# STUDY OF STRESS STRENGHT RELIABILITY RELATIONSHIP WHEN STRESS A GENERALIZED INVERSE CLASS OF DISTRIBUTIONS FACING POWER FUNCTION DISTRIBUTION STENGTH

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**Abstract:** Stress–strength reliability is studied though establishing a relationship among the parameters of the stress and strength distributions. In this article it is considered that stress follows a generalized class of inverse distributions proposed by Chaturvedi et.al (2007) [4] and strength follows a power function distribution.

**Keywords:** Generalized Inverse class of distributions, Inverse Pareto distribution, Power function distribution, Stress-Strength reliability.

**Introduction:** In reliability engineering the practice of stress-strength testing is an important topic of connotation for researchers. Under estimation and over estimation of factors associated with reliability may engender great losses. One of the statistical models of the Stress-Strength testing is the probability $P(Y > X)$, which represents the performance of an item of strength Y facing stress X, where X and Y are taken to be non –negative independent continuous random variables. The term Stress- Strength was first introduced by Church and Harries (1970) [5]. Since then lot of work has been done in this direction by various authors. For a brief review, one may refers to Downton (1973) [8], Tong (1974) [11], Kelly (1976) [9], Sathe and Shah (1981) [10], Chaturvedi and Rani (1997) [6], Chao (1982) [3], Award (1986) [2], Chaturvedi and Surinder (1999) [7]. In this paper, the problem of determining the total unreliability of the items is considered. It is considered that the stress X follows a generalized class of inverse distributions and Strength Y follows a power function distribution.

$$f(x,p,\sigma) = (\frac{p}{\sigma x^2}) g'(x^{-1}) g^{(p-1)}(x^{-1}) \exp\left(\frac{-g^p(x^{-1})}{\sigma}\right), \ 0 < x < \frac{1}{a},$$

... (1.1)

where $p$ and $\sigma$ are the parameters, $p$ is known but $\sigma$ is unknown.

Note that (1.1) represents generalized inverse class of distributions proposed by Chaturvedi and Sharma (2007) [4]. It represents a family of inverse distributions which covers following probabilistic models as specific cases:

(i) For $g(x) = x,\ a = 0$ and $p = 1$, we get inverse exponential distribution.

(ii) For $g(x) = x$ and $a = 0$, we have inverse Weibull distribution.

(iii)For $g(x) = \log(1 + x^b).\ b > 0,\ a = 0$ and $p = 1$ it gives inverse Burr distribution.

(iv) For $g(x) = \log(x/a)$ and $p = 1$, it leads us to inverse Pareto distribution.

(v)For $g(x) = x,\ a = 0\ \ g(x) = x, a = 0$ and $p = 2$, we obtain inverse Rayleigh distribution.

In the present paper, it is assumed that the random variable X represents the stress that an item faces, follows the distribution having the probability density function (p.d.f) (1.1) and strength Y follows Power function distribution with p.d.f

$$g(y,\theta) = \left(\frac{\mu}{\theta}\right)\left(\frac{y}{\theta}\right)^{\mu-1}; \qquad 0 < y < \theta,\ \mu > 0$$

... (1.2)

**2. Strength reliability for finite Strength**

A finite time distribution should be capable of describing the random variations in failure time of equipment. The strength of an equipment should limited to a finite range. This is due to the fact that manufactured product which are the function of a set of several subcomponents are not likely to have an infinite lifetime. Thus, the maximum possible value of strength distribution is $\theta$. The total unreliability of the items is therefore, obtained by $P(X > \theta)$. Alam and Roohi (2003) [1] have termed it as probability of disaster.

**Theorem 2.1:** If the random variables X and Y follows generalized inverse class of distributions given at (1.1) and Power function distribution given at (1.2) respectively, then $P(X>\theta)$ is given by

$$P\left(X>\theta\right)=1-\exp(-m)$$

, ... (2.1)

where $m=\exp\left(\frac{g^{p}(\theta^{-1})}{\sigma}\right)$.

**Proof:**

$$P(X>\theta)=\int_{\theta}^{\frac{1}{a}}\left(\frac{p}{\sigma x^{2}}\right)g'(x^{-1})g^{p-1}(x^{-1})\exp\left(\frac{-g^{p}(x^{-1})}{\sigma}\right)dx$$

... (2.2)

On substituting $t=\left(\frac{g^{p}(x^{-1})}{\sigma}\right)$ in (2.2), we get,

$P(X>\theta)=1-\exp(-m)$

.

Hence the theorem follows.

**Remark (2.1):** Table (2.1) depicts the probability of disaster i.e. $P(X>\theta)$, for generalized inverse class of distributions. It is interested to note that the probability of disaster increases with the increase in value of m.

## 3. Stress and Strength Reliability

For the stress-strength model, the probability $P\left(Y>X\right)$, when the random variable X and Y follows the (p.d.f.'s) (1.1) and (1.2), respectively is given by the following theorem.

