ESTIMATING COMPRESSIVE STRENGTH OF CONCRETE USING RIDGE REGRESSION

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Abstract: This paper deals with development of models for compressive strength of concrete using experimentally generated data that is non-orthogonal. Generalized Ridge Regression (GRR) and Traditional Ridge Regression (TRR) techniques are used to estimate model parameters. Differential Evolution (DE) strategy is used to find optimal ridge parameters according to the prediction error. The results obtained are compared with those achieved by Ordinary Least Square Regression (OLSR) technique. The results demonstrate that performance of pure quadratic GRR model is best among the developed models.

Keywords: concrete, compressive strength, ridge regression.

Introduction

Concrete, commonly made by mixing Portland cement with water, sand or fine aggregates, and crushed rock or coarse aggregates, is the most widely used man made construction material. Every year billion tons of cement is converted into concrete world-wide. Although it is neither as strong nor as tough as steel, still it is the most widely used construction material. This may be attributed to many factors like excellent resistance to water, ease with which it can be cast into variety of shapes and sizes, and to add to it is the cheapest and most readily available material on the job. Compared to most other engineering materials, the production of concrete requires considerably less energy output (Mehta and Monteiro [1]). Compressive strength is regarded as one of the most important mechanical characteristics of concrete because concrete is primarily meant to withstand compressive stresses. By varying the material ingredients in different proportions, it is possible to design concrete mixes of desired strength and durability.

There are many variants of regression methods employed by researchers to develop prediction models for compressive strength of concrete. Namyong *et al.* [2] proposed multiple linear regression models based on Ordinary Least Square Regression (OLSR) technique for predicting compressive strength of concrete for 7 days and 28 days curing period. Abd and Zain [3] applied logarithmic transformation to all the variables and built power models to predict slump and compressive strength of concrete at different curing ages. Abdullahi *et al*. [4] proposed polynomial models to predict slump, air-dry density and compressive strength of lightweight concrete. Qadi *et al.* [5] modeled modulus of elasticity of self-compacting concrete and compressive strength of concrete at various curing ages in a quadratic manner using central composite response surface method. Wu *et al.* [6] developed a parsimonious model to predict compressive strength of concrete. You *et al*. [7] developed dynamics modulus prediction models for asphalt mixes using weighted least square nonlinear multiple regression method.

The success of prediction depends both on proper form of the model and on the proper values of the parameters of the model. The parameters are usually estimated from the experimental data. The purpose of the parameter estimation in these cases is to not just to fit experimental data, but to find parameters as close to the true ones as possible. Scientists and engineers traditionally rely on different variants of the method of ordinary least square regression for estimating model parameters. This method leads to unbiased estimators. The unbiased property is meaningful only if the fitted model is the true model, and most often this may not be guaranteed and as such unbiased property should not be over emphasized (Ngo *et al.*[8]). Also, Hoerl and Kennard [9] argued that, in multiple regression, parameter estimates based on minimum residual sum of squares have a high probability of being far away from true parameter values, if prediction variables are not orthogonal. They proposed Ridge Regression (RR) technique that belongs to the class of biased estimators. This method leads to smaller values of Mean Square

Error (MSE) function (which is the measure of goodness of estimators) for estimating parameters of linear models having non-orthogonal predictor variables.

In the present study, Traditional Ridge regression (TRR) and Generalized Ridge Regression (GRR) techniques proposed by Hoerl and Kennard [9] are implemented to develop linear compressive strength models. Also, the methods are extended to develop second order polynomial models as well. Differential Evolution (DE) algorithm [10] is employed to obtain optimal ridge parameters. The prediction accuracy of the developed models is compared with OLSR linear and OLSR polynomial models.

2. Experimental Dataset

The Compressive strength data explored in this study was generated by Kumar [11] by conducting experiments under controlled laboratory conditions. The concrete mixes were proportioned using four basic ingredients, namely, water, cement, coarse aggregate and fine aggregate. The proportions of materials for concrete mixes were determined by DoE method of mix design [12]. Ordinary Portland cement of 43 grade having specific gravity of 3.14 was used. The 7 and 28 days compressive strength of cement was 35.6 MPa and 45.5 MPa, respectively. The fine aggregate was river bed sand with a fineness modulus of 2.09 and specific gravity of 2.54. Three types of coarse aggregate viz. CA-I, CA-II and CA-III, were used in different proportions in order to increase the density of resulting mix. Table 1 sums up the salient properties of these aggregates. The coarse aggregates were divided into three zones, namely, A, B and C, based on the percentage of different types of aggregates used. Table 2 summarizes details of these zones. Also, the water content variation for each zone of aggregate is shown in table 2.

