FAMILY OF ESTIMATORS OF RATIO OF A FINITE POPULATION USING AUXILIARY INFORMATION UNDER RANDOM NON-RESPONSE

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Abstract: This paper defines a general family of estimators for estimating ratio of two mean of a finite population using auxiliary information is suggested and studied under two different situation of random non-response suggested by Tracy and Osahan (1994). Asymptotic expression of biases and mean squared error of the proposed family is derived.

Keywords: Auxiliary information, Random non-response, Bias, Mean squared error

Introduction: In estimation of population characteristics ancillary information closely related to the main characteristics plays a very important role. In estimation of population characteristics, the population parameters can be estimated more accurately by making use information on an auxiliary variable that is correlated with study variable. Ratio and regression methods of estimation are good examples in this context. In sampling theory, it is well known that the efficiency of the estimators of unknown population parameters of the study variable can be increased by suitably using known information on an auxiliary variable.

The problem of estimating the ratio and product of two means of a finite population using information on a single auxiliary variable or many of them has been discussed, among others, by Singh (1965, 1967, 1986a,1986b, 1988), Rao and Pereira (1968), Shah and Shah (1978) ,Tripathi (1980),Ray and Singh (1985), Upadhyaya and Singh (1985), Upadhyaya et al.(1985), Srivastava et al(1989) and Singh et al.(1994a, 1994b).

2. Distribution of Random Non-Response, Notations and Expectations

Let $U=(U_1,U_2,\ldots\ldots\ldots,U_N)$ denote a population of N units from which a simple random sample of size n is drawn without replacement.If $r(r=0,1,2,\ldots,(n-2))$ denotes the number of sampling units on which information could not be obtained due to a random non- response, then the remaining $(n-r)$ units can be treated as a simple random sample from U .it is assumed that r is less than $(n-1)$,i.e. $0\le r\le(n-2)$. Singh and Joarder (1998) assumed that r has the following discrete distribution as -

$$P(r)=\frac{n-r}{(nq+2p)}\binom{n-2}{r}p^r q^{n-2-r} \qquad (2.1)$$

where p is the probability of non-response, $q=1-p$ and $\binom{n-2}{r}$ represents that total number ways to obtain r non-response out of a possible ($n-2$).

Singh and Joarder (1998) obtained the following maximum likelihood estimator of p (probability of non- response), as

$$\hat{p}=\frac{(n-1+r)-\sqrt{(n-1+r)^2-\frac{4rn(n-3)}{(n-2)}}}{2(n-3)} \qquad (2.2)$$

If $r=0$ then $\hat{p}=0$, and $r=n-2$ then, $\hat{p}=1$; thus $\hat{p}$ is an admissible estimators of response probability p.

We define For the variate y_0 , y_1 and y_2

$\bar{Y}_i=N^{-1}\sum_{j=1}^{N}y_{ij}$: Population mean of the ith variate $y_i\,(i=0,1,2)$;

$R=\bar{Y}_0/\bar{Y}_1$: The population parameter under study;

$C_i\,(i=0,1,2)$: The population coefficient of variation (CV) of the variate $y_i\,(i=0,1,2)$;

$\rho_{il}=S_{il}/S_iS_l\;(i\ne l)$: The population correlation coefficient between the variates y_i and y_l $(i\ne l=0,1,2)$;

$(N-1)S_{il}=\sum_{j=1}^{N}(y_{ij}-\bar{Y}_i)(y_{lj}-\bar{Y}_l)$; $\qquad K_{il}=\rho_{il}C_i/C_l$ $(i\ne l=0,1,2)$

