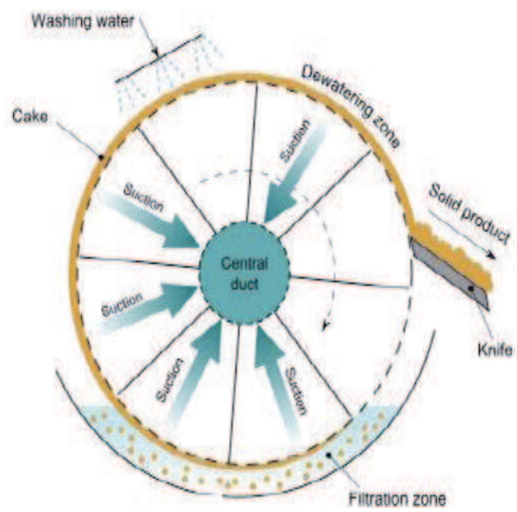
NUMERICAL SOLUTION OF MATHEMATICAL MODEL OF A WASHING ZONE OF ROTARY VACCUM WASHER USING ORTHOGONAL COLLOCATION METHOD

NISU JAIN, SHELLY

Abstract: Mathematical model for the washing zone of rotary vacuum washer is presented in terms of their fundamental parameters. Model equations consist of set of transient equations along with suitable initial and boundary conditions. The model equations solved in dimensionless form by introducing dimensionless variables. The model is solved by orthogonal collocation method (OCM) and has been solved by MATLAB 'ode15s' system solver.

Keywords: Orthogonal Collocation, Peclet Number, collocation points, axial dispersion coefficient

Introduction: The rotary vacuum washer consists of a wire mesh covered cylinder which rotates in the vat containing the slurry. A pulp mat is formed on the outer surface of the cylinder as due to the vacuum inside the cylinder and on the upper part of the cylinder wash water is applied. The process of the washing process involves the displacement of the liquor by the movement of the water plug, dispersion due to back mixing. The first and second dewatering zones are engaged in discharging much lower amounts of filtrate and air is also extracted through these zones. To increase efficiency the pulp should be washed with minimum amount of wash water and maximum removal of black liquor solutes. The results of efficient pulp treatment are: lower soda loss, reduce in volume of spent liquor, lower cost of the black liquor treatment. The schematic representation of a rotary vacuum washer is:



The control head divides the filter drum into the different sections for filtration, washing, suction drying and cake discharge, so that in the course of one revolution each point of the drum area passes through these zones in succession. The filtrate (clarified liquid) runs off through the separator receiver and is discharged by pumping. The filtered solid layer emerges from the suspension as the drum rotates, and following its emergence is washed (to remove impurities or to extract more product), suction-dried and discharged from the filter cloth. The wash liquid is fed onto the cake either directly by means of wash devices such as weirs or spray nozzles, or of a wash belt lying on top of the cake. The filtrate from the wash zone can be drained off separately from the mother filtrate. The filter cake is discharged by means of a discharge device of some kind, which covers the entire drum and which is specially suited to the cake thickness, consistency and structure [3].

Mathematics is a subject that has been studied for several years and is considered to be the base of all the basic sciences and engineering. No branch of engineering is considered to be complete without Mathematics. Numerous practical problems are solved by using mathematical equations and mathematical techniques. A variety of mathematical models has been used by industry to improve production, increase profits and generally improve understanding of complicated processes.

Increasing applications of mathematics in solving real life problems by using mathematical equations has given rise to the study of a new subject of mathematical modeling. It has also increased the applications of mathematics in industry and developed the concept of industrial mathematics.

A number of technical problems have been formulated via mathematical models and there by analyzed more efficiently. The first step of mathematical modelling is to consider a practical or real life problem and then express it in terms of mathematical equations and then analyze it using mathematical tools.

Development of Model: The washing zone phenomenon consists of liquor displacement by the movement of water plug controlled by the mechanics of fluid and dispersion due to back mixing. The amount of wash water applied is the key variable for the washing zone. Many industrial problems like paper making needs extensive use of mathematical techniques to study this process and each of its sub-process in an elaborative manner. Once a physical system has been analyzed keeping in mind the basic features of the problem, it is convenient to describe the system in terms of time and space variables using model equations with suitable initial and boundary conditions. The motivation behind mathematical modelling is to understand the physical phenomenon of the problem and to simulate the actual process.

In present study the modeling of washing zone of a rotary vacuum washer is to be dealt with. Model equations consist of set of transient equations along with suitable initial and boundary conditions.

