CLASS OF ESTIMATORS FOR ESTIMATING RATIO OF TWO POPULATION MEANS USING TWO AUXILIARY VARIABLES UNDER MEASUREMENT ERRORS

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Abstract: In this paper, we have proposed a generalized (on the lines of Abu Dayyeh (2003)) estimator for the estimation ratio of two population means in the presence of measurement errors using two auxiliary variables. The expression of bias and mean square error are derived under measurement errors. Further in order to control the bias an unbiased estimator using the jack-knife technique is proposed in presence of measurement errors. The concluding remarks are given showing some important estimators are the special cases of the proposed class and can be preferred in the sense of unbiasedness.

Keywords: Auxiliary information, Jack-knife, Bias, Mean square error, Ratio type.

Introduction: The properties of estimators based on data usually a priori assume that observations are correct measurements on characteristics under investigation. Unfortunately, this ideal situation does not met in practice for various innumerable. Sometimes the measurement errors are negligible then the statistical inferences based on observed data may remain valid to a certain extent. But when the measurement errors are appreciable, the inferences may simply be highly erroneous and may also lead to unexpected, undesirable and unfortunate consequences (see Shalabh (1997)). Various authors like Allen etal, Manisha and Singh (2001, 2002), Srivastava and Shalabh (2001), Singh and Karpe (2008, 2009), Kumar etal, (2011a, 2011b) have studied some aspects of the problem especially estimation of population mean μ_Y of the study variable y using auxiliary information in the presence of measurements errors.

We all know that in various practical situations auxiliary information is helpful to improve the efficiency of an estimator of unknown population parameter of interest in sampling. The ratio, product and regression estimators are widely utilized in many surveys sampling situation, when auxiliary information is used at the estimation stage. In many practical situations the estimation of the ratio (or product) of two population means may be of considerable interest, e.g. the crop production per hectare for different crops, ratio of male to female in working force, ratio of income to expenditure, the ratio of the liquid assets to total assets, profitability rate (profit/ investment) etc. one may be interested in estimating the total value of sales prom the price and the volume of sales. The estimation of population parameters like population ratio or population product has rarely been studied under the measurement error models in survey sampling. Though, it has been studied by Singh and Karpe (2009a, 2009b) and few others.

Suggested estimators for ratio of two population means: For a simple random sample of size n, let $\left(x_1, x_2\right)$ be the pair of values instead of the true values, $\left(X_1, X_2\right)$ and $\hat{R}$ be the estimated mean of the ratio of $\left(Y_1,\ Y_2\right)$ of the characteristics $\left(X_1, X_2\right)$ and $\left(Y_1,\ Y_2\right)$ respectively.

Let the observational or measurement error be

$u_{1j} = y_{1j} - Y_{1j},\ u_{2j} = y_{2j} - Y_{1j},\ v_{1,i} = x_{1,i} - X_{1,i},$

$v_{2,i} = x_{2,i} - X_{2,i},$

wherei=j=1,2,...n.

which are stochastic in nature and are uncorrelated with mean zero and variances $\sigma^2_{u_1}$, $\sigma^2_{u_2}$, $\sigma^2_{v_1}$ and $\sigma^2_{v_1}$ respectively. Further, let population means of $\left(X_1,\ Y_1\right)$, $\left(X_2,\ Y_2\right)$ be (μ_{X_1}, μ_{Y_1}) and (μ_{X_2}, μ_{Y_2}) population variance of $\left(X_1,\ Y_1\right)$, $\left(X_2,\ Y_2\right)$ be $(\sigma^2_{X_1}, \sigma^2_{Y_1})$, $(\sigma^2_{X_2}, \sigma^2_{Y_2})$ respectively. $\sigma_{X_1Y_1}$, $\sigma_{X_2Y_1}$, $\sigma_{X_1Y_2}$ aand $\sigma_{X_2Y_2}$, $\sigma_{X_1X_2}$, $\sigma_{Y_1Y_2}$ be the population correlation coefficient between (X_1, Y_1), (X_2, Y_1), (X_1, Y_2) and (X_2, Y_2), (X_1, X_2), (Y_1, Y_2) respectively.