$K=(K_{02}+K_{12})$, $d=(\lambda_{102}C_0+\lambda_{012}C_1)$

$\lambda_{w_1w_2w_3}=\mu_{w_1w_2w_3}\Big/(\mu_{200})^{\frac{w_1}{2}}(\mu_{020})^{\frac{w_2}{2}}(\mu_{002})^{\frac{w_3}{2}}$

$$\left(N-1\right)\mu_{w_1w_2w_3} = \sum_{j=1}^{N}\left(y_{0j}-\overline{Y}_0\right)^{w_1}\left(y_{1j}-\overline{Y}_1\right)^{w_2}\left(y_{2j}-\overline{Y}_2\right)^{w_3}$$

$$B = C_2^2K^2 + \frac{\left(C_2K\lambda_{003}-d\right)^2}{\left(\lambda_{004}-1-\lambda_{003}^2\right)}$$

$$B = \left(C_2^2K^2+\Delta B_1^2\right),\ K=\left(K_{01}+K_{12}\right),\ d=\left(\lambda_{102}+\lambda_{012}C_1\right)$$

$$B_1 = \frac{\Delta_2}{\Delta},\ \Delta_2 = \left(C_2K\lambda_{003}-d\right) \text{ and } \Delta=\left(\lambda_{004}-1-\lambda_{003}^2\right)$$

For the variates y_0, y_1 and y_2 in the sample:

where, $\overline{y}_i = n^{-1}\sum_{j=1}^{n} y_{ij}$, $(i=0,1,2)$, $\hat{R}=\overline{y}_0/\overline{y}_1$,

$\overline{y}_i^* = (n-r)^{-1}\sum_{j=1}^{n-r} y_{ij}$ $s_i^2 = (n-1)^{-1}\sum_{j=1}^{n}\left(y_{ij}-\overline{y}_i\right)^2$,

$s_i^{*2} = (n-r-1)^{-1}\sum_{j=1}^{n-r}\left(y_{ij}-\overline{y}_i^*\right)^2$ are conditionally unbiased estimators of $S_i^2\ (i=0,1,2)$ respectively.

Where $\theta^* = \left(\frac{1}{nq+2p}-\frac{1}{N}\right)$ and $\theta=\left(\frac{1}{n}-\frac{1}{N}\right)$

If $r=0$ then $\hat{p}=0$, and $r=n-2$ then, $\hat{p}=1$; thus $\hat{p}$ is an admissible estimators of response probability p.

Let us define

$\overline{y}_0^* = \overline{Y}_0\left(1+\varepsilon_0\right)$, $\overline{y}_1^* = \overline{Y}_1\left(1+\varepsilon_1\right)$, $\overline{y}_2^* = \overline{Y}_2\left(1+\varepsilon_2\right)$, $\overline{y}_2 = \overline{Y}_2\left(1+\varepsilon_3\right)$, $s_2^{*2} = S_2^2\left(1+\varepsilon_4\right)$ $s_2^2 = S_2^2\left(1+\varepsilon_5\right)$

Then under the model: $E\left(\varepsilon_i\right)=0, (i=0,1,2)$

$E(\varepsilon_0^2)=\theta^*C_0^2$, $E(\varepsilon_1^2)=\theta^*C_1^2$, $E(\varepsilon_2^2)=\theta^*C_2^2$,
$E(\varepsilon_3^2)=\theta C_2^2$, $E(\varepsilon_4^2)=\theta^*\left(\lambda_{004}-1\right)$ $E(\varepsilon_5^2)=\theta\left(\lambda_{004}-1\right)$
, $E(\varepsilon_0\varepsilon_1)=\theta^*\rho_{01}C_0C_1$ $E(\varepsilon_0\varepsilon_2)=\theta^*\rho_{02}C_0C_2$
, $E(\varepsilon_0\varepsilon_3)=\theta\rho_{02}C_0C_2$ $E(\varepsilon_0\varepsilon_4)=\theta^*\lambda_{102}C_0$
, $E(\varepsilon_0\varepsilon_5)=\theta\lambda_{102}C_0$, $E(\varepsilon_1\varepsilon_2)=\theta^*\rho_{12}C_1C_2$
, $E(\varepsilon_1\varepsilon_3)=\theta\rho_{12}C_1C_2$, $E(\varepsilon_1\varepsilon_4)=\theta^*\lambda_{012}C_1$ $E(\varepsilon_1\varepsilon_5)=\theta\lambda_{012}C_1$
, $E(\varepsilon_2\varepsilon_3)=\theta C_2^2$, $E(\varepsilon_2\varepsilon_4)=\theta^*\lambda_{003}C_2$, $E(\varepsilon_2\varepsilon_5)=\theta\lambda_{003}C_2$,
$E(\varepsilon_3\varepsilon_4)=\theta\lambda_{003}C_2$, $E(\varepsilon_3\varepsilon_5)=\theta\lambda_{003}C_2$,
$E(\varepsilon_4\varepsilon_5)=\theta\left(\lambda_{004}-1\right)$.