**Theorem 3.1:** $P\left(Y>X\right)$ is given by

$$P(Y>X)=\exp\left(-m\right)-\frac{1}{(\theta)^{\mu}}\int_{m}^{\infty}x^{\mu}\exp(-t)dt$$

... (3.1)

**Proof: -**

$$P\left(Y>X\right)=\int_{x}^{\theta}\int_{0}^{\theta}f(x)g(y)dxdy$$

--- (3.2)

$$=\int_{x}^{\theta}\int_{0}^{\theta}\left(\frac{p}{\sigma x^{2}}\right)g'(x^{-1})g^{p-1}(x^{-1})\exp\left(\frac{-g^{p}(x^{-1})}{\sigma}\right)\left(\frac{\mu}{\theta}\right)\left(\frac{y}{\theta}\right)^{\mu-1}dxdy$$

On substituting $t=\left(\frac{g^{p}(x^{-1})}{\sigma}\right)$, we get

$$P(Y>X)=\exp\left(-m\right)-\frac{1}{(\theta)^{\mu}}\int_{m}^{\infty}x^{\mu}\exp(-t)dt,$$

... (3.3)

where $m=\exp\left(\frac{g^{p}(\theta^{-1})}{\sigma}\right)$.

Hence the theorem follows. .

**Remark (3.1):** On taking $g(x)=x,\ a=0$ and $p=1$, in eq. (3.3) and make the interpretation

$$g(x)=x,$$

$$\frac{g^{p}(x^{-1})}{\sigma}=\frac{x^{-p}}{\sigma},$$

gives $x=\left(\sigma t\right)^{\frac{-1}{p}}$, where $t=\left(\frac{g^{p}(x^{-1})}{\sigma}\right)$,

we get

$$P(Y>X)=\exp\left(-m\right)-\frac{1}{(\theta\sigma)^{\mu}}\int_{m}^{\infty}t^{-\mu}\exp(-t)dt,$$

... (3.4)

The probability of $(Y>X)$, when strength Y follows power function distribution and stress X follows inverse exponential distribution and $\mu,\ m>0,\ \sigma=1,\ \theta=10$ is shown in Table (3.1).

**Remark (3.2):** On taking $g(x)=x,\ a=0$ in eq. (3.3) and make the following interpretation

$$g(x)=x,$$

$$\frac{g^{p}(x^{-1})}{\sigma}=\frac{x^{-p}}{\sigma},$$

gives $x=\left(\sigma t\right)^{\frac{-1}{p}}$, where $t=\left(\frac{g^{p}(x^{-1})}{\sigma}\right)$,

we get

$$P(Y>X)=\exp\left(-m\right)-\frac{\sigma^{\left(\frac{-\mu}{p}\right)}}{(\theta)^{\mu}}\int_{m}^{\infty}t^{\left(\frac{-\mu}{p}\right)}\exp(-t)dt,$$

... (3.5)

The probability of $(Y>X)$, when strength Y follows power function distribution and stress X follows inverse Weibull distribution and $\mu,\ m>0,\ p=0.5,\ \sigma=1,\ \theta=10$ is shown in Table (3.2).

**Remark (3.3):** On taking $g(x)=\log\left(1+x^{b}\right),\ b>0\ and\ a=0$ in eq. (3.3) and make the following interpretation

$$g(x)=\log\left(1+x^{b}\right),$$

$$\frac{g^{p}(x^{-1})}{\sigma}=\frac{\left[\log\left(1+x^{-b}\right)\right]^{p}}{\sigma},$$

gives $x=\left[\exp(\sigma t)^{\left(\frac{1}{p}\right)}-1\right]^{\left(\frac{1}{-b}\right)}$,

where $t=\left(\frac{g^{p}(x^{-1})}{\sigma}\right)$

we get

$$P(Y>X)=\exp\left(-m\right)-\frac{1}{(\theta)^{\mu}}\int_{m}^{\infty}\left[\exp(\sigma t)-1\right]^{\left(\frac{-\mu}{b}\right)}\exp(-t)dt,$$

... (3.6)

The probability of $(Y>X)$, when strength Y follows power function distribution and stress X follows inverse Burr distribution and $\mu,\ m>0,\ b=0.5,\ \sigma=1,\ \theta=10$ is shown in Table (3.3).

**Remark (3.4):** On taking $g(x)=\log\left(\frac{x}{a}\right)$ *and* $p=1$ in eq. (3.3) and make the following interpretation

$$g(x)=\log\left(\frac{x}{a}\right),$$

$$\frac{g^{p}(x^{-1})}{\sigma}=\frac{\log\left(\frac{x^{-1}}{a}\right)}{\sigma},$$

Give $x=\frac{1}{a\exp(\sigma t)}$, where $t=\left(\frac{g^{p}(x^{-1})}{\sigma}\right)$,

we get

$$P(Y>X)=\exp\left(-m\right)-\frac{1}{(a\theta)^{\mu}}\int_{m}^{\infty}\left[\exp-t\left(\sigma\mu+1\right)\right]dt,$$

... (3.7)

The probability of $(Y>X)$, when strength Y follows power function distribution and stress X follows inverse Pareto distribution and $\mu,\ m>0,\ a=2,\ \sigma=1,\ \theta=10$ is shown in Table (3.4).

