NOVEL METHOD FOR SENSITIVITY ANALYSIS OF FUZZY LINEAR PROGRAMMING PROBLEMS

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Abstract : To the best of our knowledge, till now there is no method in the literature to deal with sensitivity analysis of such fuzzy linear programming problems in which: (i) All the elements of the coefficient matrix of the constraints are represented by real numbers, coefficients of the decision variables in the objective function and requirement vectors are represented by fuzzy numbers and decision variables are represented by fuzzy variables. (ii) Decision variables are represented by fuzzy variables and rest the parameters are represented by fuzzy numbers. In this paper, a new method is proposed for the same. To show the advantages of the proposed method over existing methods, some fuzzy sensitivity analysis problems which can not be solved by the existing methods are solved by using the proposed method.

Keywords**:** Fuzzy linear programming problems; Ranking function; Sensitivity analysis; Trapezoidal fuzzy numbers.

Introduction The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka, Okuda, Asai [1] in the framework of the fuzzy decision of Bellman and Zadeh [2]. The first formulation of fuzzy linear programming is proposed by Zimmermann [3]. Afterwards, many authors have considered various kinds of fuzzy linear programming problems and have proposed several approaches for solving these problems. Maleki, Tata, Mashinchi [4] proposed a new method for solving the fuzzy number linear programming problem and used its solution to obtain the fuzzy solution of the fuzzy variable linear programming problem. Mahdavi-Amiri and Nasseri [5] extended the concepts of duality in fuzzy number linear programming problems. Ganesan and Veeramani [6] introduced a new method for solving a kind of linear programming problem involving symmetric trapezoidal fuzzy numbers without converting them to crisp linear programming problems. Mahdavi-Amiri and Nasseri [7] used a certain linear ranking function to define the dual of fuzzy number linear programming problems and proposed the dual simplex algorithm for solving fuzzy variables linear programming problems. Moreover, Ebrahimnejad and Nasseri [8] used the complementary slackness to solve fuzzy number linear programming problems and fuzzy variables linear programming problems without the need of a simplex table. Hosseinzadeh Lotfi, Allahviranloo, Jondabeh [9] proposed a method to obtain the approximate solution of fully fuzzy linear programming problems. Ebrahimnejad, Nasseri, Hosseinzadeh Lotfi, [10] proposed a method namely primal–dual simplex algorithm to obtain a fuzzy solution of fuzzy variables linear programming problems. Nasseri and Ebrahimnejad [11] applied a fuzzy primal simplex method to solve flexible linear programming problems directly without solving any auxiliary problem. Ebrahimnejad, Nasseri, Hosseinzadeh Lotfi [12] developed a method for solving such fuzzy linear programming problems in which some or all fuzzy decision variables are bounded. Kheirfam and Hasani [13] studied the basis invariance sensitivity analysis for fuzzy linear programming problems. Ebrahimnejad [14] generalized the concept of sensitivity analysis in fuzzy number linear programming problems by applying fuzzy simplex algorithms and using the general linear ranking function on fuzzy numbers. Nasseri and Ebrahimnejad [15] proposed a method for sensitivity analysis on linear programming problem with trapezoidal fuzzy variables.

In this paper, the limitations of existing methods [13, 14, 15] are pointed out and to overcome these limitations a new method is proposed. To show the advantages of the proposed method over existing methods some fuzzy sensitivity analysis problems which can not be solved by the existing methods are solved by the proposed method.

The rest of this paper is organized as follows: In Section 2, some basic definitions, arithmetic operations and Yager's ranking approach for comparing trapezoidal fuzzy numbers are presented. In Section 3, applicability and limitations of the existing methods [13, 14, 15] are pointed out. In Section 4, a new method is proposed to deal with sensitivity analysis of fuzzy linear programming problems with fuzzy parameters. In Section 5, to show the advantages of the proposed method over existing methods a fuzzy sensitivity analysis problem which can not be solved by any of the existing methods and two fuzzy sensitivity analysis problem, chosen from literature, are solved by the proposed method and the obtained results are compared. Finally conclusion is presented in Section 6.

2. Preliminaries

In this section, some basic definitions, arithmetic operations of trapezoidal fuzzy numbers and an existing ranking approach for comparing trapezoidal fuzzy numbers are presented.

2.1 Basic definitions

In this section, some basic definitions are presented [16].

Definition 2.1 A fuzzy number defined on the universal set of real numbers R, denoted as $\widetilde{A}=(a,b,c,d)$, is said to be a trapezoidal fuzzy number if its membership function $\mu_{\widetilde{A}}(x)$, is given by:

$$\mu_{\widetilde{A}}(x)=\begin{cases}\dfrac{(x-a)}{(b-a)}, & a\le x<b\\ 1\quad , & b\le x\le c\\ \dfrac{(x-d)}{(c-d)}, & c<x\le d\\ 0\quad , & \text{otherwise}\end{cases}$$

where, $a,b,c,d\in R$

Definition 2.2 A trapezoidal fuzzy number (a,b,c,d), is said to be a non-negative trapezoidal fuzzy number if and only if $a\ge 0$.