Estimated value of the terms are defined as-

$\hat{\Delta}=\left(\hat{\lambda}_{004}-\hat{\lambda}_{003}^2-1\right)>0$, $\hat{\Delta}^*=\left(\hat{\lambda}_{004}^*-\hat{\lambda}_{003}^{*2}-1\right)$,

$\hat{\Delta}_1^* = \left[\hat{d}^*\hat{\lambda}_{003}^* - \hat{K}^*\left(\hat{\lambda}_{004}^*-1\right)C_2\right]$

$\hat{\Delta}_2^* = \left[\hat{d}^*\hat{\lambda}_{003}^*C_2-\hat{d}^*\right]$, $\hat{K}^*=\left(\hat{K}_{02}^*-\hat{K}_{12}^*\right)$, $\hat{K}_{02}^* = \hat{\rho}_{02}^*\hat{C}_0^*/C_2$, $\hat{K}_{12}^*=\hat{\rho}_{12}^*\hat{C}_1^*/C_2$,

$\hat{\rho}_{02}^* = \hat{\mu}_{101}^*/\sqrt{\hat{\mu}_{200}^*\hat{\mu}_{002}^*}$, $\hat{\rho}_{12}^* = \hat{\mu}_{011}^*/\sqrt{\hat{\mu}_{020}^*\hat{\mu}_{002}^*}$,

$\hat{K}_{02}=\hat{\rho}_{02}\hat{C}_0^*/C_2$, $\hat{K}_{12}=\hat{\rho}_{12}\hat{C}_1^*/C_2$, $\hat{C}_0^*=\sqrt{\hat{\mu}_{200}^*/\overline{y}_0^*}$,

$\hat{C}_1^*=\sqrt{\hat{\mu}_{020}^*/\overline{y}_1^*}$, $\hat{C}_2^*=\sqrt{\hat{\mu}_{002}^*/\overline{y}_2^*}$, $\hat{\rho}_{02}=\hat{\mu}_{101}^*/\sqrt{\hat{\mu}_{200}^*\mu_{002}}$,

$\hat{\rho}_{12}=\hat{\mu}_{011}^*/\sqrt{\hat{\mu}_{020}^*\hat{\mu}_{002}}$ and $\hat{C}_2=\sqrt{\hat{\mu}_{002}/\overline{y}_2}$

where

$$\left(n-r-1\right)\hat{\mu}_{w_1w_2w_3} = \sum_{j=1}^{n-r}\left(y_{0j}-\overline{y}_0^*\right)^{w_1}\left(y_{1j}-\overline{y}_1^*\right)^{w_2}\left(y_{2j}-\overline{y}_2^*\right)^{w_3}$$

$$\hat{B}^* = C_2^2\hat{K}^{*2} + \left(\hat{C}_2\hat{K}^*\hat{\lambda}_{003}-\hat{d}^*\right)^2 \Big/ \left(\hat{\lambda}_{004}-\hat{\lambda}_{003}^2-1\right)$$

Let $y_i\ (i=0,1)$ be the study character with population mean $\overline{Y}_i\ (i=0,1)$, and let y_2 be an auxiliary character (correlated with study character y_i with a known population mean $\overline{Y}_2$. Assume that a simple random sample of size n is drawn without replacement and $\left(y_{0i}, y_{1i}, y_{2i}\right)$, $i=1,2,....,n,$ are observed.

