METHODS TO FIND THE RANK AND MULTIPLICATIVE INVERSE OF FULLY FUZZY MATRICES

JAGDEEP KAUR

Abstract: A fully fuzzy matrix is an extension of crisp matrix with fuzzy numbers as its elements. In this paper, under the general framework of fuzzy matrix theory, we have proposed some methods to find the rank and multiplicative inverse of a fully fuzzy matrix. To illustrate the proposed methods some numerical examples are solved and it can be easily seen that the multiplicative inverse of a fully fuzzy matrix is also a fully fuzzy matrix as well.

Keywords: Fully fuzzy matrix, Ranking function, Trapezoidal fuzzy numbers.

1. IntroductionFuzzy matrix has at least two meanings in literature (1) $A = [a_{ij}]_{m\times n}$ is called a fuzzy matrix if $a_{ij} \in [0,1]$, (2) A is called a fuzzy matrix if all its entries are fuzzy numbers. The first class of fuzzy matrices has been first described by Kim and Roush [1]. After that many researchers have used this concept [2-4]. Some authors [5-9] have discussed on the second class of matrices. To check the invertibility and find the inverse of fuzzy matrix many researchers [10-15] have converted the fuzzy matrix into interval matrix by using α-cut.

Dehghan et al. [16] proposed two main ideas based on employing real scenarios and arithmetic operators and proposed a method to find the ϵ-inverse of a fuzzy matrix. The problems of checking invertibility, finding the multiplicative inverse of square fuzzy matrix and its applications have been warm topics for the recent researchers.

In this paper, methods are proposed to find the rank and multiplicative inverse of such fully fuzzy matrices in which all the elements of matrix are fuzzy numbers.

The remaining paper is organized as follows. In Section 2, some basic definitions and arithmetic operations are presented. In Section 3, two methods are proposed to find the rank of a fully fuzzy matrix. In Section 4, a method is proposed to find the multiplicative inverse of a fully fuzzy matrix. In Section 5, numerical examples are solved and the obtained results are discussed in Section 6. In Section 7, conclusions are discussed.

2. Preliminaries

In this section, some necessary backgrounds and notions of fuzzy set theory are presented.

2.1. Basic Definitions

In this section some basic definitions are presented.

Definition 2.1. [17] A fuzzy number $\widetilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x < b \\ 1, & b \le x \le c \\ \dfrac{x-d}{c-d}, & c < x \le d \\ 0, & otherwise \end{cases}$$

Definition 2.2. [17] A trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d)$ is said to be non-negative trapezoidal fuzzy number if and only if $a \ge 0$.

Definition 2.3. [18] A ranking function is a function $\Re: F(\mathbb{R}) \to \mathbb{R}$, where $\mathrm{F(R)}$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\widetilde{\mathrm{A}} = (\mathrm{a, b, c, d})$ be a trapezoidal fuzzy number then $\mathrm{R}(\widetilde{\mathrm{A}}) = \frac{\mathrm{a+b+c+d}}{4}$.

Let $\widetilde{\mathrm{A}} = (\mathrm{a_1, b_1, c_1, d_1})$ and $\widetilde{\mathrm{B}} = (\mathrm{a_2, b_2, c_2, d_2})$ be two trapezoidal fuzzy numbers, then

(i) $\widetilde{\mathrm{A}} \le \widetilde{\mathrm{B}}$ iff $\Re(\tilde{A}) \le \Re(\tilde{B})$

(ii) $\widetilde{\mathrm{A}} \ge \widetilde{\mathrm{B}}$ iff $\mathrm{R}(\widetilde{\mathrm{A}}) \ge \mathrm{R}(\widetilde{\mathrm{B}})$

(iii) $\widetilde{\mathrm{A}} =_{\mathrm{R}} \widetilde{\mathrm{B}}$ iff $\mathrm{R}(\widetilde{\mathrm{A}}) = \mathrm{R}(\widetilde{\mathrm{B}})$

2.2. Arithmetic Operations

In this section, arithmetic operations between two trapezoidal fuzzy numbers, defined on universal set of real numbers R , are presented [19].