**Remark (3.5):** On taking $g(x)=x,\ a=0$ and $p=1$, in eq. (3.3) and make the following interpretation $g(x)=x$,

$$\frac{g^{p}(x^{-1})}{\sigma}=\frac{x^{-p}}{\sigma},$$

gives $x=\left(\sigma t\right)^{\frac{-1}{p}}$, where $t=\left(\frac{g^{p}(x^{-1})}{\sigma}\right)$,

we get

$$P(Y>X)=\exp\left(-m\right)-\frac{\sigma^{\left(\frac{-\mu}{2}\right)}}{(\theta)^{\mu}}\int_{m}^{\infty}t^{\left(\frac{-\mu}{2}\right)}\exp(-t)dt,$$

... (3.8)

The probability of $(Y>X)$, when strength Y follows power function distribution and stress X follows inverse Rayleigh distribution and $\mu,\ m>0,\ \sigma=1,\ \theta=10$ is shown in Table (3.5)

**Table (2.1)**

| $m$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $P(X>\theta)$ | 0.6321 | 0.8647 | 0.9502 | 0.9817 | 0.9933 | 0.9976 | 0.9991 | 0.9996 | 0.9998 | 0.9999 |

**Table (3.1)**

| m / $\mu$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.01801 | 0.01769 | 0.01698 | 0.01611 | 0.01516 | 0.01418 | 0.01322 | 0.01227 | 0.01137 |
| 0.03 | 0.05275 | 0.05177 | 0.04968 | 0.04709 | 0.04429 | 0.04142 | 0.03857 | 0.03580 | 0.03314 |
| 0.05 | 0.08585 | 0.08424 | 0.08078 | 0.07659 | 0.07192 | 0.06721 | 0.06255 | 0.05802 | 0.05369 |
| 0.07 | 0.11741 | 0.11516 | 0.11037 | 0.10448 | 0.09818 | 0.09164 | 0.08523 | 0.07902 | 0.07307 |

**Table (3.2)**

| m \ $\mu$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.01534 | 0.01668 | 0.01706 | 0.01692 | 0.01649 | 0.01585 | 0.01511 | 0.01430 | 0.01347 |
| 0.03 | 0.04439 | 0.04847 | 0.04963 | 0.04923 | 0.04791 | 0.04603 | 0.04383 | 0.04145 | 0.03899 |
| 0.05 | 0.07137 | 0.07829 | 0.08023 | 0.07956 | 0.07738 | 0.07428 | 0.07067 | 0.06676 | 0.06274 |
| 0.07 | 0.09643 | 0.10627 | 0.10900 | 0.10807 | 0.10503 | 0.10075 | 0.09576 | 0.09093 | 0.08487 |

**Table (3.3)**

| m \ $\mu$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.02636 | 0.02753 | 0.02771 | 0.02733 | 0.02659 | 0.02563 | 0.02336 | 0.02336 | 0.02215 |
| 0.03 | 0.07452 | 0.07820 | 0.07875 | 0.07760 | 0.07541 | 0.07258 | 0.06936 | 0.06593 | 0.02382 |
| 0.05 | 0.11743 | 0.12365 | 0.12958 | 0.12267 | 0.11906 | 0.11443 | 0.10358 | 0.10358 | 0.09789 |
| 0.07 | 0.15571 | 0.16455 | 0.16586 | 0.16320 | 0.15821 | 0.15183 | 0.14463 | 0.13699 | 0.12918 |

**Table (3.4)**

| m \ $\mu$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.03626 | 0.03360 | 0.03111 | 0.02879 | 0.02664 | 0.02462 | 0.02276 | 0.02101 | 0.01940 |
| 0.03 | 0.10426 | 0.09652 | 0.08929 | 0.08256 | 0.07629 | 0.07047 | 0.06506 | 0.06003 | 0.05538 |
| 0.05 | 0.16666 | 0.15413 | 0.14246 | 0.13161 | 0.12152 | 0.11214 | 0.10344 | 0.09537 | 0.08789 |
| 0.07 | 0.02223 | 0.20693 | 0.19110 | 0.17638 | 0.16272 | 0.15003 | 0.13827 | 0.12138 | 0.11729 |

**Table (3.5)**

| m \ $\mu$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.01932 | 0.01817 | 0.01693 | 0.01568 | 0.01448 | 0.01334 | 0.01226 | 0.01125 | 0.01032 |
| 0.03 | 0.05666 | 0.05326 | 0.04960 | 0.04595 | 0.04241 | 0.03905 | 0.03588 | 0.03292 | 0.03016 |
| 0.05 | 0.09236 | 0.08678 | 0.08077 | 0.07478 | 0.06900 | 0.06350 | 0.05833 | 0.05345 | 0.04900 |
| 0.07 | 0.12649 | 0.11878 | 0.11050 | 0.10227 | 0.09432 | 0.08677 | 0.07867 | 0.07305 | 0.06689 |

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