The generalized estimator of ratio of population means using $R=\bar{Y}_1/\bar{Y}_2$ based on Abu Dayyeh et al (2003) using two study variables (Y_1, Y_2).

$$\hat{R}_a=\hat{R}\left(\frac{\bar{x}_1}{\mu_1}\right)^{a_1}\left(\frac{\bar{x}_2}{\mu_2}\right)^{a_2} \qquad (1)$$

we have

We have

$$\bar{y}_1=\frac{1}{n}\left[\sum_{j=1}^{n}Y_{1,j}\right]$$

$$=\frac{1}{n}\left[\sum_{j=1}^{n}u_{1,j}+Y_{1,j}-\mu_{Y_1}+\mu_{Y_1}\right]$$

$$=\frac{1}{\sqrt{n}}\left[W_{u_1}+W_{Y_1}\right]+\mu_{Y_1}$$

Similarly,

$$\bar{y}_2=\frac{1}{\sqrt{n}}\left[W_{u_2}+W_{Y_2}\right]+\mu_{Y_2}$$

$$\bar{x}_1=\frac{1}{\sqrt{n}}\left[W_{v_1}+W_{x_1}\right]+\mu_1$$

$$\bar{x}_2=\frac{1}{\sqrt{n}}\left[W_{v_2}+W_{x_2}\right]+\mu_2$$

Putting these values in (1) and expanding using Maclaurin's series expansion and ignore the terms having powers greater than two. We have

$$\hat{R}_a-\hat{R}=\hat{R}\left[\frac{a_1\left(W_{v_1}+W_{x_1}\right)}{n^{1/2}\mu_1}+\frac{a_2\left(W_{v_2}+W_{x_2}\right)}{n^{1/2}\mu_2}\right.$$
$$+\frac{a_1(a_1-1)\left(W_{v_1}+W_{x_1}\right)^2}{2n\mu_1^2}+\frac{a_2(a_2-1)\left(W_{v_2}+W_{x_2}\right)^2}{2!n\mu_2^2}$$
$$+\frac{a_1\left(W_{v_1}+W_{x_1}\right)}{n^{1/2}\mu_1}\frac{a_2\left(W_{v_2}+W_{x_2}\right)}{n^{1/2}\mu_2}+\frac{\left(W_{u_1}+W_{y_1}\right)}{n^{1/2}\mu_{Y_1}}$$
$$-\frac{\left(W_{u_2}+W_{y_2}\right)}{n^{1/2}\mu_{Y_2}}+\frac{\left(W_{u_2}+W_{y_2}\right)^2}{2!n\mu_{Y_2}^2}$$
$$-\frac{\left(W_{u_1}+W_{y_1}\right)}{n^{1/2}\mu_{Y_1}}\frac{\left(W_{u_2}+W_{y_2}\right)}{n^{1/2}\mu_{Y_2}}$$
$$+\frac{\left(W_{u_1}+W_{y_1}\right)}{n^{1/2}\mu_{Y_1}}\frac{a_1\left(W_{v_1}+W_{x_1}\right)}{n^{1/2}\mu_1}$$
$$+\frac{\left(W_{u_1}+W_{y_1}\right)}{n^{1/2}\mu_{Y_1}}\frac{a_2\left(W_{v_2}+W_{x_2}\right)}{n^{1/2}\mu_2}$$
$$-\left\{\frac{\left(W_{u_2}+W_{y_2}\right)}{n^{1/2}\mu_{Y_2}}\frac{a_1\left(W_{v_1}+W_{x_1}\right)}{n^{1/2}\mu_1}\right.$$
$$\left.\left.+\frac{\left(W_{u_2}+W_{y_2}\right)}{n^{1/2}\mu_{Y_2}}\frac{a_2\left(W_{v_2}+W_{x_2}\right)}{n^{1/2}\mu_2}\right\}\right] \qquad (2)$$