Let $\widetilde{\mathrm{A}} = (\mathrm{a, b, c, d})$ and $\widetilde{\mathrm{B}} = (\mathrm{e, f, g, h})$ be two trapezoidal fuzzy numbers then

(i) $\widetilde{\mathrm{A}} \oplus \widetilde{\mathrm{B}} = (\mathrm{a+e, b+f, c+g, d+h})$

(ii) $\tilde{A} \ominus \tilde{B} = (a-h, b-g, c-f, d-e)$

(iii) $\widetilde{A} \otimes \widetilde{B} = (a\frac{(e+f+g+h)}{4}, b\frac{(e+f+g+h)}{4}, c\frac{(e+f+g+h)}{4}, d\frac{(e+f+g+h)}{4})$ $if\ \Re(\tilde{B}) \geq 0$

(iv) $\widetilde{A} \otimes \widetilde{B} = (d\frac{(e+f+g+h)}{4}, c\frac{(e+f+g+h)}{4}, b\frac{(e+f+g+h)}{4}, a\frac{(e+f+g+h)}{4})$ $if\ \Re(\tilde{B}) \leq 0$

(v) $\frac{\widetilde{A}}{\widetilde{B}} = \left(\frac{4a}{e+f+g+h}, \frac{4b}{e+f+g+h}, \frac{4c}{e+f+g+h}, \frac{4d}{e+f+g+h}\right)$ $if\ \Re(\tilde{B}) > 0$

(vi) $\frac{\widetilde{A}}{\widetilde{B}} = \left(\frac{4d}{e+f+g+h}, \frac{4c}{e+f+g+h}, \frac{4b}{e+f+g+h}, \frac{4a}{e+f+g+h}\right)$ $if\ \Re(\tilde{B}) < 0$

3. Proposed Methods to Find the Rank of Fully Fuzzy Matrices

In this section, two new methods are proposed to find the rank of fully fuzzy matrices.

3.1. Proposed Method to Find the Rank of Fully Fuzzy Matrices using Determinants

This method can be used only to find the rank of square fully fuzzy matrix. Let $\widetilde{A}$ be any $m \times m$ fully fuzzy matrix i.e., $\widetilde{A} = [\tilde{a}_{ij}]$, where $i = 1,2,\ldots,m; j = 1,2,\ldots,m$. If $\Re(\tilde{a}_{ij}) = 0\ \forall i = 1,2,\ldots,m; j = 1,2,\ldots,m$ then the matrix is null matrix, otherwise non-null matrix. If $\widetilde{A}$ is null matrix then rank$(\tilde{A}) = 0$. If matrix is non-null then take a variable k and initialize it as $k = 1$. Now use the following steps to find the rank of fully fuzzy matrix $\widetilde{A}$ using determinants:

Step 1 Check the determinant$(\widetilde{A}) =_R \tilde{0}$, or $\neq_R \tilde{0}$, now the following two cases arise:

Case (i) If determinant$(\tilde{A}) \neq_{\Re} \tilde{0}$ then rank$(\widetilde{A}) = m$.

Case (ii) If determinant$(\widetilde{A}) =_R \tilde{0}$ then go to Step 2.

Step 2 Assume $m = m - k$ and check all the minors of order m . Now the following two cases arise:

Case (i) If at least one minor of order m is non-zero, then rank$(\widetilde{A}) = m$.

Case (ii) If rank of all the minors of order m are zero, then increase the value of k by 1 i.e., $k = k + 1$ and repeat the Step 2.

3.2. Proposed Method to Find the Rank of Fully Fuzzy Matrices using Row Echelon Form

This method can be used to find the rank of square and non-square fully fuzzy matrix. Let $\widetilde{A}$ be any fully fuzzy matrix of order $m \times n$ i.e., $\widetilde{A} = [\tilde{a}_{ij}]$, where $i = 1,2,\ldots,m; j = 1,2,\ldots,m$. If $\Re(\tilde{a}_{ij}) = 0\ \forall i = 1,2,\ldots,m; j = 1,2,\ldots,m$ then the matrix is null matrix, otherwise non-null matrix. If $\widetilde{A}$ is null matrix then rank$(\widetilde{A}) = 0$. If matrix is non-null then take two variables j and k and initialize these as $j = 1, k = 2$. Now follow the steps given below to find the rank of fully fuzzy matrix $\widetilde{A}$ using row echelon form:

Step 1 Check the R$(\tilde{a}_{ij})$, now the following two cases arise:

Case (i) If R$(\tilde{a}_{ij}) \neq 0$ then go to Step 2.