2.2 Arithmetic operations

In this section, arithmetic operations between two trapezoidal fuzzy numbers, defined on universal set of real numbers R, are presented [16].

Let $\widetilde{A}_1=(a_1,b_1,c_1,d_1)$ and $\widetilde{A}_2=(a_2,b_2,c_2,d_2)$ be two trapezoidal fuzzy numbers then

(i) $\widetilde{A}_1\oplus\widetilde{A}_2=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$

(ii) $\widetilde{A}_1\ominus\widetilde{A}_2=(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2)$

(iii) $\widetilde{A}_1\otimes\widetilde{A}_2\approx(a',b',c',d')$ where,

$a'=\text{minimu}(a_1a_2,a_1d_2,a_2d_1,d_1d_2)$,

$b'=\text{minimum }(b_1b_2,b_1c_2,b_2c_1,c_1c_2)$,

$c'=\text{maximum}(b_1b_2,b_1c_2,b_2c_1,c_1c_2)$,

$d'=\text{maximum }(a_1a_2,a_1d_2,a_2d_1,d_1d_2)$

(iv) $\lambda\widetilde{A}=\begin{cases}(\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1) & \lambda\ge 0\\ (\lambda d_1,\lambda c_1,\lambda b_1,\lambda a_1) & \lambda\le 0\end{cases}$

2.3 Yager's ranking approach

A number of ranking approaches have been proposed for comparing fuzzy numbers. A relatively simple computational and easily understandable ranking approach, proposed by Yager [17] and used in existing methods [13, 14, 15], is considered for comparing fuzzy numbers in this paper. Yager [17] proposed a procedure for ordering fuzzy sets in which a ranking index $\Re(\widetilde{A})$ is calculated for the fuzzy number $\widetilde{A}=(a,b,c,d)$ from its λ-cut $[b-(b-a)\lambda,c+(d-c)\lambda]$ according to the following formula:

$$\Re(\widetilde{A})=\frac{1}{2}\left(\int_0^1(b-(b-a)\lambda)\,d\lambda+\int_0^1(c+(d-c)\lambda)\,d\lambda\right)=\frac{a+b+c+d}{4}$$

Let $\widetilde{A}$ and $\widetilde{B}$ be two fuzzy numbers then

(i) $\widetilde{A}\underset{\Re}{\ge}\widetilde{B}$ if $\Re(\widetilde{A})\ge\Re(\widetilde{B})$

(ii) $\widetilde{A}\underset{\Re}{>}\widetilde{B}$ if $\Re(\widetilde{A})>\Re(\widetilde{B})$

(iii) $\widetilde{A}\underset{\Re}{=}\widetilde{B}$ if $\Re(\widetilde{A})=\Re(\widetilde{B})$

3. Applicability and limitations of the existing methods

In this section, the applicability of existing methods [13, 14, 15] for solving fuzzy sensitivity analysis problems are discussed and also on the basis of applicability the limitations of the existing methods are pointed out.

3.1 Applicability of the existing methods

In this section, the applicability of the existing methods [13, 14, 15] are discussed.

(1) The existing methods [13, 15] can be used only to deal with the sensitivity analysis of fuzzy linear programming problem (P_0) in which the elements of coefficient matrix of the constraints and coefficients of the decision variables in the

objective function are represented by real numbers, requirement vectors are represented by fuzzy numbers and decision variables are represented by fuzzy variables.

Maximize (or Minimize) $(C^T \otimes \tilde{X})$

Subject to

$$A \otimes \tilde{X} \leq_{\Re} \text{ or } =_{\Re} \text{ or } \geq_{\Re} \tilde{b} \qquad (\mathrm{P}_0)$$

where, $\tilde{b} = [\tilde{b}_j]_{m\times 1}$, $\tilde{X} = [\tilde{X}_j]_{n\times 1}$, $A = [a_{ij}]_{m\times n}$, $C^T = [c_j]_{1\times n}$, $\Re$ is a linear ranking function, $\tilde{b}_j$ is a trapezoidal fuzzy number, c_j and a_{ij} are real numbers and $\tilde{X}_j$ is a non-negative trapezoidal fuzzy variable.

(2) The existing method [14] can be used only to deal with the sensitivity analysis of fuzzy linear programming problems (P_1) in which the decision variables are represented by real variables and rest of the parameters are represented by fuzzy numbers.