The usual estimator of the ratio $R=\overline{Y}_0/\overline{Y}_1$ is defined by $\hat{R}=\overline{y}_0/\overline{y}_1$ (2.3)

Singh et al (2007) suggested a family of estimator under non- response model

$$\hat{d}_1 = \hat{R}^* g\left(u^*, v^*\right) \quad (2.4)$$

Where $g\left(u^*,v^*\right)$ is a function of u^* and v^* such that $g(1,1)=1$ and satisfying certain regularity conditions, as defined below. To the first order of approximation, MSE of d_1 is given by,

Thus the minimum $MSE\left(\hat{d}_1\right)$ is given by

$$\min.MSE\left(\hat{d}_1\right) = MSE\left(\hat{R}^*\right) - \left(\theta^*-\theta\right)R^2B \quad (2.5)$$

where

$$MSE\left(\hat{R}^*\right) = R^2\theta^*\left[C_0^2+C_1^2\left(1-2K_{01}\right)\right]$$

The following families of estimators of $\hat{R}^*$ is also defined as

$$d_1^{(1)} = \hat{R}^* g\left(u^*\right) \quad (2.6)$$

$$d_1^{(2)} = \hat{R}^* g\left(v^*\right) \quad (2.7)$$

may be defined as members of the family $\hat{d}_1$, minimum MSE of $\hat{d}_1^{(1)}$ and $\hat{d}_1^{(2)}$ are given by

$$\min.MSE\left(\hat{d}_1^{(1)}\right) = \left[MSE\left(\hat{R}^*\right) - \left(\theta^*-\theta\right)R^2C_2^2K^2\right] \quad (2.8)$$

$$\min.MSE\left(\hat{d}_1^{(2)}\right) = \left[MSE\left(\hat{R}^*\right) - \left(\theta^*-\theta\right)\frac{R^2d^2}{\left(\lambda_{004}-1\right)}\right] \quad (2.9)$$

Any parametric function $g\left(u^*,v^*\right)$, satisfying the regularity conditions, can generate as asymptotically acceptable estimator. The following estimators

$d_{1(1)} = \hat{R}^* u^*\ v^*$

$$d_{1(2)} = \hat{R}^* \frac{\left\{1+\alpha_1\left(u^*-1\right)\right\}}{\left\{1+\alpha_2\left(v^*-1\right)\right\}}$$

$$d_{1(3)} = \hat{R}^*\left[1-\alpha_1\left(u^*-1\right)-\alpha_2\left(v^*-1\right)\right]^{-1}$$

$$d_{1(4)} = \hat{R}^*\left[1-\alpha_1\left(u^*-1\right)-\alpha_2\left(v^*-1\right)\right]$$

$d_{1(5)} = \hat{R}^*\left(2-u^{*\alpha_1}v^{*\alpha_2}\right)$, and

$d_{1(6)} = \hat{R}^*\left[\alpha_1 u^* + (1-\alpha_1)v^{*\alpha_2}\right]$, etc

Of the parameter R are members of the family of estimators $\hat{d}_4$, the resulting estimators will have the same minimum MSE given by (2.5). Furthermore, we note that the class of estimators $\hat{d}_4$ does not include ratio cum regression type estimators

$$d_{1(7)} = \hat{R}^*\left(\frac{\bar{y}_2^*}{\bar{y}_2}\right)^{\alpha}\left\{1+\beta\left(s_2^{*2}-s_2^2\right)\right\} \qquad (2.10)$$

In this paper, we proposed a general class of family in which we shown that estimators proposed in this paper include the estimators proposed by Singh et al (2007) and also include estimator as (2.10).