Case (ii) If R$(\tilde{a}_{ij}) = 0$ then go to Step 3.

Step 2 Change the rows $R_i\ \ \forall i = k, \ldots m$ by using the following operation:

$R_i \to R_i \ominus \frac{\tilde{a}_{ij}}{\tilde{a}_{jj}} \otimes R_j\ \forall i = k, \ldots m.$

Now change $j = j + 1, k = k + 1$. If $j = m + 1$ then exit, otherwise go to Step 1.

Step 3 If R$(\tilde{a}_{jj}) = 0$ then the following two cases arise:

Case (i) If R$(\tilde{a}_{kj}) \neq 0$ for some $k \in j + 1, \ldots, m$ then interchange the j^{th} row with k^{th} row and go to Step 2.

Case (ii) If R$(\tilde{a}_{kj}) = 0\ \forall\ k \in j + 1, \ldots, m$ then change $j = j + 1$, $k = k + 1$. If $j = m + 1$ then exit, otherwise go to Step 1.

After applying the above method we get a fully fuzzy matrix in row echelon form. Now the rank of fully fuzzy matrix will be equal to the number of non-zero rows in the row echelon matrix on the basis of rank.

4. Proposed Method to Find the Multiplicative Inverse of a Fully Fuzzy Matrix

In this section a new method is proposed to find the multiplicative inverse of a fully fuzzy matrix. Let

$$\tilde{A} = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1m} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mm} \end{pmatrix}$$

be a $m \times m$ fully fuzzy matrix. where, $\tilde{a}_{ij}$ are trapezoidal fuzzy numbers for all $i = 1,2,\ldots,m; j = 1,2,\ldots,m$. Then find the determinant of fully fuzzy matrix by using the arithmetic operations defined in Section 2.2. Let determinant$(\tilde{A})$ be $\tilde{a}$. If $\tilde{a} =_{\Re} \tilde{0}$ then the fully fuzzy matrix is not invertible but if $\tilde{a} \neq_{\Re} \tilde{0}$ then matrix is invertible and use the following steps to find its multiplicative inverse.

Step 1 Find the co-factor of each element $\tilde{a}_{ij}$ of matrix $\tilde{A}$. To find the co-factor of $\tilde{a}_{ij}$ delete the entire i^{th} row and j^{th} column from the matrix $\tilde{A}_{m\times m}$ then we get a matrix $\tilde{B}_{(m-1)\times(m-1)}$ and the co-factor of $\tilde{a}_{ij} = (-1)^{i+j}$determinant$(\tilde{B}) = \tilde{b}_{ij}\ \forall\ i = 1,2,\dots,m; j = 1,2,\dots,m$.

Step 2 Find the co-factor matrix of the fully fuzzy matrix $\tilde{A}$ denoted by co-factor$(\tilde{A})$ i.e.,

$$\text{co-factor}(\tilde{A}) = \begin{pmatrix} \tilde{b}_{11} & \tilde{b}_{12} & \cdots & \tilde{b}_{1m} \\ \tilde{b}_{21} & \tilde{b}_{22} & \cdots & \tilde{b}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{b}_{m1} & \tilde{b}_{m2} & \cdots & \tilde{b}_{mm} \end{pmatrix}$$

Step 3 Find the adjoint matrix of fully fuzzy matrix $\tilde{A}$ denoted by adjoint$(\tilde{A})$. Using adjoint$(\tilde{A})$ = transpose of (co-factor$(\tilde{A})$) = (co-factor$(\tilde{A}))^T$ we get

$$\text{adjoint}(\tilde{A}) = \begin{pmatrix} \tilde{b}_{11} & \tilde{b}_{21} & \cdots & \tilde{b}_{m1} \\ \tilde{b}_{12} & \tilde{b}_{22} & \cdots & \tilde{b}_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{b}_{1m} & \tilde{b}_{2m} & \cdots & \tilde{b}_{mm} \end{pmatrix}$$