Mathematical Model: The non-ideal flow pattern is described by the dimensional axial dispersion model given as:

$$\frac{\partial c}{\partial t} = D_L \frac{\partial^2 c}{\partial x^2} - u \frac{\partial c}{\partial x}$$

Boundary Condition

$$c = 0 \qquad at\ x = 0$$

$$\frac{\partial c}{\partial x} = 0 \qquad at\ x = L$$

where c = c(x,t) is the solute concentration, t is time from commencement of the displacement, D_L is the axial dispersion coefficient, u is average interstitial velocity, x is the distance from the point of introduction of the displacing fluid, L is the length of the bed, c_o is initial solute concentration.

The above equations can be put in dimensionless form by introducing the following dimensionless variables:

$$C = \frac{c}{c_0}; \quad T = \frac{ut}{L}\ ; \ P = \frac{uL}{D_L};\ \xi = \frac{x}{L}$$

where dimensionless, T,is equal to the ratio of total fluid volume introduced to the free volume of the bed, ξ is dimensionless axial coordinate,P is Peclet number, dimensionless. Using this, the dimensionalless form of the given model is

$$\frac{\partial C}{\partial T} = \frac{1}{P}\frac{\partial^2 C}{\partial \xi^2} - \frac{\partial C}{\partial \xi}$$

Boundary Condition

$$C = 0 \qquad at\ \xi = 0$$

$$\frac{\partial C}{\partial \xi} = 0 \qquad at\ \xi = 1$$

Orthogonal Collocation Method: The model is solved numerically by orthogonal collocation. The orthogonal collocation method (OCM) is one of the axial dispersion coefficient weighted residual methods. This method was first developed by Villadsen and Stewart [1967] [8]. Due to its simplicity and easy application, this method has attained popularity in the field of numerical simulation. In orthogonal collocation method, the dependent variable y is approximated by a series expansion containing n undetermined parameters. The function y(x) is chosen which satisfies the linear differential equation and the corresponding boundary conditions. The residual is set equal to zero at n collocation points. Finlayson [1971] has followed the orthogonal collocation method to solve the model equations of a packed bed reactor. The model equations were governed by radial temperature and concentration gradients. The orthogonal collocation method was compared with finite difference method and found to converge faster than finite difference method [5]. Lefevre [2000] has presented the properties of orthogonal collocation in context of orthogonal polynomials [6]. Curtis & Beard [2001] have presented the method of successive collocation instead of global collocation [4].

To solve the problem numerically using orthogonal collocation method,the firststep is to approximate the given function c(x,t) by an approximating function $\tilde{c}(x,t)$ such that:$c(x,t) \approx \tilde{c}(x,t) = \sum_{i=1}^{n} l_i(x) c_i(t)$ where

$l_i(x) = \prod_{j=1}^{n} \frac{(x-x_j)}{(x_i-x_j)}$ is a Lagrangian interpolation polynomial of order n and $c_i(t)$ be undefined constants. Since $\tilde{c}(x,t)$ is differential being a polynomial function, its first and secondorder derivative can be defined as : $\frac{\partial \tilde{c}}{\partial x} = \sum_{i=1}^{n} l_i'(x) c_i(t)$
At j^{th} collocation point x_j , $R(x_j,t)=0$,there fore after implementing orthogonal collocation we have,

$\frac{dc_j}{dt} = \frac{1}{P}\sum_{i=1}^{n} B_{ji}c_i - \sum_{i=1}^{n} A_{ji}c_i$ i=2,…,n-1

Boundary conditions are:

$$c_1 = 0$$

$$\sum_{i=1}^{n} A_{ni}c_i = 0$$

Initially $c_j = 1 \quad \forall t = 0, j = 1, \dots, n$

Collocation points are taken as roots of Legendre polynomial were chosen [2]. The resulting ordinary differential equation is solved using MATLAB with 'ode15s' system solver.

Results and Discussion: The given model is discussed for different values of Peclet Number.

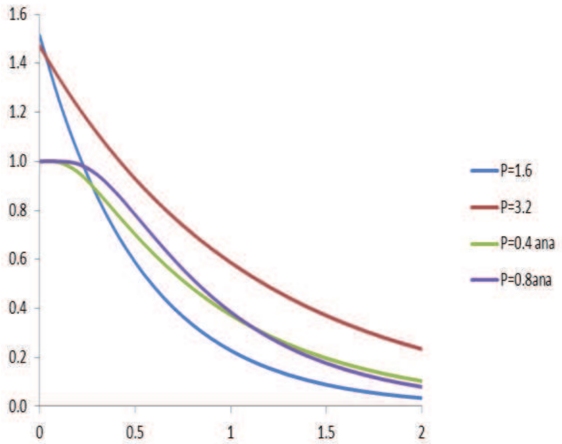


Fig.1 For different values of Peclet Number

It can be seen that for small values of P, the curve converges to steady state condition.However for large value of P, i.e., for P>5 and for small number of collocation points the curve oscillates firstly than starts converging to steady state condition.This is due to the stiffness of the problem. This situation can be overcome by increasing number of collocation points.The relative error of the model is also checked for different values of Peclet Number, and for different number of collocation point.