Maximize (or Minimize) $(\tilde{C}^T \otimes X)$

Subject to

$$\tilde{A} \otimes X \leq_{\Re} \text{ or } =_{\Re} \text{ or } \geq_{\Re} \tilde{b} \qquad (\mathrm{P}_1)$$

where, $\tilde{b} = [\tilde{b}_j]_{m\times 1}$, $X = [X_j]_{n\times 1}$, $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$, $\tilde{C}^T = [\tilde{c}_j]_{1\times n}$, $\Re$ is a linear ranking function, $\tilde{c}_j$, $\tilde{a}_{ij}$ and $\tilde{b}_j$ are trapezoidal fuzzy numbers and X_j is a non-negative real variable.

3.2 Limitations of the existing methods

In this section, the limitations of the existing methods [13, 14, 15] are pointed out.

(1) Fuzzy linear programming problem (P_2) in which all the elements of the coefficient matrix of the constraints are represented by real numbers, coefficients of the decision variables in the objective function and requirement vectors are represented by fuzzy numbers and decision variables are represented by fuzzy variables.

Maximize (or Minimize) $(\tilde{C}^T \otimes \tilde{X})$

Subject to

$$A \otimes \tilde{X} \leq_{\Re} \text{ or } =_{\Re} \text{ or } \geq_{\Re} \tilde{b} \qquad (\mathrm{P}_2)$$

where, $\tilde{b} = [\tilde{b}_j]_{m\times 1}$, $\tilde{X} = [\tilde{X}_j]_{n\times 1}$, $A = [a_{ij}]_{m\times n}$, $\tilde{C}^T = [\tilde{c}_j]_{1\times n}$, $\Re$ is a linear ranking function, a_{ij} is a real number, $\tilde{c}_j$, $\tilde{b}_j$ are trapezoidal fuzzy numbers and $\tilde{X}_j$ is a non-negative trapezoidal fuzzy variable.

(1) Fuzzy linear programming problem (P_3) in which decision variables are represented by fuzzy variables and rest the parameters are represented by fuzzy numbers.

Maximize(or Minimize) $(\tilde{C}^T \otimes \tilde{X})$

Subject to

$$\tilde{A} \otimes \tilde{X} \leq_{\Re} \text{ or } =_{\Re} \text{ or } \geq_{\Re} \tilde{b} \qquad (\mathrm{P}_3)$$

where, $\tilde{b} = [\tilde{b}_j]_{m\times 1}$, $\tilde{X} = [\tilde{X}_j]_{n\times 1}$, $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$, $\tilde{C}^T = [\tilde{c}_j]_{1\times n}$, $\Re$ is a linear ranking function, $\tilde{c}_j$, $\tilde{a}_{ij}$ and $\tilde{b}_j$ are trapezoidal fuzzy numbers and $\tilde{X}_j$ is a non-negative trapezoidal fuzzy variable.

4. Proposed method

In this section, to overcome the limitations of the existing methods [13, 14, 15], discussed in Section 3, a new method is proposed to deal with the sensitivity analysis of fuzzy linear programming problems (P_3). The same method can also be used to deal with the sensitivity analysis of fuzzy linear programming problems (P_0), (P_1), (P_2) and (P_3).

The main advantage of the proposed method over the existing methods [13, 14, 15] is that the fuzzy sensitivity analysis problems, which can not be solved by the existing methods, can be solved by the proposed method. Also, the fuzzy sensitivity analysis problems that can be solved by the existing methods, can also be solved by the proposed method.

The steps of the proposed method are as follows:

Step 1 Convert the fuzzy linear programming problem (P_3) into the crisp linear programming problem (P_4):

Maximize (or Minimize) $\Re(\tilde{C}^T \otimes \tilde{X})$

Subject to

$$\Re(\tilde{A} \otimes \tilde{X}) \leq or = or \geq \Re(\tilde{b}) \qquad (\mathrm{P}_4)$$

$$x_j \geq 0,\ w_j - z_j \geq 0,\ y_j - x_j \geq 0$$

$$z_j - y_j \geq 0 \quad j = 1,2,3,\ldots,n.$$

Step 2 Solve the crisp linear programming problem (P_4), to find the optimal solution $\{x_j, y_j, z_j, w_j\}$.

Step 3 Find the fuzzy optimal solution $\tilde{X} = [\tilde{X}_j]_{n\times 1}$, by putting the values of x_j, y_j, z_j

and w_j, obtained from Step 2, in $\tilde{X}_j = (x_j, y_j, z_j, w_j)$ and the fuzzy optimal value by putting the values of $\tilde{X}$ in $\tilde{C}^T \otimes \tilde{X}$.

Step 4 Check that which of the following case is to be considered:

(i)Change in the fuzzy cost vector,

(ii) Change in the fuzzy requirement vector,

(iii)Addition of a new fuzzy variable,

(iv)Change in the coefficient matrix of the constraints,

(v)Addition of new fuzzy constraint.