Step 4 We know that $\tilde{A}^{-1} = \frac{adjoint(\tilde{A})}{determinant(\tilde{A})}$ so the multiplicative inverse of $\tilde{A}$ will be equal to

$$\tilde{A}^{-1} = \begin{pmatrix} \frac{\tilde{b}_{11}}{\tilde{a}} & \frac{\tilde{b}_{21}}{\tilde{a}} & \cdots & \frac{\tilde{b}_{m1}}{\tilde{a}} \\ \frac{\tilde{b}_{12}}{\tilde{a}} & \frac{\tilde{b}_{22}}{\tilde{a}} & \cdots & \frac{\tilde{b}_{m2}}{\tilde{a}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\tilde{b}_{1m}}{\tilde{a}} & \frac{\tilde{b}_{2m}}{\tilde{a}} & \cdots & \frac{\tilde{b}_{mm}}{\tilde{a}} \end{pmatrix}$$

5. Numerical Example

In this section, to illustrate the methods, proposed in Section 3 and 4, numerical examples are solved.

Example 5.1 Find the rank of following fully fuzzy matrix:

$$\begin{pmatrix} (-4,-3,2,5) & (1,2,3,4) & (2,3,5,6) \\ (2,3,4,5) & (-3,-2,-1,2) & (2,4,6,8) \\ (-2,-1,1,2) & (-3,-2,1,4) & (-2,-1,0,3) \end{pmatrix}$$

Solution: The rank of the chosen fully fuzzy matrix by using the method, proposed in Section 3.1, can be obtained as follows.

As discussed in Step 1 of the proposed method we check that determinant$(\tilde{A}) =_{\Re} \tilde{0}$ or $\neq_{\Re} \tilde{0}$.

determinant$(\tilde{A})$ =
$(-4,-3,2,5) \otimes [(-3,-2,-1,2) \otimes (-2,-1,0,3) \ominus (-3,-2,1,4) \otimes (2,4,6,8)] \ominus (1,2,3,4) \otimes [(2,3,4,5) \otimes (-2,-1,0,3) \ominus (-2,-1,1,2) \otimes (2,4,6,8)] \oplus (2,3,5,6) \otimes [(2,3,4,5) \otimes (-3,-2,1,4) \ominus (-2,-1,1,2) \otimes (-3,-2,-1,2)]$

After simplifying we get determinant$(\tilde{A}) = (0,0,0,0) =_{\Re} \tilde{0}$.

Since, determinant of the fully fuzzy matrix is equal to zero, so using Step 2 of the proposed method check all the minors of order 2. Take the first minor $\tilde{A}_1$ as follows:

$$\tilde{A}_1 = \begin{pmatrix} (-4,-3,2,5) & (1,2,3,4) \\ (2,3,4,5) & (-3,-2,-1,2) \end{pmatrix}$$

determinant$(\tilde{A}_1) = (\frac{-35}{2}, -12, \frac{-9}{2}, -1) \neq_{\Re} \tilde{0}$.

According to the Step 2 of the proposed method, rank$(\tilde{A}) = 2$.

Similarly, the rank of matrix can be obtained by using the row echelon form, proposed in Section 3.2.

Example 5.2 Find the multiplicative inverse of the following fully fuzzy matrix:

$$\tilde{A} = \begin{pmatrix} (0,1,3,4) & (2,4,6,7) \\ (-2,0,2,4) & (3,4,6,7) \end{pmatrix}$$

Solution: Using the proposed method, firstly we check the matrix $\tilde{A}$ is invertible or not.

determinant$(\tilde{A}) = (-19, \frac{-9}{2}, 15, \frac{59}{2}) \neq_{\Re} \tilde{0}$

determinant$(\tilde{A}) \neq_{\Re} \tilde{0}$ so the matrix is invertible.

Using Step 1 of the proposed method the co-factor of each element of the matrix are:

co-factor of $(0,1,3,4) = (-1)^{1+1}(3,4,6,7) = (3,4,6,7)$

co-factor of $(2,4,6,7) = (-1)^{1+2}(-2,0,2,4) = (-4,-2,0,2)$

co-factor of $(-2,0,2,4) = (-1)^{2+1}(2,4,6,7) = (-7,-6,-4,-2)$

co-factor of $(3,4,6,7) = (-1)^{2+2}(0,1,3,4) = (0,1,3,4)$

Using Step 2 of the proposed method the co-factor$(\tilde{A})$ is

$$\text{co-factor}(\tilde{A}) = \begin{pmatrix} (3,4,6,7) & (-4,-2,0,2) \\ (-7,-6,-4,-2) & (0,1,3,4) \end{pmatrix}$$