Table 1. Properties of coarse aggregates

Type of aggregate	Unit mass (compact) (kg/m^3)	Specific gravity	Percentage absorption (%)
CA-I	1.58	2.68	1.80
CA-II	1.48	2.68	1.18
CA-III	2.15	2.60	1.20

Table 2. Zones of aggregates

Zone	Percentage passing 20 mm sieve and retained on 10 mm sieve (CA-I)	Percentage passing 10 mm sieve and retained on 4.75 mm sieve (CA-II)	Percentage passing 4.75 mm sieve and retained on 2.36 mm sieve (CA-III)	Water content requirement (kg/m^3)
A	67	33	-	180 – 210
B	50	50	-	190 – 220
C	-	50	50	200 – 230

A set of 49 concrete mixes were prepared by varying water-cement ratios, cement contents and aggregates fractions (Kumar [11]). Water-cement content ratio was kept between 0.42 and 0.55. Out of these 49 mixes, 18, 17 and 14 mixes were prepared using zone A, zone B and zone C of coarse aggregates, respectively. For each mix, 15 cubes of 150 mm size were cast and were tested at 28 days of curing period. Thus, an extensive data bank was generated and the same has been used in the present work for analyzing compressive strength of concrete.

3. Overview of ridge regression

In matrix notation, the multiple linear regression model is given as:

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \quad (1)$$

where $\boldsymbol{Y}$ is a $n \times 1$ vector for response variable, $\boldsymbol{X}$ is a $n \times (p+1)$ matrix. First column of $\boldsymbol{X}$ consists of ones and remaining p columns are for explanatory variables or predictors, $\boldsymbol{\beta}$ is a $(p+1) \times 1$ vector for unknown regression coefficients and $\boldsymbol{\varepsilon}$ is a $n \times 1$ vector of experimental errors with mean 0 and variance σ^2. OLS estimators of regression coefficients are given as:

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X'X})^{-1}\boldsymbol{X'Y} \qquad (2)$$

When the predictor variables are non orthogonal or the data is multicollinear, the matrix $(X'X)^{-1}$ is ill conditioned. A unique solution to equation (2) exists in these nearly singular cases but the solution is very unstable. Small changes in independent or dependent variables can cause drastic changes in the estimates of regression coefficients (Rawling *et al.*[13]). To tackle the problem of multicollinearity, Hoerl and Kennard [9] proposed a ridge regression technique to estimate $\boldsymbol{\beta}$.

Generalized Ridge Regression (GRR) estimates of the regression coefficients proposed by Hoerl and Kennard [9] are given as:

$$\widehat{\boldsymbol{\beta}}_{\boldsymbol{RR}} = (\boldsymbol{X'X} + \boldsymbol{K})^{-1}\,\boldsymbol{X'Y} \qquad (3)$$

where $\boldsymbol{K}$ is a $(p+1)\times(p+1)$ diagonal matrix. Diagonal entries of $\boldsymbol{K}$ are called ridge parameters. If all the ridge parameters are taken to be same, then ridge estimates obtained using equation (3) are referred to as Traditional Ridge Regression (TRR) estimates. For $\boldsymbol{K} = \boldsymbol{0}$, the ridge regression coefficients are identical to least square regression coefficients.

It is worth mentioning here that ridge regression is a linear regression technique. But, exact form of relationship between compressive strength of concrete and its ingredients is still not known. In the present work, it is intended to develop first and second order approximation models for compressive strength of concrete. It is known that all polynomial models regardless of their degree, are linear in the parameters. So, sample space is expanded to accommodate quadratic terms. Then, ridge regression technique is implemented to develop quadratic models.

Further, performance of ridge regression models depends on the selection of ridge parameters. Many methods to estimate the optimal value of ridge parameter for TRR are proposed in literature. Hoerl and Kennard [9] used ridge trace to decide the value of k. This is a graph of the standardized regression coefficient estimates against k. The value of k is selected for which parameter estimates tend to stabilize. Many other methods to find optimal value of k has been suggested in literature [14-17]. In the present work, cross validation criterion employed by Yan [18] has been used to find optimal ridge parameters for TRR and GRR models.