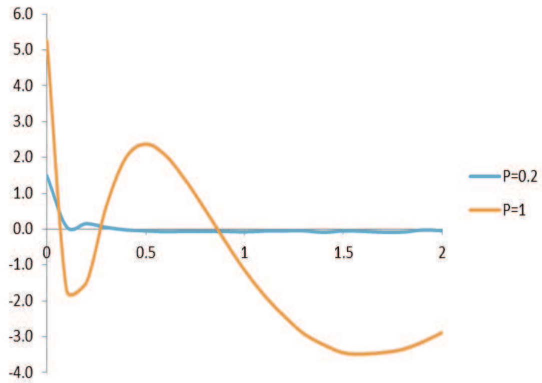


Fig.2 Error analysis for two collocation points

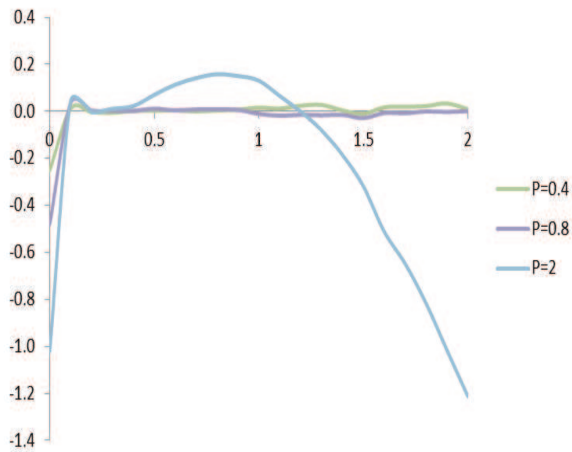


Fig.3 Error analysis for five collocation points

1. References

1. R. A. Adomaitis, and Y. Lin, "A technique for accurate collocation residual calculations", Chem. Eng. J., vol. 71, 1998, pp. 127-134.
2. Manoj Solanki, Arvind Bohare, Common Fixed Point theorem in Complex Valued Metric; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 655-657
3. S. Arora, and F. Potucek, "Modelling of displacement washing of packed bed of fibers", Brazilian J. of Chemical Engineering, vol. 26, 2009, pp. 385-393.

4. S. Arora, S. S. Dhaliwal, V. K. Kukreja "Mathematical modelling of washing zone of an industrial rotary vacuum washer", Indian Journal of Chemical Technology, Vol.15, 2008 pp. 332-340.
5. *K.S Balamurugan , S.V.K Varma,* Chemical Reaction and thermo Diffusion Effects on MHD ; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 504-512
6. J. W. Curtis, and R. W. Beard, "Successive collocation: An approximation to optimal linear control" Proceedings of the American Control Conference, Arlington, VA June 2001, pp. 25-27.
7. B. A. Finlayson, "Packed bed reactor analysis by orthogonal collocation", Chem. Eng. Sci., vol. 26, 1971, pp. 1081-1091.
8. *J.Jothikumar, V.Sumathy,* Estimation of Survival Time of Patients With Uterine Fibroid; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 327-332
9. L. Lefevre, D. Dochain, S. F. Azevedo, and A. Magnus, "Optimal selection of orthogonal polynomials applied to the integration of chemical reactor equations by collocation methods" Comp. & Chem. Eng., vol. 24, 2000, pp. 2571-2588.
10. *Indira Routaray,Sudhir Kumar Sahu,* An Eoq Model for Quadratic Demand Items With Shortages; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 560-565
11. N. S. Raghvan, and D. M. Ruthven, "Numerical simulation of a fixed bed adsorption column by the method of orthogonal collocation", AIChE, vol. 29(6), 1983, pp. 922-925.
12. *P. Srikanth Rao, D. Bharathi,* Common Fixed Point theorems in 2- Metric Spaces With Sub ; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 530-537
13. J. Villadsen, and W. E. Stewart, "Solution of boundary value problems by orthogonal collocation", Chem. Eng. Sci., vol. 22, 1967, pp. 1483-1501.

***

Author1: Nisu Jain, Research Scholars, Punjabi University Patiala,
e-mail: jainnisu@yahoo.co.in
Author2: Shelly, Assistant Professor, Punjabi University Patiala