Using Step 3 of the proposed method adjoint$(\tilde{A})$ is

$$\text{adjoint}(\tilde{A}) = \begin{pmatrix} (3,4,6,7) & (-4,-2,0,2) \\ (-7,-6,-4,-2) & (0,1,3,4) \end{pmatrix}^T = \begin{pmatrix} (3,4,6,7) & (-7,-6,-4,-2) \\ (-4,-2,0,2) & (0,1,3,4) \end{pmatrix}$$

Using Step 4 of the proposed method multiplicative inverse of the given matrix $\tilde{A}$ is given by $\tilde{A}^{-1} = \frac{adjoint(\tilde{A})}{determinant(\tilde{A})}$ i.e.,

$$\tilde{A}^{-1} = \begin{pmatrix} \frac{(3,4,6,7)}{(-19,\frac{-9}{2},15,\frac{59}{2})} & \frac{(-7,-6,-4,-2)}{(-19,\frac{-9}{2},15,\frac{59}{2})} \\ \frac{(-4,-2,0,2)}{(-19,\frac{-9}{2},15,\frac{59}{2})} & \frac{(0,1,3,4)}{(-19,\frac{-9}{2},15,\frac{59}{2})} \end{pmatrix}$$

$$= \begin{pmatrix} (\frac{12}{21},\frac{16}{21},\frac{24}{21},\frac{28}{21}) & (\frac{-28}{21},\frac{-24}{21},\frac{-16}{21},\frac{-8}{21}) \\ (\frac{-16}{21},\frac{-8}{21},0,\frac{8}{21}) & (0,\frac{4}{21},\frac{12}{21},\frac{16}{21}) \end{pmatrix}$$

Verification : The multiplicative inverse of the chosen fully fuzzy matrix can be verified as follows:

$\tilde{A} \otimes \tilde{A}^{-1} =$

$$\begin{pmatrix} (\frac{-28}{21},\frac{-4}{21},\frac{44}{21},\frac{72}{21}) & (\frac{-60}{21},\frac{-25}{21},\frac{29}{21},\frac{56}{21}) \\ (\frac{-68}{21},\frac{-24}{21},\frac{24}{21},\frac{68}{21}) & (\frac{-52}{21},\frac{-6}{21},\frac{48}{21},\frac{94}{21}) \end{pmatrix} =_{\Re} \tilde{I}$$

$\tilde{A}^{-1} \otimes \tilde{A} =$

$$\begin{pmatrix} (\frac{-4}{21},\frac{8}{21},\frac{32}{21},\frac{48}{21}) & (\frac{-83}{21},\frac{-44}{21},\frac{34}{21},\frac{93}{21}) \\ (\frac{-32}{21},\frac{-12}{21},\frac{12}{21},\frac{32}{21}) & (\frac{-76}{21},\frac{-18}{21},\frac{60}{21},\frac{118}{21}) \end{pmatrix} =_{\Re} \tilde{I}$$

Example 5.3. Find the multiplicative inverse of the following fully fuzzy matrix by using the proposed method:

$$\tilde{A} = \begin{pmatrix} (0,1,2,3) & (2,4,6,8) & (-2,0,2,4) \\ (3,4,6,7) & (0,1,3,4) & (-4,2,4,6) \\ (2,4,4,6) & (1,2,3,4) & (0,2,4,6) \end{pmatrix}$$

Solution: Using the proposed method, firstly we check the matrix $\tilde{A}$ is invertible or not.

determinant$(\tilde{A}) = (-65,-41,-17,7) \neq_{\Re} \tilde{0}$ so the matrix is invertible.