3.1. Optimal ridge parameters

Given data set is randomly divided into two parts. One part is called modeling sample $\boldsymbol{S}$ and is used to develop model. The other part is called test sample $\boldsymbol{S_t}$. Let the test sample contains n_t pairs of observations. Let $\boldsymbol{X_t}$ and $\boldsymbol{y_t}$ denote design matrix of independent variables and vector of dependent variable, respectively. Given the diagonal matrix $\boldsymbol{K}$, GRR estimates $\widehat{\boldsymbol{\beta}}_{\boldsymbol{RR}}$ are obtained using equation (3). The prediction value $\widetilde{\boldsymbol{y}}_{\boldsymbol{t}}$ for $\boldsymbol{y_t}$ are calculated as follows:

$$\widetilde{\boldsymbol{y}}_{\boldsymbol{t}} = \boldsymbol{X_t}\widehat{\boldsymbol{\beta}}_{\boldsymbol{RR}} \qquad (4)$$

The mean square prediction error $MSE(\boldsymbol{K})$ corresponding to the given diagonal matrix $\boldsymbol{K}$ is defined as

$$MSE(\boldsymbol{K}) = (\boldsymbol{y_t} - \widetilde{\boldsymbol{y}}_{\boldsymbol{t}})'(\boldsymbol{y_t} - \widetilde{\boldsymbol{y}}_{\boldsymbol{t}})/n_t \qquad (5)$$

Thus, $MSE(\boldsymbol{K})$ is a function of $\boldsymbol{K}$. Considering that the prediction power is one of the most important qualities of the model, the objective is to find optimal ridge parameters that minimizes equation (5) directly. DE is employed for the minimization of equation (5). DE algorithm developed by Storn and Price [10] performs a group based search in place of a point to point search to find optimal solution.

4. Prediction models for compressive strength of concrete

The variables used for prediction are water-cement content ratio (w/c), fine aggregate-cement content ratio (fa/c), coarse aggregate-cement content ratio (ca/c) and cement content (c). Compressive strength of concrete is measured in MPa and cement content is measured in kg/m^3. The total sample set consists of 49 concrete mix composition observations. Total sample set is divided into the modeling sample $\boldsymbol{S}$ of 16 observations and the test sample $\boldsymbol{S_t}$ of 33 observations. To check the accuracy of ridge regression model developed by a small data set, test sample is taken larger than the modeling sample.

4.1. Sample data analysis

To analyze multicollinearity among the sample data, Ryan [19] suggested to examine the correlations between the pairs of predictor variables and the Variance Inflation Factor (VIF) of predictor variables. A pair wise correlation matrix of predictor variables might be insufficient to identify collinearity problem because linear dependencies may exist among combinations of predictors, hence, it is necessary to examine VIFs also. Following Ryan [19], the VIF_i of i^{th} predictor variable x_i (say) has been considered as:

$$VIF_i = \frac{1}{1 - R_i^2} \tag{6}$$

where R_i^2 is the squared multiple correlation coefficient that results from regression of x_i against all other predictors. It is clear that if x_i has a strong linear relationship with other predictor variables, R_i^2 is close to 1 and VIF value tends to be very high. In the absence of linear relationship among predictor variables, R_i^2 is zero and VIF equals 1. As a rule of thumb, multicollinearity is said to exist if VIF value for a predictor variable is more than 10.

The pair wise correlation between independent variables and VIF values are listed in table 3. It can be seen that all the pair wise correlations are numerically greater than 0.500. Three pairs of variables have very high level of correlation. The pair w/c and fa/c has highest correlation (0.960). The other two pairs with high correlation are that of ca/c and c (-0.776) and that of w/c and c (-0.734). These results indicate that the given data set suffers from multicollinearity. Also, it can be noted from the Table that VIF values for w/c and fa/c exceed 10 and thus provide an evidence for presence of multicollinearity. Further, in quadratic models, strong multicollinearity is present because of form of the models.