Bias and Mean square error of the proposed class of estimators are defined as,

On taking expectation on both side of (2)

$$\text{Bias}\left(\hat{R}_a\right)=\frac{R}{n}\left[\frac{1}{2\mu_{Y_2}^2}\sigma_{Y_2}^2+\frac{a_1(a_1-1)}{2\mu_1^2}\sigma_{X_1}^2\right.$$
$$+\frac{a_2(a_2-1)}{2\mu_2^2}\sigma_{X_2}^2+\frac{a_1a_2}{\mu_1\mu_2}\sigma_{X_1X_2}$$
$$-\frac{1}{\mu_{Y_1}\mu_{Y_2}}\sigma_{Y_1Y_2}+\left\{\frac{a_1}{\mu_{Y_1}\mu_1}\sigma_{X_1Y_1}\right.$$
$$\left.+\frac{a_2}{\mu_{Y_1}\mu_2}\sigma_{X_2Y_1}\right\}-\left\{\frac{a_1}{\mu_{Y_2}\mu_1}\sigma_{X_1Y_2}\right.$$
$$\left.\left.+\frac{a_2}{\mu_{Y_1}\mu_2}\sigma_{X_2Y_2}\right\}\right]+\frac{\hat{R}}{n}\left[\frac{\sigma_{u_2}^2}{2\mu_{Y_2}^2}\right.$$
$$\left.+\frac{a_1(a_1-1)}{2\mu_1^2}\sigma_{v_1}^2+\frac{a_2(a_2-1)}{2\mu_2^2}\sigma_{v_2}^2\right]$$
$$=B_1+B_2 \qquad (3)$$

where B_1 *is the amount of bias without measurement error and* B_2 *is the amount of biasness in the proposed class of estimator.*

Now squaring (2) on both side and then taking expectation we have

$$\text{MSE}\left(\hat{R}_a\right)=\frac{R^2}{n}\left[\left\{\frac{1}{\mu_{Y_1}^2}\sigma_{Y_1}^2+\frac{1}{\mu_{Y_2}^2}\sigma_{Y_2}^2\right.\right.$$
$$\left.-\frac{2}{\mu_{Y_1}\mu_{Y_2}}\sigma_{Y_1Y_2}\right\}+\left\{\frac{a_1^2}{\mu_1^2}\sigma_{X_1}^2\right.$$

$$+\frac{a_2^2}{\mu_2^2}\sigma_{X_2}^2+\frac{2a_1a_2}{\mu_1\mu_2}\sigma_{X_1X_2}\Bigg\}$$

$$+\frac{2}{\mu_{Y_1}}\left\{\frac{a_1}{\mu_1}\sigma_{X_1Y_1}+\frac{a_2}{\mu_2}\sigma_{X_2Y_1}\right\}$$

$$-\frac{2}{\mu_{Y_2}}\left\{\frac{a_1}{\mu_1}\sigma_{X_1Y_2}+\frac{a_2}{\mu_2}\sigma_{X_2Y_2}\right\}\Bigg]$$

$$+\frac{R^2}{n}\left[\frac{1}{\mu_{Y_1}^2}\sigma_{u_1}^2+\frac{1}{\mu_{Y_2}^2}\sigma_{u_2}^2\right.$$

$$\left.+\frac{a_1^2}{\mu_1^2}\sigma_{v_1}^2+\frac{a_2^2}{\mu_2^2}\sigma_{v_2}^2\right]$$

$$=M_1+M_2 \qquad (4)$$

where M_1 is the MSE without measurement error and M_2 is the term of measurement error in the proposed class of estimator.

It is observed that in consider class of estimator was already biased (see Abu Dayyeh etal.) and its bias has been aggravated by quantity B_2 consisting of the measurement error. But this bias can be removed by using jack-knife technique and so the next session is devoted to the same problem.