Using Step 1 of the proposed method the co-factor of each element of the matrix are:

co-factor of $(0,1,2,3) = (-8,-3,5,10)$

co-factor of $(2,4,6,8) = (-17,-10,-4,3)$

co-factor of $(-2,0,2,4) = (\frac{-9}{2},2,7,\frac{27}{2})$

co-factor of $(3,4,6,7) = (-23,-16,-9,-2)$

co-factor of $(0,1,3,4) = (-6,-1,2,7)$

co-factor of $(-4,2,4,6) = (\frac{5}{2},15,\frac{35}{2},30)$

co-factor of $(2,4,4,6) = (0,5,11,16)$

co-factor of $(1,2,3,4) = (-3,0,4,7)$

co-factor of $(0,2,4,6) = (-35,-28,-16,-9)$

Using Step 3 of the proposed method co-factor$(\tilde{A})$ is

$$\begin{pmatrix} (-8,-3,5,10) & (-17,-10,-4,3) & (\frac{-9}{2},2,7,\frac{27}{2}) \\ (-23,-16,-9,-2) & (-6,-1,2,7) & (\frac{5}{2},15,\frac{35}{2},30) \\ (0,5,11,16) & (-3,0,4,7) & (-35,-28,-16,-9) \end{pmatrix}$$

Using Step 4 of the proposed method adjoint$(\tilde{A})$ is

$$\begin{pmatrix} (-8,-3,5,10) & (-23,-16,-9,-2) & (0,5,11,16) \\ (-17,-10,-4,3) & (-6,-1,2,7) & (-3,0,4,7) \\ (\frac{-9}{2},2,7,\frac{27}{2}) & (\frac{5}{2},15,\frac{35}{2},30) & (-35,-28,-16,-9) \end{pmatrix}$$

Using Step 5 of the proposed method multiplicative inverse of the matrix $\tilde{A}$ is $\tilde{A}^{-1} = \frac{adjoint(\tilde{A})}{determinant(\tilde{A})}$ i.e.,

$$\begin{pmatrix} \left(\frac{-10}{29},\frac{-5}{29},\frac{3}{29},\frac{8}{29}\right) & \left(\frac{2}{29},\frac{9}{29},\frac{16}{29},\frac{23}{29}\right) & \left(\frac{-16}{29},\frac{-11}{29},\frac{-5}{29},0\right) \\ \left(\frac{-3}{29},\frac{4}{29},\frac{10}{29},\frac{17}{29}\right) & \left(\frac{-7}{29},\frac{-2}{29},\frac{1}{29},\frac{6}{29}\right) & \left(\frac{-7}{29},\frac{-4}{29},0,\frac{3}{29}\right) \\ \left(\frac{-27}{58},\frac{-7}{29},\frac{-2}{29},\frac{9}{58}\right) & \left(\frac{-30}{29},\frac{-35}{58},\frac{-15}{29},\frac{-5}{58}\right) & \left(\frac{9}{29},\frac{16}{29},\frac{28}{29},\frac{35}{29}\right) \end{pmatrix}$$

6. Results and Discussion

In this section the results of the chosen problems, obtained by using the proposed methods, are discussed.

(i) The rank of the fully fuzzy matrix, obtained by using both the proposed methods, determinant method as well as row echelon form, is same.

(ii) The multiplicative inverse of the fully fuzzy matrix, obtained by using the proposed method, is a fully fuzzy matrix as well.

(iii) The multiplicative inverse of the fully fuzzy matrix, obtained by using the proposed method, is unique on the basis of rank.

Remark 1. In the proposed method we can also use the following fuzzy product:

$$A \otimes \widetilde{B} = (\frac{(a+b+c+d)}{4}e, \frac{(a+b+c+d)}{4}f, \frac{(a+b+c+d)g}{4}g, \frac{(a+b+c+d)}{4}h) \; if \; \Re(\tilde{A}) \geq 0$$

$$A \otimes \widetilde{B} = (\frac{(a+b+c+d)}{4}h, \frac{(a+b+c+d)}{4}g, \frac{(a+b+c+d)g}{4}f, \frac{(a+b+c+d)}{4}e) \; if \; \Re(\tilde{A}) \leq 0$$

The multiplicative inverse of a fully fuzzy matrix can be obtained by using any of the above two arithmetic and it will be unique on the basis of rank.

7. **Conclusion**

This paper is an attempt to deal with the problems of finding the rank and multiplicative inverse of a fully fuzzy matrix. Some methods are proposed to find the rank and multiplicative inverse of fully fuzzy matrix which are useful to solve the real life problems. The inverse matrix, obtained by using the proposed method, is unique on the basis of rank.

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Jagdeep Kaur, School of Mathematics and Computer Applications, Thapar University,
Patiala-147 004, India, sidhu.deepi87@gmail.com