Case 1: Change in the fuzzy cost vector

If in the given fuzzy linear programming problem (P_3) the cost vector is changed from $\tilde{C}^T$ to $\tilde{C}'^T$ then replace $\Re(\tilde{C}^T \otimes \tilde{X})$ by $\Re(\tilde{C}'^T \otimes \tilde{X})$ in crisp linear programming (P_4) to obtain (P_5):

Maximize (or Minimize) $\Re(\tilde{C}'^T \otimes \tilde{X})$

Subject to

$$\Re(\tilde{A} \otimes \tilde{X}) \le or = or \ge \Re(\tilde{b}) \qquad (\mathrm{P}_5)$$

$$x_j \ge 0 \; w_j - z_j \ge 0, y_j - x_j \ge 0$$

$$z_j - y_j \ge 0 \quad j = 1,2,3,\ldots,n$$

Case 2: Change in fuzzy requirement vector $\tilde{b}$

If in the fuzzy linear programming problem (P_3) the requirement vector is changed from $\tilde{b}$ to $\tilde{b}'$ then replace $\Re(\tilde{b})$ by $\Re(\tilde{b}')$ in crisp linear programming problem (P_4) to obtain (P_6):

Maximize (or Minimize) $\Re(\tilde{C}^T \otimes \tilde{X})$

Subject to

$$\Re(\tilde{A} \otimes \tilde{X}) \le or = or \ge \Re(\tilde{b}') \qquad (\mathrm{P}_6)$$

$$x_j \ge 0, \; w_j - z_j \ge 0, y_j - x_j \ge 0,$$

$$z_j - y_j \ge 0 \quad j = 1,2,3,\ldots,n$$

Case 3: Addition of a new fuzzy variable

Suppose a new non-negative fuzzy variable, say $\tilde{X}_{n+1}$, is added in (P_3). Assume that $\tilde{c}_{n+1}$ is the cost and $\tilde{A}_{n+1}$ is the column associated with $\tilde{X}_{n+1}$, then replace $\Re(\tilde{A} \otimes \tilde{X})$ by $\Re(\tilde{A} \otimes \tilde{X} \oplus \tilde{A}_{n+1} \otimes \tilde{X}_{n+1})$ and $\Re(\tilde{C}^T \otimes \tilde{X})$ by $\Re(\tilde{C}^T \otimes \tilde{X} \oplus \tilde{c}_{n+1} \otimes \tilde{X}_{n+1})$ in (P_4) to obtain the crisp linear programming problem (P_7):

Maximize(orMinimize) $\Re(\tilde{C}^T \otimes \tilde{X} \oplus \tilde{c}_{n+1} \otimes \tilde{X}_{n+1})$

Subject to

$$\Re(\tilde{A} \otimes \tilde{X} \oplus \tilde{A}_{n+1} \otimes \tilde{X}_{n+1}) \le or = or \ge \Re(\tilde{b}) \quad (\mathrm{P}_7)$$

$$x_j \ge 0, \; w_j - z_j \ge 0, y_j - x_j \ge 0,$$

$$z_j - y_j \ge 0 \quad j = 1,2,3,\ldots,n+1$$

Case 4: Addition of a new fuzzy constraint

Suppose a new fuzzy constraint is added in the original fuzzy linear programming problem (P_3) then, replace $\Re(\tilde{A} \otimes \tilde{X}) \le or = or \ge \Re(\tilde{b})$ by $\Re(\tilde{A}' \otimes \tilde{X}) \le or = or \ge \Re(\tilde{b}')$ in (P_4) to obtain crisp linear programming problem (P_8):

Maximize (or Minimize) $\Re(\tilde{C}^T \otimes \tilde{X})$

Subject to

$$\Re(\tilde{A}' \otimes \tilde{X}) \le or = or \ge \Re(\tilde{b}') \qquad (\mathrm{P}_8)$$

$$x_j \ge 0, \; w_j - z_j \ge 0, y_j - x_j \ge 0,$$

$$z_j - y_j \ge 0 \quad j = 1,2,3,\ldots,n$$

Case 5: Change in coefficient matrix of the constraints

Suppose the column of the coefficient matrix of the constraints, corresponding to the fuzzy variable $\tilde{X}_j$, is changed from $\tilde{A}_j$ to $\tilde{A}_j'$ in the original fuzzy linear programming problem (P_3) then, replace $\Re(\tilde{A} \otimes \tilde{X}) \le or = or \ge \Re(\tilde{b})$ by $\Re(\tilde{A}' \otimes \tilde{X}) \le or = or \ge \Re(\tilde{b})$ in (P_4) to obtain new crisp linear programming problem (P_9):

Maximize (or Minimize) $\Re(\tilde{C}^T \otimes \tilde{X})$

Subject to

$$\Re(\tilde{A}' \otimes \tilde{X}) \le or = or \ge \Re(\tilde{b}) \qquad (\mathrm{P}_9)$$

$$x_j \ge 0, \; w_j - z_j \ge 0, y_j - x_j \ge 0,$$

$$z_j - y_j \ge 0 \quad j = 1,2,3,\ldots,n$$

Step 5: Now apply the existing sensitivity analysis technique to find the optimal solution of $(\mathrm{P}_5),(\mathrm{P}_6),(\mathrm{P}_7),(\mathrm{P}_8)$ and (P_9) with the help of optimal solution of (P_4) and use Step 3 of the proposed method to find the fuzzy optimal solution of the resulting fuzzy linear programming problem.