Jack-knife estimators : In order to introduce the jack-knife estimator we consider the following set of simple random sample of size $n=2m$ drawn from the finite population of size N by SRSWOR is split randomly into two sub-samples each of size m. let $(\bar{x}_{1,n},\bar{x}_{2,n})$ be the sample mean of values (X_1,X_2) and $\hat{R}_n$ be the estimated mean of ratio of study variable (Y_1, Y_2) respectively for the entire sample of size n=2m and $(\bar{x}_{1,m}^{(1)},\bar{x}_{2,m}^{(1)})$ $(\bar{x}_{1,m}^{(2)},\bar{x}_{2,m}^{(2)})$ be the sample means respectively of (X_1,X_2) for the first and second sub-samples and $\hat{R}_m^{(1)}$, $\hat{R}_m^{(2)}$ be estimated population means of (Y_1, Y_2) respectively for two randomly split sub-sample of size m.

Let $\hat{R}_j^{(3)}$ is the generalized estimator for the complete sample for the entire sample of size $n=2m$ and $\hat{R}_j^{(1)}$ & $\hat{R}_j^{(2)}$ be the generalized estimators for the two randomly split sub-samples of size m each based on the considered estimators.

$$\hat{R}_j^{(3)}=\hat{R}_{2n}\left(\frac{\bar{x}_{1,n}}{\mu_1}\right)^{a_1}\left(\frac{\bar{x}_{2,n}}{\mu_2}\right)^{a_2} \qquad (5)$$

$$\hat{R}_j^{(1)}=\hat{R}_m^{(1)}\left(\frac{\bar{x}_{1,m}^{(1)}}{\mu_1}\right)^{a_1}\left(\frac{\bar{x}_{2,m}^{(1)}}{\mu_2}\right)^{a_2} \qquad (6)$$

$$\hat{R}_j^{(2)}=\hat{R}_m^{(2)}\left(\frac{\bar{x}_{1,m}^{(2)}}{\mu_1}\right)^{a_1}\left(\frac{\bar{x}_{2,m}^{(2)}}{\mu_2}\right)^{a_2} \qquad (7)$$

Proceeding on the lines of Sukhatme and Sukhatme (Chapter IV, page 162), for N to be large for simplicity, the Jack-knifed generalized estimator $\hat{R}_{aj}$ is given by,

$$\hat{R}_{aj}=2\hat{R}_j^{(3)}-\frac{1}{2}\left(\hat{R}_j^{(1)}+\hat{R}_j^{(2)}\right) \qquad (8)$$

Substituting the values of error terms in $\hat{R}_j^{(3)}$, $\hat{R}_j^{(1)}$ and $\hat{R}_j^{(2)}$ further solve (8) using (6), (7) and (8) for bias and mean square error we have

$$E\left(\hat{R}_{Rj}-\hat{R}\right)=2R\left[\frac{a_1(a_1-1)\left(\sigma_{v_1}^2+\sigma_{X_1}^2\right)}{2!n\mu_1^2}\right.$$

$$+\frac{a_2(a_2-1)\left(\sigma_{v_2}^2+\sigma_{X_2}^2\right)}{2!n\mu_2^2}$$

$$+\frac{a_1a_2\sigma_{x_1x_2}}{m\mu_1\mu_2}+\frac{\left(\sigma_{u_2}^2+\sigma_{Y_2}^2\right)}{2!n\mu_{Y_2}^2}$$

$$-\frac{\sigma_{Y_1Y_2}}{n\mu_{Y_1}\mu_{Y_2}}+\frac{a_1\sigma_{X_1Y_1}}{n\mu_{Y_1}\mu_1}+\frac{a_2\sigma_{X_2Y_1}}{n\mu_{Y_1}\mu_2}$$

$$\left.-\left\{\frac{a_1\sigma_{X_1Y_2}}{n\mu_{Y_2}\mu_1}+\frac{a_2\sigma_{X_2Y_2}}{n\mu_{Y_2}\mu_2}\right\}\right]$$

