APPLICATION OF SIGNED GRAPH IN SOCIAL PSYCHOLOGY

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Abstract: A signed graph may be defined as an ordered pair (G, σ) where G= (V, E) is the graph and σ is a function called signature or sign mapping from the edge set E to the sign group {+, -}. Two terms "friendship matrix" or and "character point (C.P)" are introduced in this paper. Using the concept of friendship matrix and C.P in various fields the characteristics or performances could be evaluated. Two problems have been considered and analysed with the help of signed graph. This paper has been written, inspired by the works of the American mathematician Frank Harary and psychologist Dorwin Cartwright who used the concept of signed graph in social psychology. Peoples or friends are considered as the vertices and the relationships between these friends are considered as the edges. '+' sign is given to the edges for good relationship and '-' sign for bad relationship between the friends.

Key words: character point, friendship matrix, half edge, loose edge, marked signed graph, signed graph.

Introduction

A "signed graph" is associated with two signs '+' and '-'. The signature of S, assigns a '+' or '-' sign to every edge, accordingly designating it as being either positive or negative. The graph may have loops and multiple edges as well as half edges and loose edges. A half or loose edge does not get a sign. Signed graphs were first introduced by Frank Harary to handle a problem in social psychology along with the psychologist Dorwin Cartwright. Basically, a **friendship matrix** or **judgement matrix** is an m×n matrix whose elements can be +1 and -1 only. The matrix should always be associated with character points because a friendship matrix without character points will be considered as invalid. "**Character point C.P is a point or score which determines the characteristics or nature of person, place, time, object or action**." Generally, the signed graph associated with the matrix is a complete graph. The matrix gives the best result if the number of rows and columns in the matrix are odd because in that case, C.P will never be zero. The disadvantage of C.P is that, no definite conclusion can be drawn if C.P = 0. The main objective of this paper is to apply the concept of signed graph in decision making.

Definition 1: [1] A **marked signed graph** is an ordered pair (S, μ) where S = (G, σ) is a signed graph and μ: V (G) → {+1, -1} is a function called marking of S from the vertex set V (G) of G to the set {+1, -1}.

Definition 2: [2] A **half edge** is an edge having only one end point.

Definition 3: [2] A **loose edge** is an edge having no end points.

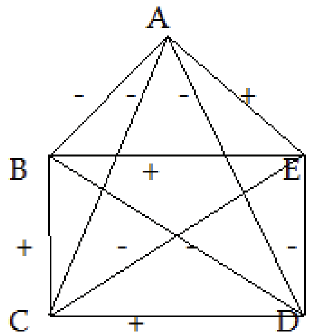
Definition 4: [2] A **loop** is an edge having two equal end points.

Applications

1 Representation of 'fair' friendship

The objective of this problem is to determine whether a person is good or bad on the basis of his relationship with other friends.

The diagram given below is a signed graph where the vertices represent the five friends A, B, C, D and E and the edges represent the mutual relationships among the five friends. Here '+' sign represents good relationship and '-' sign represents bad relationship among the two friends. We also see that the graph is a complete graph.



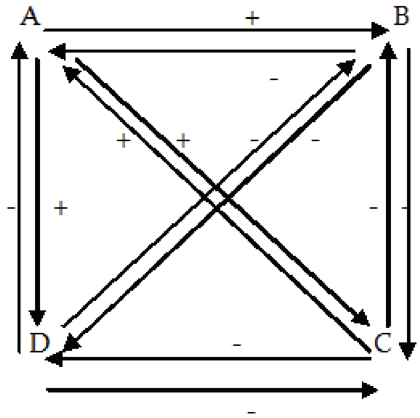
Corresponding *friendship matrix* with C.P is given below, where +1 is given for a good relationship between friends and -1 otherwise.

	A	B	C	D	E	C.P
A	+1	-1	-1	-1	+1	-1
B	-1	+1	+1	-1	+1	+1
C	-1	+1	+1	+1	-1	+1
D	-1	-1	+1	+1	-1	-1
E	+1	+1	-1	-1	+1	+1
C.P	-1	+1	+1	-1	+1	TOTAL C.P = +1

Obviously, a person loves himself and so all the diagonal elements here are +1.
Sum of the elements of the A-row or A-column gives the character point of A and similarly for B, C, D and E.
Total C.P = sum of the C.Ps of the rows or of the columns = -1+1+1-1+1 = +1.
Here we are trying to determine whether a person is good or bad on the basis of his relationship with other friends, i.e., we make an assumption that if a person has a good relationship with almost all his friends then he is good but if a person has a bad relationship with most of his friends, then he is considered as bad because he must have some negative qualities in him due to which his relationship is bad with most of his friends.
As per our assumption, B, C and E are good persons as they have C.P +1(>0) and A, D are bad as they have C.P -1(<0). Now the total character point tells us about the friend circle or the whole group. Here the total C.P is +1, i.e., the friend circle or the group A-B-C-D-E is good.
Knowing a person properly is not a very easy job in this world but here we have just made a mathematical attempt to do so.
Conclusion: Thus we can see that, by observing the type of relationship of a person with other persons we can easily determine his nature, in fact, the nature of the whole friend circle with the help of our newly introduced matrix called "friendship matrix" which is based on the concept of signed graph. We also note that the matrix obtained here is a symmetric matrix.

2 Representation of 'unfair' friendship

In our previous application, we have considered only one-way relationships but now we will consider two-way relationships. The objective of this problem is same as that of section 1. Suppose there are two friends A and B. It may happen that, A likes B but B does not like A. The "friendship matrix" in this case will not be symmetric. Let us consider the following example. Given below is a graph, where the edges represent the relationship among two friends and the vertices denote the friends respectively but in this section the graph is a directed graph.



Now, let us construct the ***friendship matrix*** of the above graph.

	A	B	C	D	C.P
A	+1	+1	+1	+1	+4
B	-1	+1	-1	-1	-2
C	+1	-1	+1	-1	0
D	-1	-1	-1	+1	-2
C.P	0	0	0	0	total C.P = 0

Here, the sum of the elements of the A-row gives the character point of A as +4 but as the matrix is not symmetric, sum of the elements of the A-column gives a different character point of A as 0.This is similar in case of B, C and D. So,
C.P of A = +4 + 0 = +4 (i.e., sum of the C.P of the row and column of A)
C.P of B = -2 + 0 = -2
C.P of C = 0 + 0 = 0
C.P. of D = -2 + 0 = -2.

Total C.P = sum of the C.Ps of the rows or the columns = +4 – 2 + 0 - 2 = 0+0+0+0 = 0.
So, by our assumption of section 1, A is the best person among the four, B and D are worse persons among the four but we cannot say anything about C. So we may consider C as a neutral person i.e., neither good nor bad.
Total C.P = 0. That means the friend circle or the group A-B-C-D is neither good nor bad. This is one disadvantage of character point, i.e., if C.P = 0, we cannot say anything properly.

Conclusion: Thus we can see that, in case of a two-way relationship we are also able to determine the nature of a person from the character points after constructing our *friendship matrix*. In fact, we are able to determine the nature of the whole group or the friend circle.

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