Remark 4.1 The other cases i.e., deletion of fuzzy variables, deletion of fuzzy constraints, simultaneous change in coefficients of the decision variables in objective function and requirement vectors etc. can also be solved by using the proposed method.

5. Advantage of the proposed method

To show the advantage of the proposed method over existing methods [13, 14, 15], the fuzzy sensitivity analysis problem, chosen in Example 5.1 which can't be solved by using any of the existing methods, is solved by using the proposed method.

Example 5.1 Consider the following fuzzy linear programming problem,

Minimize $[(-4,-3,-1,0)\otimes \tilde{X}_1 \oplus(-1,0,2,3)\otimes \tilde{X}_2 \oplus(-3,-2,0,1)\otimes \tilde{X}_3]$

Subject to

$$\left(-\frac{1}{2},0,2,\frac{5}{2}\right)\otimes \tilde{X}_1 \oplus\left(-\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{5}{2}\right)\otimes \tilde{X}_2 \oplus(-1,0,2,3)\otimes \tilde{X}_3 \leq_{\Re} (3,5,7,9)$$

$$(-2,-2,0,0)\otimes \tilde{X}_1 \oplus\left(\frac{1}{2},1,3,\frac{7}{2}\right)\otimes \tilde{X}_2 \leq_{\Re} (2,3,5,6) \quad (E_1)$$

$\tilde{X}_1, \tilde{X}_2, \tilde{X}_3$ are non-negative trapezoidal fuzzy variables.

(a) Discuss the effect of changing the cost coefficients $(-4,-3,-1,0)$, $(-1,0,2,3)$ and $(-3,-2,0,1)$ of the fuzzy decision variables $\tilde{X}_1, \tilde{X}_2$ and $\tilde{X}_3$ to $(-1,0,3,5)$, $(-6,-5,-1,0)$ and $(-2,-1,0,2)$ respectively on the fuzzy optimal solution and fuzzy optimal value of resulting fuzzy linear programming problem.

(b) Discuss the effect of changing the requirement vectors from $(3,5,7,9)$, $(2,3,5,6)$ to $(1,2,4,5)$, $(2,3,5,6)$ on the fuzzy optimal solution and fuzzy optimal value of resulting fuzzy linear programming problem.

(c) Find the effect of addition of a new non-negative fuzzy variable $\tilde{X}_4$ with cost $(-1,0,2,3)$ and column vectors $[(-2,-2,0,0),(0,1,3,4)]^T$ on the current fuzzy optimal solution and fuzzy optimal value of resulting fuzzy linear programming problem.

(d) Discuss the effect of changing column of the coefficient matrix of the constraints, corresponding to the fuzzy variable $\tilde{X}_2$, by $[(-3,-2,0,1), (0,1,3,4)]^T$ on the fuzzy optimal solution and fuzzy optimal value of resulting fuzzy linear programming problem.

(e) Find the effect of addition of a new fuzzy constraint $(0,1,3,4)\otimes \tilde{X}_1 \Theta (-1,0,2,3)\otimes \tilde{X}_2 \leq_{\Re} (1,2,4,5)$ on the current fuzzy optimal solution and fuzzy optimal value of resulting fuzzy linear programming problem.

Solution: The solution of the fuzzy sensitivity analysis problem, chosen in Example 5.1, by using the proposed method can be obtained as follows:

Assuming $\tilde{X}_1=(x_1, y_1, z_1, w_1)$, $\tilde{X}_2=(x_2, y_2, z_2, w_2)$ and $\tilde{X}_3=(x_3, y_3, z_3, w_3)$ the fuzzy linear programming problem, chosen in Example 5.1, can be written as:

Minimize $[(-4,-3,-1,0)\otimes (x_1, y_1, z_1, w_1) \oplus(-1,0,2,3)\otimes (x_2, y_2, z_2, w_2) \oplus (-3,-2,0,1)\otimes (x_3, y_3, z_3, w_3)]$

Subject to

$$\left(-\frac{1}{2},0,2,\frac{5}{2}\right)\otimes (x_1, y_1, z_1, w_1) \oplus\left(-\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{5}{2}\right)\otimes (x_2, y_2, z_2, w_2) \oplus(-1,0,2,3)\otimes (x_3, y_3, z_3, w_3) \leq_{\Re} (3,5,7,9) \quad (E_2)$$

$$(-2,-2,0,0)\otimes (x_1, y_1, z_1, w_1) \oplus\left(\frac{1}{2},1,3,\frac{7}{2}\right)\otimes (x_2, y_2, z_2, w_2) \leq_{R} (2,3,5,6)$$

(x_1, y_1, z_1, w_1) (x_2, y_2, z_2, w_2) and (x_3, y_3, z_3, w_3) are non-negative trapezoidal fuzzy numbers.