3. Suggested estimators

We consider the situation when information on study variables y_0 and y_1 can-not be obtained for r units while information on the auxiliary variable y_2 is obtained for all the sample units. Class of estimator defined as follows

(a) The population mean $\bar{Y}_2$ and the population variance S_2^2 of the auxiliary character y_2 are not known. $t_r = \hat{R}^* f\left(\bar{y}_2^*, \bar{y}_2, s_2^{*2}, s_2^2\right)$ (3.1)

where $f\left(\bar{y}_2^*, \bar{y}_2, s_2^{*2}, s_2^2\right)$ are functions of $\bar{y}_2^*, \bar{y}_2, s_2^{*2}$ and s_2^2 , such that, $f\left(\bar{Y}_2, \bar{Y}_2, S_2^2, S_2^2\right)=1$ satisfying following regularity conditions:

1. Whatever the sample $\left(\bar{y}_2^*, \bar{y}_2, s_2^{*2}, s_2^2\right)$, assume values in a bounded, closed convex subset, S, of the two- dimensional real space containing the point $\left(\bar{Y}_2, \bar{Y}_2, S_2^2, S_2^2\right)$.
2. In S the function is continuous and bounded.
3. The first and second order partial derivatives of $f\left(\bar{y}_2^*, \bar{y}_2, s_2^{*2}, s_2^2\right)$ exist as well as is continuous and bounded in S.

Theorem 1. To the first degree of approximation the bias and min.MSE of t_r are, respectively, given by $B(t_r) = \Big[B\left(\hat{R}^*\right)+\left(\theta^*-\theta\right)R\left\{\bar{Y}_2 f_1 C_2^2 K + S_2^2 f_3 d\right.$

$$+\frac{R}{2}\left\{\bar{Y}_2^2 f_{11}\theta^* C_2^2 + \bar{Y}_2^2 f_{22}\theta C_2^2 + S_2^4 f_{33}\,\theta^*\left(\lambda_{004}-1\right)\right.$$
$$+S_2^4 f_{44}\theta\left(\lambda_{004}-1\right) + 2\bar{Y}_2^2 f_{12}\theta C_2^2 + 2\bar{Y}_2 S_2^2 f_{13}\theta^*\lambda_{003}C_2$$
$$+2\bar{Y}_2 S_2^2 f_{14}\theta\lambda_{003}C_2 + 2\bar{Y}_2 S_2^2 f_{23}\theta\lambda_{003}C_2 + 2\bar{Y}_2 S_2^2 f_{24}$$
$$\left.\left.\theta\lambda_{003}C_2 + 2S_2^4 f_{34}\theta\left(\lambda_{004}-1\right)\right\}\right] \qquad (3.2)$$

$$MSE(t_r) = \left[MSE\left(\hat{R}^*\right)-\left(\theta^*-\theta\right)R^2 B\right] \qquad (3.3)$$

Remark:- It is noteworthy to mention that $t_{r_1} = \hat{R}^* f\left(\bar{y}_2^*, \bar{y}_2\right)$ and $t_{r_2} = \hat{R}^* f\left(s_2^{*2}, s_2^2\right)$ are the subclass of the above family. Which contains the family of estimators of Singh et al (2007) .The population mean $\bar{Y}_2$ and population variance S_2^2 of the auxiliary character y_2 are not known.

Theorem 2.

Bias and min.MSE of the above subclass estimators are defined as

$$B\left(t_{r_1}\right) = \Big[B\left(\hat{R}^*\right)+\left(\theta^*-\theta\right)R\bar{Y}_2 f_1 C_2^2 K + \frac{R}{2}\left\{\bar{Y}_2^2 f_{11}\theta^* C_2^2\right.$$
$$\left.+\bar{Y}_2^2 f_{22}\theta C_2^2 + \bar{Y}_2^2 f_{12}\theta C_2^2\right\}\Big] \qquad (3.4)$$

