfectly-$S_g^*$-continuous, $f^{-1}(g^{-1}$(O)) is both open and closed in X. Hence $gof$ is perfectly-continuous.

**Theorem 5.10:** If $f: X \to Y$ is perfectly-continuous and $g: Y \to Z$ is strongly-$S_g^*$-continuous, then $gof: X \to Z$ is perfectly-$S_g^*$-continuous.

**Proof:** Let O be any $S_g^*$-open set in $Z$. Since $g$ is strongly-$S_g^*$-continuous, $g^{-1}$(O) is open in Y. Since $f$ is perfectly-continuous, $f^{-1}(g^{-1}$(O)) is both open and closed in X. Hence $gof$ is perfectly-$S_g^*$-continuous.

**6.On $S_g^*$-open map and $S_g^*$-closed map:**

**Theorem 6.1:** Every $S_g^*$-open map is semi-open map.

**Proof:** Let $f: X \to Y$ be a $S_g^*$-open map and let U be any open set in X then $f$(U) is $S_g^*$-open in Y. Since every $S_g^*$-open set is semi-open,$f$(U) is semi-open in Y. Hence $f$ is a semi-open map.

**Remark 6.2:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 6.3:** Let $X = Y = \{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{b\},\{d\},\{b,d\},\{a,b,c\}\}$ and $\sigma$=$\{\emptyset, Y, \{b\},\{c\},\{b,c\}\}$. Define $f:(X,\tau) \to (Y,\sigma)$ by $(a) = a$ , $f(b) = b, f(c) = d$ and $f(d) = c$. Here $SO$(Y ,$\sigma$)=

$$\left\{\begin{matrix}\emptyset, Y, \{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\}, \\ \{a,b,d\},\{a,c,d\},\{b,c,d\}\end{matrix}\right\}$$

and $S_g^*O$(Y ,$\sigma$)=$\{\emptyset, Y, \{b\},\{c\},\{b,c\},\{a,b,c\},\{b,c,d\}\}$.

Here $f\{a,b,c\} = \{a,b,d\}$ is semi-open but not $S_g^*$-open in Y. Hence $f$ is not $S_g^*$-open map.

**Theorem 6.4:** Every $S_g^*$-open map is semi-generalised open map.

**Proof:** Let $f: X \to Y$ be a $S_g^*$-open map and let U be any open set in X then $f$(U) is $S_g^*$-open in Y. Since every $S_g^*$-open set is semi-generalised open, $f$(U) is semi-generalised open in Y. Hence $f$ is a semi-generalised open map.

**Remark 6.5:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 6.6:** Let $X = Y = \{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{a\},\{b,c\},\{a,b,c\}\}$ and $\sigma$=$\{\emptyset, Y, \{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}\} = S_g^*O$(Y ,$\sigma$). Define $f:(X,\tau) \to (Y,\sigma)$ by $(a) = b$ , $f(b) = a, f(c) = d$ and $f(d) = c$. Here $SgO$(Y ,$\sigma$)=

$$\left\{\begin{matrix}\emptyset, Y, \{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\}, \\ \{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\end{matrix}\right\}$$

. Here $f\{b,c\} = \{a,d\}$ and $f\{a,b,c\} = \{a,b,d\}$ are semi-generalised open but not $S_g^*$-open in Y. Hence $f$ is not $S_g^*$-open map.

**Remark 6.7:** The composition of two $S_g^*$-open maps need not be a $S_g^*$-open map as can be seen from the following example.

**Example 6.8:** Let $X = Y = Z = \{a,b,c,d\}$. $\tau = \{\emptyset, X, \{a\},\{a,b\}\}$, $\sigma = \{\emptyset, Y, \{a\},\{a,b\}\}$ and $\eta = \{\emptyset, Z, \{a,b\}\}$.

$S_g^*O$(Y ,$\sigma$) = $\{\emptyset, Y, \{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$.

$S_g^*O(Z ,\eta) = \{\emptyset, Z, \{a,b\},\{a,b,c\},\{a,b,d\}\}$

Define $f:(X,\tau) \to$ (Y ,$\sigma$) by $f(a) = a, f(b) = c, f(c) = b$ and $f(d) = d$ and define $g$: (Y ,$\sigma$) $\to (Z ,\eta)$ as an identity map. Here $f$ and $g$ are $S_g^*$-open map. But $gof(\{a,b\}) = g(f(\{a,b\}) = g(\{a,c\})$=$\{a,c\}$ which is not $S_g^*$-open in $(Z ,\eta)$ Hence $gof$: (X ,$\tau$) $\to (Z ,\eta)$ is not $S_g^*$-open.