Using Step 1 of the proposed method the fuzzy linear programming problem (E_2) is converted into the crisp linear programming problem (E_3):

Minimize

$$\left(\frac{1}{4}(-y_1-3z_1+2z_2-2z_3-4w_1+2w_2-2w_3)\right)$$

Subject to

$$\frac{1}{2}y_2+2z_1+\frac{3}{2}z_2+2z_3+2w_1+2w_2+3w_3 \leq 24$$

$$\frac{1}{2}x_2+2y_2-2z_1+3z_2-2w_1+\frac{7}{2}w_2 \leq 16 \quad (E_3)$$

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

$$y_1-x_1 \geq 0,\ z_1-y_1 \geq 0, w_1-z_1 \geq 0$$

$$y_2-x_2 \geq 0,\ z_2-y_2 \geq 0, w_2-z_2 \geq 0$$

$$y_3-x_3 \geq 0,\ z_3-y_3 \geq 0, w_3-z_3 \geq 0.$$

The optimal solution of the crisp linear programming problem (E_3) is:

$x_1 = 0, x_2 = 0, x_3 = 0, y_1 = 0, y_2 = 0, y_3 = 0, z_1 = 0, z_2 = 0,$
$z_3 = 0, w_1 = 12, w_2 = 0, w_3 = 0$

and the optimal value is -12.

Using Step 3 of the proposed method the fuzzy optimal solution is given by $\tilde{X}_1 = (0,0,0,12), \tilde{X}_2 = (0,0,0,0), \tilde{X}_3 = (0,0,0,0)$ and the fuzzy optimal value is $(-48,0,0,0)$.

(a) Since the cost coefficients corresponding to the variables $\tilde{X}_1, \tilde{X}_2$ and $\tilde{X}_3$ is changed from $(-4,-3,-1,0)$, $(-1,0,2,3)$ and $(-3,-2,0,1)$ to $(-1,0,3,5)$, $(-6,-5,-1,0)$ and $(-2,-1,0,2)$ respectively in the original fuzzy linear programming problem (E_1) so replacing crisp linear programming problem (E_3) by (E_4):

Minimize

$$\left(\frac{1}{4}(-y_2 + 3z_1 - 5z_2 - z_3 + 4w_1 - 6w_2)\right)$$

Subject to

$$\frac{1}{2}y_2 + 2z_1 + \frac{3}{2}z_2 + 2z_3 + 2w_1 + 2w_2 + 3w_3 \le 24$$
$$\frac{1}{2}x_2 + 2y_2 - 2z_1 + 3z_2 - 2w_1 + \frac{7}{2}w_2 \le 16 \quad (\text{E}_4)$$
$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$$
$$y_1 - x_1 \ge 0,\ z_1 - y_1 \ge 0, w_1 - z_1 \ge 0$$
$$y_2 - x_2 \ge 0,\ z_2 - y_2 \ge 0, w_2 - z_2 \ge 0$$
$$y_3 - x_3 \ge 0,\ z_3 - y_3 \ge 0, w_3 - z_3 \ge 0.$$

Now applying the existing sensitivity analysis techniques, the optimal solution of resulting crisp linear programming problem (E_4) is:

$x_1 = 0, x_2 = 0, x_3 = 0, y_1 = 0, y_2 = 0, y_3 = 0, z_1 = 0,$
$z_2 = 0, z_3 = 2.97, w_1 = 0, w_2 = 4.57, w_3 = 2.97$

and the optimal value is -7.6

Using Step 3 of the proposed method the fuzzy optimal solution is given by $\tilde{X}_1 = (0,0,0,0), \tilde{X}_2 = (0,0,0,4.57), \tilde{X}_3 = (0,0,2.97,2.97)$ and the fuzzy optimal value is $(-33.36, -27.42, 0, 5.94)$.

(b) Since the requirement vector is changed from (3,5,7,9) and (2,3,5,6) to (1,2,4,5) and (2,3,5,6) respectively in the original fuzzy linear programming problem (E_1) so replacing $\Re(3,5,7,9)$ and $\Re(2,3,5,6)$ by $\Re(1,2,4,5)$ and $\Re(2,3,5,6)$ respectively i.e., 24 and 16 by 12 and16 respectively in (E_3).