$$B\left(t_{r_2}\right) = \Big[B\left(\hat{R}^*\right)+\left(\theta^*-\theta\right)RS_2^2 f_3 d + \frac{1}{2}\left\{S_2^4 f_{33}\,\theta^*\left(\lambda_{004}-1\right)\right.$$
$$\left.+S_2^4 f_{44}\theta\left(\lambda_{004}-1\right) + 2S_2^4 f_{34}\left(\lambda_{004}-1\right)\right\}\Big] \qquad (3.5)$$

$$\min.MSE\left(t_{r_1}\right) = \left[MSE\left(\hat{R}^*\right)-\left(\theta^*-\theta\right)R^2 C_2^2 K^2\right] \qquad (3.6)$$

$$\min.MSE\left(t_{r_2}\right) = \left[MSE\left(\hat{R}^*\right)-\left(\theta^*-\theta\right)\frac{R^2 d^2}{\left(\lambda_{004}-1\right)}\right] \qquad (3.7)$$

4. Estimators proposed in this paper with estimated optimum parameters

It is to be noted that the optimum values of the parameters included in estimators depend on unknown population parameters such as λ_{003} , λ_{004} , λ_{102} , λ_{012} , C_2 , K_{01} , K_{02} ,.....,etc. Thus to use such an estimator one has to use estimated values of these parameters. Estimated values of population parameters can be obtained from either past data or experience. If the estimated values are not known then it is advisable to use sample data at hand to estimate these parameters. Thus estimated optimum value of f_1 and f_3 is given by

$\hat{f}_1(\bar{Y}_2, \bar{Y}_2, S_2^2, S_2^2) = \hat{A}$ and $\hat{f}_3(\bar{Y}_2, \bar{Y}_2, S_2^2, S_2^2) = \hat{B}$ (4.1)

Under the regularity conditions, for t_r , we consider a function $f\left(\bar{y}_2^*, \bar{y}_2, s_2^{*2}, s_2^2\right)$ such that $f(\bar{Y}_2, \bar{Y}_2, S_2^2, S_2^2) = 1$,

$f_1(\bar{Y}_2,\bar{Y}_2,S_2^2,S_2^2)=\left.\frac{\partial f\left(\bar{y}_2^*,\bar{y}_2,s_2^{*2},s_2^2\right)}{\partial \bar{y}_2^*}\right|_{(G)}=A$,

$f_3(\bar{Y}_2,\bar{Y}_2,S_2^2,S_2^2)=\left.\frac{\partial f\left(\bar{y}_2^*,\bar{y}_2,s_2^{*2},s_2^2\right)}{\partial s_2^{*2}}\right|_{(G)}=B$ which indicates that the function $f\left(\bar{y}_2^*,\bar{y}_2,s_2^{*2},s_2^2\right)$ will contain not only $\bar{y}_2^*,\bar{y}_2,s_2^{*2},s_2^2$ but A and B as well, and, thus ,we require a function $f^*\left(\bar{y}_2^*,\bar{y}_2,s_2^{*2},s_2^2,A,B\right)$, such that $f^*(G)=1$, $f_1^*(G)=A$ and $f_3^*(G)=B$. Since in such function $f^*\left(\bar{y}_2^*,\bar{y}_2,s_2^{*2},s_2^2,A,B\right)$ so required, A and B are unknown, we may take $f^{**}\left(\bar{y}_2^*,\bar{y}_2,s_2^{*2},s_2^2,\hat{A},\hat{B}\right)=f^*\left(\bar{y}_2^*,\bar{y}_2,s_2^{*2},s_2^2,\hat{A},\hat{B}\right)$, (replacing A and B by their estimated values). Now, $f^*(G)=1$, $f_1^*(G)=A$ and $f_3^*(G)=B$. Now, we may consider

To develop a family of estimators t_r^* and associated MSE's analogous to the class t_r when optimum values are unknown. Then we get the following estimator, given by

$$t_r^*=\hat{R}^* f^*\left(\bar{y}_2^*,\bar{y}_2,s_2^{*2},s_2^2,\hat{A},\hat{B}\right) \qquad (4.2)$$