$$-\frac{1}{2}\left[R\left\{\frac{a_1(a_1-1)\left(\sigma_{v_1}^2+\sigma_{X_1}^2\right)}{2!m\mu_1^2}\right.\right.$$

$$+\frac{a_2(a_2-1)\left(\sigma_{v_2}^2+\sigma_{X_2}^2\right)}{2!m\mu_2^2}$$

$$+\frac{a_1a_2\sigma_{X_1X_2}}{m\mu_1\mu_2}+\frac{\left(\sigma_{u_2}^2+\sigma_{Y_2}^2\right)}{2!m\mu_{Y_2}^2}$$

$$-\frac{\sigma_{Y_1Y_2}}{m\mu_{Y_1}\mu_{Y_2}}+\frac{a_1\sigma_{X_1Y_1}}{m\mu_{Y_1}\mu_1}+\frac{a_2\sigma_{X_2Y_1}}{m\mu_{Y_1}\mu_2}-\left(\frac{a_1\sigma_{X_1Y_2}}{m\mu_{Y_2}\mu_1}\right.$$

$$+\frac{a_2\sigma_{X_2Y_2}}{m\mu_{Y_2}\mu_2}\Bigg)\Bigg\}+R\Bigg\{\frac{a_1(a_1-1)\left(\sigma^2_{v_1}+\sigma^2_{X_1}\right)}{2!m\mu_1^2}$$

$$+\frac{a_2(a_2-1)\left(\sigma^2_{v_2}+\sigma^2_{X_2}\right)}{2!m\mu_2^2}+\frac{a_1a_2\sigma_{x_1x_2}}{m\mu_1\mu_2}+\frac{\left(\sigma^2_{u_2}+\sigma^2_{Y_2}\right)}{2!m\mu^2_{Y_2}}$$

$$-\frac{\sigma_{Y_1Y_2}}{m\mu_{Y_1}\mu_{Y_2}}+\frac{a_1\sigma_{X_1Y_1}}{m\mu_{Y_1}\mu_1}$$

$$+\frac{a_2\sigma_{X_2Y_1}}{m\mu_{Y_1}\mu_2}-\left(\frac{a_1\sigma_{X_1Y_2}}{m\mu_{Y_2}\mu_1}+\frac{a_2\sigma_{X_2Y_2}}{m\mu_{Y_2}\mu_2}\right)\Bigg\}\Bigg]$$

$$\text{Bias}(\hat{R}_{aj})=0 \tag{9}$$

$$\text{MSE}\left(\hat{R}_{Rj}\right)=\frac{R^2}{n}\Bigg[\Bigg\{\frac{1}{\mu^2_{Y_1}}\sigma^2_{Y_1}+\frac{1}{\mu^2_{Y_2}}\sigma^2_{Y_2}$$

$$-\frac{2}{\mu_{Y_1}\mu_{Y_2}}\sigma_{Y_1Y_2}\Bigg\}+\Bigg\{\frac{a_1^2}{\mu_1^2}\sigma^2_{X_1}$$

$$+\frac{a_2^2}{\mu_2^2}\sigma^2_{X_2}+\frac{2a_1a_2}{\mu_1\mu_2}\sigma_{X_1X_2}\Bigg\}$$

$$+\frac{2}{\mu_{Y_1}}\left\{\frac{a_1}{\mu_1}\sigma_{X_1Y_1}+\frac{a_2}{\mu_2}\sigma_{X_2Y_1}\right\}$$

$$-\frac{2}{\mu_{Y_2}}\left\{\frac{a_1}{\mu_1}\sigma_{X_1Y_2}+\frac{a_2}{\mu_2}\sigma_{X_2Y_2}\right\}\Bigg]$$