Minimize

$$\left(\frac{1}{4}(-y_1 - 3z_1 + 2z_2 - 2z_3 - 4w_1 + 2w_2 - 2w_3)\right)$$

Subject to

$$\frac{1}{2}y_2 + 2z_1 + \frac{3}{2}z_2 + 2z_3 + 2w_1 + 2w_2 + 3w_3 \le 12$$
$$\frac{1}{2}x_2 + 2y_2 - 2z_1 + 3z_2 - 2w_1 + \frac{7}{2}w_2 \le 16$$
$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0 \qquad (\text{E}_5)$$
$$y_1 - x_1 \ge 0,\ z_1 - y_1 \ge 0, w_1 - z_1 \ge 0$$
$$y_2 - x_2 \ge 0,\ z_2 - y_2 \ge 0, w_2 - z_2 \ge 0$$
$$y_3 - x_3 \ge 0,\ z_3 - y_3 \ge 0, w_3 - z_3 \ge 0.$$

Now applying the existing sensitivity analysis techniques, the optimal solution of crisp linear programming problem (E_5) is:

$x_1 = 0, x_2 = 0, x_3 = 0, y_1 = 0, y_2 = 0, y_3 = 0, z_1 = 0,$
$z_2 = 0, z_3 = 0, w_1 = 6, w_2 = 0, w_3 = 0$

and the optimal value is -6.

Using Step 3 of the proposed method the fuzzy optimal solution is given by $\tilde{X}_1 = (0,0,0,6), \tilde{X}_2 = (0,0,0,0), \tilde{X}_3 = (0,0,0,0)$ and the fuzzy optimal value is $(-24, 0, 0, 0)$.

(c) Since a new non-negative fuzzy variable $\tilde{X}_4$ with cost $(-1,0,2,3)$ and column $[(-2,-2,0,0),(0,1,3,4)]^{\text{T}}$ is added in the original fuzzy linear programming problem (E_1) so replacing crisp linear programming problem (E_3) by (E_6):

Minimize

$$\left(\frac{1}{4}(-y_1 - 3z_1 + 2z_2 - 2z_3 + 2z_4 - 4w_1 + 2w_2 - 2w_3 + 2w_4)\right)$$

Subject to

$$\frac{1}{2}y_2 + 2z_1 + \frac{3}{2}z_2 + 2z_3 - 2z_4 + 2w_1 + 2w_2 + 3w_3 - 2w_4 \le 24$$
$$\frac{1}{2}x_2 + 2y_2 + y_4 - 2z_1 + 3z_2 + 3z_4 - 2w_1 + \frac{7}{2}w_2 + 4w_4 \le 16$$
$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0\ x_4 \ge 0 \qquad (\text{E}_6)$$
$$y_1 - x_1 \ge 0,\ z_1 - y_1 \ge 0, w_1 - z_1 \ge 0$$
$$y_2 - x_2 \ge 0,\ z_2 - y_2 \ge 0, w_2 - z_2 \ge 0$$
$$y_3 - x_3 \ge 0,\ z_3 - y_3 \ge 0, w_3 - z_3 \ge 0$$

$$y_4 - x_4 \geq 0,\ z_4 - y_4 \geq 0, w_4 - z_4 \geq 0.$$

Now apply the existing sensitivity analysis techniques, the optimal solution of crisp linear programming problem (E_6) is:

$x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, y_1 = 0, y_2 = 0, y_3 = 0,$
$y_4 = 0, z_1 = 0, z_2 = 0, z_3 = 0, z_4 = 13.33,$
$w_1 = 38.66, w_2 = 0, w_3 = 0, w_4 = 13.33$ and the optimal value is -25.33

Using Step 3 of the proposed method the fuzzy optimal solution is given by $\tilde{X}_1 = (0,0,0,38.66), \tilde{X}_2 = (0,0,0,0), \tilde{X}_3 = (0,0,0,0), \tilde{X}_4 = (0,0,13.33,13.33)$ and the fuzzy optimal value is $(-167.97, 0, 26.66, 39.99)$.

(d) Since the column of the coefficient matrix of the constraints, corresponding to the fuzzy variable $\tilde{X}_2$, is changed from $[(-\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{5}{2}), (\frac{1}{2},1,3,\frac{7}{2})]^T$ to $[(-3,-2,0,1), (0,1,3,4)]^T$ in the original fuzzy linear programming problem (E_1) so replacing the crisp linear programming problem (E_3) by (E_7):

Minimize

$$\left(\frac{1}{4}(-y_1 - 3z_1 + 2z_2 - 2z_3 - 4w_1 + 2w_2 - 2w_3)\right)$$

Subject to

$$\frac{1}{2}y_2 + 2z_1 - 2z_2 + 2z_3 + 2w_1 - 2w_2 + 3w_3 \leq 24$$
$$b_2 - 2c_1 + 4c_2 - 2d_1 + 4d_2 \leq 16 \qquad (E_7)$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$
$$y_1 - x_1 \geq 0,\ z_1 - y_1 \geq 0, w_1 - z_1 \geq 0$$
$$y_2 - x_2 \geq 0,\ z_2 - y_2 \geq 0, w_2 - z_2 \geq 0$$
$$y_3 - x_3 \geq 0,\ z_3 - y_3 \geq 0, w_3 - z_3 \geq 0.$$

Now applying the existing sensitivity analysis techniques, the optimal solution of crisp linear programming problem (E_7) is:

$x_1 = 0, x_2 = 0, x_3 = 0, y_1 = 0, y_2 = 0, y_3 = 0, z_1 = 0, z_2 = 10,$
$z_3 = 0,\ w_1 = 32,\ w_2 = 10,\ w_3 = 0$ and the optimal value is -22.