**Theorem 6.9:** A map $f:(X,\tau) \to$ (Y ,$\sigma$) is a $S_g^*$-open map if and only if $f(Int(A)) \subseteq s_g^*Int(f(A))$ for each set A in X.

**Proof:** Suppose that $f$ is a $S_g^*$-open map. Since $Int(A) \subseteq A, f(Int(A)) \subseteq f(A)$ . By hypothesis $f(Int(A))$ is $S_g^*$-open and $s_g^*Int(f(A))$ is the largest $S_g^*$-open set contained in $f(A)$. Hence $f(Int(A)) \subseteq s_g^*Int(f(A))$.

Conversely suppose A is an open set in $X$. Then $f(Int(A)) \subseteq s_g^*Int(f(A))$. Since $Int(A) = A$, $f(A) \subseteq$

$s_g^*Int(f(A))$. Therefore $f(A)$ is $S_g^*$-open in $Y$. Hence $f:(X,\tau) \to (Y,\sigma)$ is a $S_g^*$-open map.

**Theorem 6.10:** Let $(X,\tau)$, $(Y,\sigma)$ and $(Z,\eta)$ be three topological spaces. Let $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\eta)$ be any two maps. Then the following are true.

1. If $gof$ is a $S_g^*$-open map and $f$ is continuous, then $g$ is a $S_g^*$-open map.
2. If $gof$ is an open map and $g$ is $S_g^*$-continuous, then $f$ is $S_g^*$-open map.

**Proof:**

1. Let U be an open set in Y. Then $f^{-1}(U)$ is an open in X. Since $gof$ is an $S_g^*$-open map, then $(gof)(f^{-1}(U))= g(f(f^{-1}(U)))=g(U)$ is $S_g^*$-open in Z.
2. Let A be an open set in X. Since $gof$ is an open map, $g(f(A))$ is an open set in Z. Also $g$ is $S_g^*$-continuous, $g^{-1}(g(f(A)))=f(A)$ is a $S_g^*$-open set in Y. Hence $f$ is $S_g^*$-open map.

**Theorem 6.11:** For any bijection $f:(X,\tau) \to (Y,\sigma)$ the following statements are equivalent:

(i) $f^{-1}:(Y,\sigma) \to (X,\tau)$ is $S_g^*$-continuous,

(ii) $f$ is a $S_g^*$-open map and

(iii) $f$ is a $S_g^*$-closed map.

**Proof:**

**(i)⟹(ii)** Let U be an open set of $(X,\tau)$. By (i) $(f^{-1})^{-1}$(U)=$f$(U) is $S_g^*$-open in $(Y,\sigma)$. Hence $f$ is a $S_g^*$-open map.

**(ii) ⟹(iii)** Let O be any closed set in $(X,\tau)$. Then $O^c$ is open in $(X,\tau)$. By assumption, $f(A^c)=(f(A))^c$ is $S_g^*$-open in $(Y,\sigma)$. Hence $f$(A) is $S_g^*$-closed in $(Y,\sigma)$.

**(iii) ⟹(i)** Let A be a closed set in $(X,\tau)$. By assumption, $f$(A)=$(f^{-1})^{-1}$(A) is $S_g^*$-closed in $(Y,\sigma)$. Hence $f^{-1}$ is $S_g^*$-continuous.

**Theorem 6.9:** A map $f:(X,\tau) \to (Y,\sigma)$ is a $S_g^*$-closed map if and only if $s_g^*Cl(f(A)) \subseteq f(Cl(A))$ for each set A in X.

**Proof:** Suppose that $f$ is a $S_g^*$-closed map. Since for each set A in X, Cl(A) is closed set in X. By hypothesis $f(Cl(A))$ is $S_g^*$-closed set in Y. Since $f(A) \subseteq f(Cl(A)$, $s_g^*Cl(f(A)) \subseteq f(Cl(A))$.

Conversely suppose A is a closed set in $X$. Since $s_g^*Cl(f(A))$ is the smallest $S_g^*$-closed set containing $f(A)$, $f(A) \subseteq s_g^*Cl(f(A)) \subseteq f(Cl(A)) = f(A)$. Hence $f(A) = s_g^*Cl(f(A))$. Therefore $f(A)$ is $S_g^*$-closed in $Y$. Hence $f:(X,\tau) \to (Y,\sigma)$ is a $S_g^*$-closed map.

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*Dr.S.Pious Missier* /Associate professor/PG and Research Department of Mathematics/ V.O. Chidambaram College, Thoothukudi. spmissier@gmail.com

*J.Arul Jesti* / Research Scholar/ PG and Research Department of Mathematics/V.O. Chidambaram College/Thoothukudi/ India/arul_jesti@yahoo.co.in