Table 3. Correlation matrix and VIF values

	w/c	fa/c	ca/c	c	VIFvalue
w/c	1.000	-0.960	0.517	-0.734	33.444
fa/c		1.000	0.546	-0.637	25.863
ca/c			1.000	-0.776	4.532
c				1.000	7.877

4.2. Development of prediction models for compressive strength of concrete

In order to illustrate the performance of developed models, MSE_p for test sample $\boldsymbol{S_t}$ is defined as

$$MSE_p = MSE(\boldsymbol{K_{opt}}) \tag{7}$$

where $MSE(\boldsymbol{K_{opt}})$ is the value of mean square error evaluated from equation (5) at optimal value of $\boldsymbol{K}$ which is obtained using DE algorithm. However, for OLSR models, $\boldsymbol{K} = \boldsymbol{0}$. Moreover, Six tests were executed as follows and six compressive strength models were developed.

- **Test 1** Employ OLSR technique to the modeling sample $\boldsymbol{S}$ to develop linear compressive strength model and obtain the value of MSE_p for the developed model.
- **Test 2** Employ OLSR technique to develop pure quadratic model for compressive strength of concrete and thus, calculating MSE_p for the developed model.
- **Test 3** Employ TRR technique for the modeling sample $\boldsymbol{S}$ to build linear compressive strength model. Value of MSE_p based on linear TRR compressive strength model is then calculated.
- **Test 4** Employ TRR technique for the modeling sample $\boldsymbol{S}$ to build pure quadratic compressive strength model. Value of MSE_p is calculated for the developed model.
- **Test 5** Employ GRR technique to the modeling sample $\boldsymbol{S}$ to develop linear compressive strength model and obtain MSE_p value of linear GRR based model.
- **Test 6** Employ GRR technique to the modeling sample $\boldsymbol{S}$ to develop pure quadratic compressive strength model and find MSE_p value of pure quadratic GRR model.

In order to obtain the optimal ridge parameters for tests 3-6, DE algorithm was employed using parameters $Np = 50$, $Cr = 0.9$,

$F = 0.85$ and $g_{max} = 500$. DEMAT, a MATLAB program developed by Price *et al.* [20] is used to carry out DE algorithm.

For tests 3 and 4, the optimal ridge parameters obtained by DE are 0.02112 and 0.04497, respectively. For test 5, five diagonal elements of optimal diagonal matrix $\boldsymbol{K}$ are 0.02052, 1, 0, 0 and 0, respectively. For test 6, nine diagonal elements of optimal diagonal matrix $\boldsymbol{K}$ are 1, 0, 1, 0.07356, 0, 1, 1, 0 and 0.00002, respectively. The regression coefficients of the developed models are summarized in table 4.

To demonstrate the performances of six developed compressive strength models, the predicted compressive strength values for the test sample $\boldsymbol{S_t}$ are plotted against the true observed values for the test sample. The graphs obtained are shown in figure 1. Further, the MSE_p values of each of the six developed models are listed in table 5.

It can be clearly seen in figure 1 that prediction values of pure quadratic GRR model distribute very close along the diagonal and the prediction power of pure quadratic GRR model is best of all the six models. It can be noted from the table 5 that the MSE_p value of pure quadratic GRR model is the lowest of all. The second lowest value of MSE_p is for linear GRR model.

Table 4. Regression coefficients for developed models

	Regression Coefficients					
	Linear OLSR model	Pure Quadratic OLSR model	Linear TRR model	Pure Quadratic TRR model	Linear GRR model	Pure Quadratic GRR model
w/c	-10.22872	-256.17107	0.94213	-0.40166	0.054915	-96.39035
fa/c	-11.71000	4.70592	-13.47313	-2.74175	-14.88389	-0.11708
ca/c	-4.62577	-13.94506	2.33996	-1.41537	3.14997	-1.19753
c	0.03831	0.47805	-0.10120	0.26039	0.102159	0.39167
$(w/c)^2$	-	228.05329	-	-0.24700	-	0.00254
$(fa/c)^2$	-	-5.24375	-	-4.25536	-	-0.24907
$(ca/c)^2$	-	2.22354	-	-0.40971	-	0.48211
c^2	-	-0.00051	-	-0.00026	-	-0.00041
$Intercept$	62.51107	30.13350	13.75796	0.03771	13.38763	0.016201

Table 5. Mean square error for test sample data set

Name of the model	MSE_p
Linear OLSR model	12.28840
Pure Quadratic OLSR model	8.18966
Linear TRR model	3.56506
Pure Quadratic TRR model	9.93054
Linear GRR model	3.07543
Pure Quadratic GRR model	1.50783

5. Conclusion

This paper presents application of ridge regression approach to estimate model parameters to predict compressive strength of concrete. OLSR, TRR and GRR techniques are used to develop compressive strength models. DE is used to find optimal ridge parameters. It is seen that pure quadratic GRR model perform best with respect to prediction accuracy of the model.