$$+\frac{R^2}{n}\left[\frac{1}{\mu^2_{Y_1}}\sigma^2_{u_1}+\frac{1}{\mu^2_{Y_2}}\sigma^2_{u_2}+\frac{a_1^2}{\mu_1^2}\sigma^2_{v_1}+\frac{a_2^2}{\mu_2^2}\sigma^2_{v_2}\right]$$

$$=M_1+M_2 \tag{10}$$

Concluding remarks: From (3) we can easily see that $\hat{R}_a$ is a biased estimator and its bias under measurement error is accentuated by B_2, is given by

$$\text{Bias}\left(\hat{R}_a\right)=\frac{R}{n}\Bigg[\frac{1}{2\mu^2_{Y_2}}\sigma^2_{Y_2}+\frac{a_1(a_1-1)}{2\mu_1^2}\sigma^2_{X_1}$$

$$+\frac{a_2(a_2-1)}{2\mu_2^2}\sigma^2_{X_2}+\frac{a_1a_2}{\mu_1\mu_2}\sigma_{X_1X_2}$$

$$-\frac{1}{\mu_{Y_1}\mu_{Y_2}}\sigma_{Y_1Y_2}+\Bigg\{\frac{a_1}{\mu_{Y_1}\mu_1}\sigma_{X_1Y_1}$$

$$+\frac{a_2}{\mu_{Y_1}\mu_2}\sigma_{X_2Y_1}\Bigg\}-\Bigg\{\frac{a_1}{\mu_{Y_2}\mu_1}\sigma_{X_1Y_2}$$

$$+\frac{a_2}{\mu_{Y_1}\mu_2}\sigma_{X_2Y_2}\Bigg\}\Bigg]+\frac{\hat{R}}{n}\Bigg[\frac{\sigma^2_{u_2}}{2\mu^2_{Y_2}}$$

$$+\frac{a_1(a_1-1)}{2\mu_1^2}\sigma^2_{v_1}+\frac{a_2(a_2-1)}{2\mu_2^2}\sigma^2_{v_2}\Bigg]$$

$$=B_1+B_2 \tag{11}$$

Also the mean square error of $\hat{R}_a$ under measurement error is accentuated by M_2, is given by, which depends on the measurement error variances.

$$\text{MSE}\left(\hat{R}_a\right)=\frac{R^2}{n}\Bigg[\Bigg\{\frac{1}{\mu^2_{Y_1}}\sigma^2_{Y_1}+\frac{1}{\mu^2_{Y_2}}\sigma^2_{Y_2}$$

$$-\frac{2}{\mu_{Y_1}\mu_{Y_2}}\sigma_{Y_1Y_2}\Bigg\}+\Bigg\{\frac{a_1^2}{\mu_1^2}\sigma^2_{X_1}$$

$$+\frac{a_2^2}{\mu_2^2}\sigma^2_{X_2}+\frac{2a_1a_2}{\mu_1\mu_2}\sigma_{X_1X_2}\Bigg\}$$

$$+\frac{2}{\mu_{Y_1}}\left\{\frac{a_1}{\mu_1}\sigma_{X_1Y_1}+\frac{a_2}{\mu_2}\sigma_{X_2Y_1}\right\}$$

$$-\frac{2}{\mu_{Y_2}}\left\{\frac{a_1}{\mu_1}\sigma_{X_1Y_2}+\frac{a_2}{\mu_2}\sigma_{X_2Y_2}\right\}\Bigg]$$

$$+\frac{R^2}{n}\Bigg[\frac{1}{\mu^2_{Y_1}}\sigma^2_{u_1}+\frac{1}{\mu^2_{Y_2}}\sigma^2_{u_2}$$

$$+\frac{a_1^2}{\mu_1^2}\sigma^2_{v_1}+\frac{a_2^2}{\mu_2^2}\sigma^2_{v_2}\Bigg]$$

$$=M_1+M_2 \tag{12}$$

In order to circumvent the problem of bias, the technique of jack-knife is employed and the bias and mean square error of jack-knife estimator are found to be