Thus the MSE of t_r^* to the first degree of approximation obtain under the conditions $f_1^*(G)=-f_2^*(G)$ $f_3^*(G)=-f_4^*(G)$ we get the $MSE\left(t_r^*\right)$ equal to the minimum $MSE\left(t_r\right)$ in(3.3),

Estimators of the minimum MSE of proposed family of estimator are given below

$$Est.\min MSE\left(t_r\right)=\left[Est.MSE\left(\hat{R}^*\right)-\left(\theta^*-\theta\right)R^2\hat{B}\right]$$
(4.3)
$$Est.\min MSE\left(t_{r_1}\right)=\left[Est.MSE\left(\hat{R}^*\right)-\left(\theta^*-\theta\right)R^2\hat{C}_2^2\hat{K}^2\right]$$
(4.4)
$$Est.\min MSE(t_{r_2})=\left[Est.MSE\left(\hat{R}^*\right)-\left(\theta^*-\theta\right)\frac{R^2\hat{d}^2}{\left(\hat{\lambda}_{004}-1\right)}\right]$$
(4.5)

5. Efficiency comparison:

Efficiency of the proposed estimators with respect to $MSE\left(\hat{R}^*\right)$ is defined as

$$MSE\left(\hat{R}_\alpha^*\right)-\min.MSE\left(t_1\right)=\left(\theta^*-\theta\right)R_\alpha^2C_2^2K_\alpha^2\geq 0$$
$$MSE\left(\hat{R}_\alpha^*\right)-\min.MSE\left(t_2\right)=\left(\theta^*-\theta\right)\frac{R_\alpha^2d_\alpha^2}{\left(\lambda_{004}-1\right)}\geq 0$$
$$MSE\left(\hat{R}_\alpha^*\right)-\min.MSE(t_3)=\left(\theta^*-\theta\right)R_\alpha^2B_\alpha\geq 0$$
$$\min.MSE\left(t_1\right)-\min.MSE\left(t_2\right)\geq 0$$
If $\frac{d_\alpha^2}{\left(\lambda_{004}-1\right)}\geq C_2^2K_\alpha^2$
$$\min.MSE\left(t_1\right)-\min.MSE(t_3)\geq 0$$
If $B_\alpha\geq C_2^2K_\alpha^2$
$$\min.MSE\left(t_2\right)-\min.MSE\left(t_3\right)\geq 0$$
If $B_\alpha\geq\frac{d_\alpha^2}{\left(\lambda_{004}-1\right)}$

6. Empirical study

For empirical study data are related to the total cultivated area during 1978-79 and the area under wheat in the two consecutive years 1978-79 and 1979-80 for a sample of 16 villages selected in the Baghpat Tehsil of Meerut District (UP) for the survey on fertilizer practices. The following parameters are given below

$N=364$, $n=16$, $r=4$, $\hat{p}=0.3037$, $\hat{q}=0.6962$, $\theta^*=0.08237$, $\theta=0.05975$, $\bar{Y}_0=422.625$, $\bar{Y}_1=411.75$, $\bar{Y}_2=1360.938$, $\hat{C}_0^*=16.8961$, $\hat{C}_1^*=16.0810$, $\hat{C}_2^*=30.3785$, $\hat{K}_{02}=0.6048$, $\hat{K}_{12}=0.5650$, $\hat{K}_{01}^*=1.03559$, $\hat{K}_{02}^*=1.1643$, $\hat{K}_{12}^*=1.0520$, $C_2=27.4371$.

Relative efficiency of the estimators proposed in this paper is given as-

$$RE\left(t_r\right)=\frac{MSE\left(\hat{R}^*\right)}{\min.MSE\left(t_r\right)}\times 100 \qquad (6.1)$$

$MSE\left(\hat{R}^*\right)$=0.001891133

S. no.	Estimators	MSE	Relative efficiency (%)
1.	t_r	0.0018807	100.554
2.	t_{r_1}	0.0018811	100.533
3.	t_{r_2}	0.0018832	100.416

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