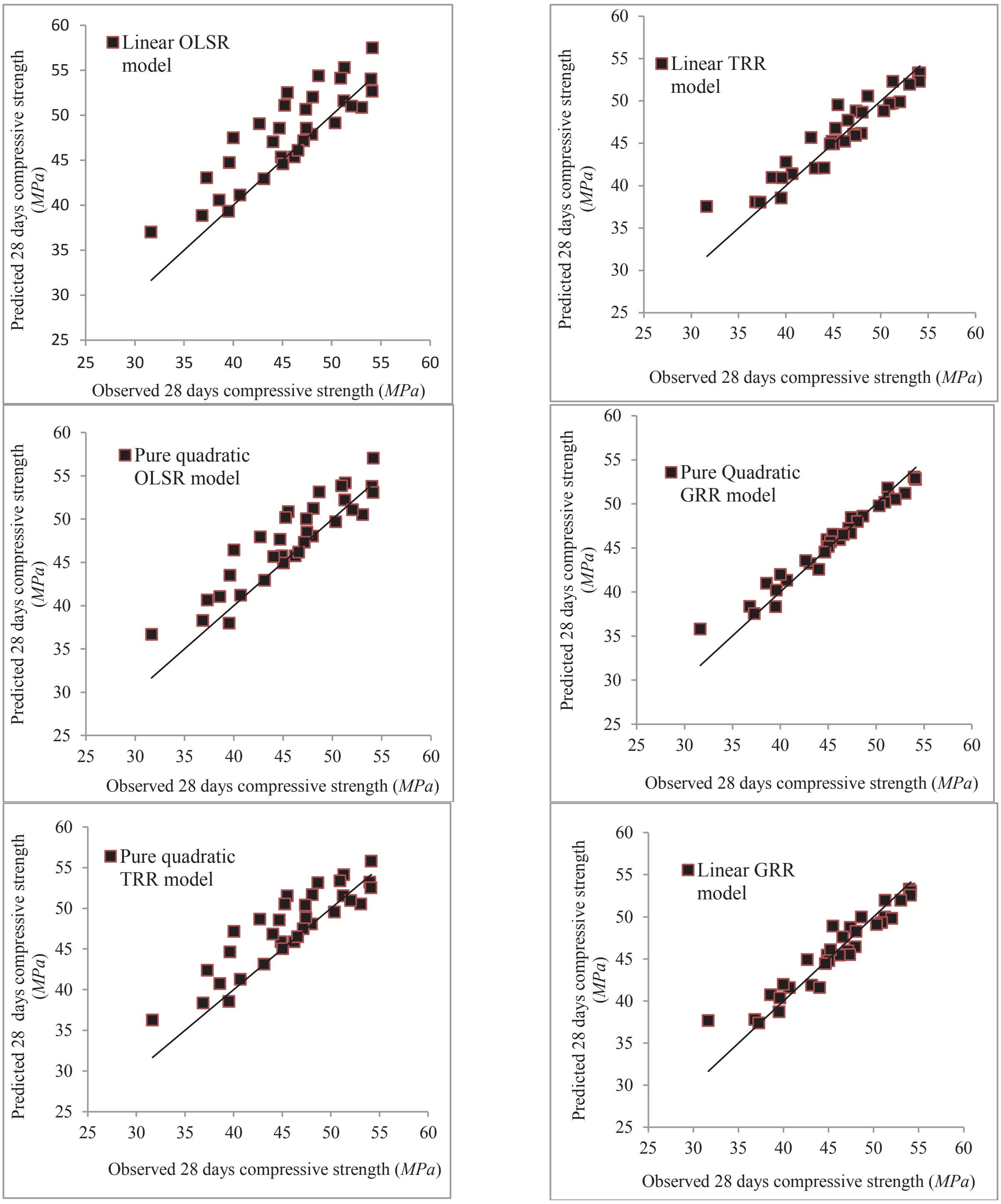

Figure 1. Performances of six compressive strength models

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