$$\text{bias}(\hat{R}_{aj})=0 \tag{13}$$

$$\text{MSE}\left(\hat{R}_{Rj}\right)=\frac{R^2}{n}\Bigg[\Bigg\{\frac{1}{\mu^2_{Y_1}}\sigma^2_{Y_1}+\frac{1}{\mu^2_{Y_2}}\sigma^2_{Y_2}$$

$$-\frac{2}{\mu_{Y_1}\mu_{Y_2}}\sigma_{Y_1Y_2}\Bigg\}+\Bigg\{\frac{a_1^2}{\mu_1^2}\sigma^2_{X_1}$$

$$+\frac{a_2^2}{\mu_2^2}\sigma^2_{X_2}+\frac{2a_1a_2}{\mu_1\mu_2}\sigma_{X_1X_2}\Bigg\}$$

$$+\frac{2}{\mu_{Y_1}}\left\{\frac{a_1}{\mu_1}\sigma_{X_1Y_1}+\frac{a_2}{\mu_2}\sigma_{X_2Y_1}\right\}$$

$$\left.-\frac{2}{\mu_{Y_2}}\left\{\frac{a_1}{\mu_1}\sigma_{X_1Y_2}+\frac{a_2}{\mu_2}\sigma_{X_2Y_2}\right\}\right]$$

$$+\frac{R^2}{n}\left[\frac{1}{\mu_{Y_1}^2}\sigma_{u_1}^2+\frac{1}{\mu_{Y_2}^2}\sigma_{u_2}^2+\frac{a_1^2}{\mu_1^2}\sigma_{v_1}^2+\frac{a_2^2}{\mu_2^2}\sigma_{v_2}^2\right]$$

$$=M_1+M_2 \qquad (14)$$

which is same as that of (12). Hence in the sense of bias $\hat{R}_{\mathbf{Rj}}$ is to be preferred of the estimators $\hat{R}_{\mathbf{a}}$ and $\hat{R}_{\mathbf{Rj}}$ have the same mean square error.

Special cases of the proposed estimator:

Furthermore, the following estimators are special case of the proposed class and their properties can be elucidated below.

1. $$\hat{R}_{\mathbf{PP}}=\hat{R}\left(\frac{\bar{x}_1}{\mu_1}\right)\left(\frac{\bar{x}_2}{\mu_2}\right) \qquad (15)$$

The product-product type estimator mentioned in (15) is a special case of proposed class of estimators. Mean square error of the estimator (15) is given by

$$MSE\left(\hat{R}_{\mathbf{PP}}\right)=\frac{R^2}{n}\left[\left\{\frac{1}{\mu_{Y_1}^2}\sigma_{Y_1}^2+\frac{1}{\mu_{Y_2}^2}\sigma_{Y_2}^2\right.\right.$$

$$\left.-\frac{2}{\mu_{Y_1}\mu_{Y_2}}\sigma_{Y_1Y_2}\right\}+\left\{\frac{1}{\mu_1^2}\sigma_{X_1}^2\right.$$

$$\left.+\frac{1}{\mu_2^2}\sigma_{X_2}^2+\frac{2}{\mu_1\mu_2}\sigma_{X_1X_2}\right\}$$

$$+\frac{2}{\mu_{Y_1}}\left\{\frac{1}{\mu_1}\sigma_{X_1Y_1}+\frac{1}{\mu_2}\sigma_{X_2Y_1}\right\}$$

$$\left.-\frac{2}{\mu_{Y_2}}\left\{\frac{1}{\mu_1}\sigma_{X_1Y_2}+\frac{1}{\mu_2}\sigma_{X_2Y_2}\right\}\right]$$

$$+\frac{R^2}{n}\left[\frac{1}{\mu_{Y_1}^2}\sigma_{u_1}^2+\frac{1}{\mu_{Y_2}^2}\sigma_{u_2}^2+\frac{a_1^2}{\mu_1^2}\sigma_{v_1}^2+\frac{a_2^2}{\mu_2^2}\sigma_{v_2}^2\right]$$

$$=M_{11}+M_{12} \qquad (16)$$

2. $$\hat{R}_{\mathbf{RR}}=\hat{R}\left(\frac{\mu_1}{\bar{x}_1}\right)\left(\frac{\mu_2}{\bar{x}_2}\right) \qquad (17)$$