Using Step 3 of the proposed method the fuzzy optimal solution is given by $\tilde{X}_1 = (0,0,0,32), \tilde{X}_2 = (0,0,10,10), \tilde{X}_3 = (0,0,0,0)$ and the fuzzy optimal value is $(-138, 0, 20, 30)$.

(e) Since a new fuzzy constraint $(0,1,3,4) \otimes \tilde{X}_1$ $(-1,0,2,3) \otimes \tilde{X}_2 \leq_{\Re} (1,2,4,5)$ is added to the original fuzzy linear programming problem (E_1) so adding the constraint $b_1 + 3c_1 - 2c_2 + 4d_1 - 2d_2 \leq 12$ to (E_3).

Minimize

$$\left(\frac{1}{4}(-y_1 - 3z_1 + 2z_2 - 2z_3 - 4w_1 + 2w_2 - 2w_3)\right)$$

Subject to

$$\frac{1}{2}y_2 + 2z_1 + \frac{3}{2}z_2 + 2z_3 + 2w_1 + 2w_2 + 3w_3 \leq 24$$
$$\frac{1}{2}x_2 + 2y_2 - 2z_1 + 3z_2 - 2w_1 + \frac{7}{2}w_2 \leq 16 \quad (E_8)$$
$$y_1 + 3z_1 - 2z_2 + 4w_1 - 2w_2 \leq 12$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$
$$y_1 - x_1 \geq 0,\ z_1 - y_1 \geq 0, w_1 - z_1 \geq 0$$
$$y_2 - x_2 \geq 0,\ z_2 - y_2 \geq 0, w_2 - z_2 \geq 0$$
$$y_3 - x_3 \geq 0,\ z_3 - y_3 \geq 0, w_3 - z_3 \geq 0.$$

Now applying the existing sensitivity analysis techniques, the optimal solution of resulting crisp linear programming problem (E_8) is:

$x_1 = 0, x_2 = 0, x_3 = 0, y_1 = 1.5, y_2 = 0, y_3 = 0, z_1 = 1.5,$
$z_2 = 0, z_3 = 3.6, w_1 = 1.5, w_2 = 0, w_3 = 3.6$
and the optimal value is -6.6

Using Step 3 of the proposed method the fuzzy optimal solution is given by $\tilde{X}_1 = (0,1.5,1.5,1.5), \tilde{X}_2 = (0,0,0,0), \tilde{X}_3 = (0,0,3.6,3.6)$ and the fuzzy optimal value is $(-16.8, -7.2, 0, 3.6)$.

5.1 Discussion

To compare the proposed method with the existing methods [13, 14, 15] the results of fuzzy sensitivity analysis problems, obtained by using existing methods as well as the proposed method, are shown in Table 1.

Table 1 Comparison of existing methods with the proposed method

Example	Existing method [14]	Existing methods [13, 15]	Proposed method
3.1 [14, pp.1882]	Applicable	Not Applicable	Applicable
1 [13, pp. 263]	Not Applicable	Applicable	Applicable
5.1	Not applicable	Not applicable	Applicable

It is obvious from the results, shown in Table 1, that the fuzzy sensitivity analysis problem, chosen in Example 3.1 [14, pp. 1882], can be solved by using both the existing method [14] as well as the proposed method but the same problem can not be solved by using the existing method [13,15]. Similarly the fuzzy sensitivity analysis problem, chosen in Example 3.2 [13, pp. 263], can be solved by using both the existing methods [13, 15] as well as the proposed method but the same problem can not be solved by using the existing method [14]. The fuzzy sensitivity analysis problem, chosen in Example 5.1 can not be solved by using any of the existing methods [13, 14, 15]. It can only be solved by using the proposed method. On the basis of above discussion, it can be suggested that it is better to use the proposed method for solving fuzzy sensitivity analysis problems instead of the existing methods [13, 14, 15].

6. Conclusions

In this paper, the limitations of the existing methods to deal with the sensitivity analysis of fuzzy linear programming problems are pointed out and to overcome the limitations, a new method is proposed to deal with the sensitivity analysis of such fuzzy linear programming problems in which decision variables are represented by fuzzy variables and rest the parameters are represented by fuzzy numbers. To show the advantage of the proposed methods over existing methods a fuzzy sensitivity analysis problem, which can not be solved by using the existing methods, as well as two fuzzy sensitivity analysis problems, which can be solved by the existing methods, are solved by the proposed method.

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