The ratio-ratio type estimator mentioned in (17) is a special case of proposed class of estimators. Mean square error of the above estimator is given by

$$MSE\left(\hat{R}_{\mathbf{RR}}\right)=\frac{R^2}{n}\left[\left\{\frac{1}{\mu_{Y_1}^2}\sigma_{Y_1}^2+\frac{1}{\mu_{Y_2}^2}\sigma_{Y_2}^2\right.\right.$$

$$\left.-\frac{2}{\mu_{Y_1}\mu_{Y_2}}\sigma_{Y_1Y_2}\right\}+\left\{\frac{1}{\mu_1^2}\sigma_{X_1}^2\right.$$

$$\left.+\frac{1}{\mu_2^2}\sigma_{X_2}^2+\frac{2}{\mu_1\mu_2}\sigma_{X_1X_2}\right\}$$

$$-\frac{2}{\mu_{Y_1}}\left\{\frac{1}{\mu_1}\sigma_{X_1Y_1}+\frac{1}{\mu_2}\sigma_{X_2Y_1}\right\}$$

$$\left.+\frac{2}{\mu_{Y_2}}\left\{\frac{1}{\mu_1}\sigma_{X_1Y_2}+\frac{1}{\mu_2}\sigma_{X_2Y_2}\right\}\right]$$

$$+\frac{R^2}{n}\left[\frac{1}{\mu_{Y_1}^2}\sigma_{u_1}^2+\frac{1}{\mu_{Y_2}^2}\sigma_{u_2}^2+\frac{a_1^2}{\mu_1^2}\sigma_{v_1}^2+\frac{a_2^2}{\mu_2^2}\sigma_{v_2}^2\right]$$

$$=M_{21}+M_{22} \qquad (18)$$

3. $$\hat{R}_{\mathbf{a_1}}=\hat{R}\left(\frac{\bar{x}_1}{\mu_1}\right)^{a_1} \qquad (19)$$

The general ratio type estimator mentioned in (19) is a special case of proposed class of estimators. Mean square error of the above estimator is given by

$$MSE\left(\hat{R}_{\mathbf{a_1}}\right)=\frac{R^2}{n}\left[\left\{\frac{1}{\mu_{Y_1}^2}\sigma_{Y_1}^2+\frac{1}{\mu_{Y_2}^2}\sigma_{Y_2}^2\right.\right.$$

$$\left.-\frac{2}{\mu_{Y_1}\mu_{Y_2}}\sigma_{Y_1Y_2}\right\}+\frac{1}{\mu_1^2}\sigma_{X_1}^2$$

$$\left.+\frac{2a_1}{\mu_{Y_1}\mu_1}\sigma_{X_1Y_1}-\frac{2a_1}{\mu_{Y_2}\mu_1}\sigma_{X_1Y_2}\right]$$

$$+\frac{R^2}{n}\left[\frac{1}{\mu_{Y_1}^2}\sigma_{u_1}^2+\frac{1}{\mu_{Y_2}^2}\sigma_{u_2}^2\right.$$

$$\left.+\frac{a_1^2}{\mu_1^2}\sigma_{v_1}^2+\frac{a_2^2}{\mu_2^2}\sigma_{v_2}^2\right]$$

$$=M_{31}+M_{32} \qquad (20)$$

Here in (16), (18) and (20) M_{11}, M_{21} and M_{31} is the terms of mean square error without measurement error. Whereas M_{12}, M_{22} and M_{32} is the term of MSE with measurement error.

It is noteworthy that since bias can be removed by using jack-knife technique hence we recommend the use of jack-knife technique in all the above estimators whereas the MSE still remain unchanged.

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