AN EOQ MODELOFDETERIORATINGITEMSWITHLOT-SIZEDEPENDENT REPLENISHMENTCOSTANDTIMEDEPENDENTDEMAND

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Abstract: In this paper authors gave an inventory model for deteriorating items over a finite time horizon where the demand increases exponentially with time. This model is established with assumption that the successive replenishment cycle lengths are the same. This has been proved several times that an optimal replenishment schedule where the replenishment cost is constant in each cycle length over the finite time horizon. Here the replenishment cost per replenishment is taken to be linearly dependent on the lot-size of the replenishment. Shortages are allowed and are fully backlogged.

Keywords: EOQ Model, Deterioration; Replenishment; Demand

Introduction: Inventory problems with deterministic time dependent demand patterns received attention of several researchers in the recent year. A finite horizon inventory problem for time dependent variable demand was discussed by Stanfel and Sivazlian [10]. Silver and Meal [6] developed an approximate solution technique of a deterministic inventory model with time varying demand. Donaldson [16] first developed an exact solution procedure for items with a linearly increasing demand rate over a finite planning horizon. Dutta and Pal [14] developed models with time proportional demand with shortages. This model was based on assumption that there was no deterioration effecting inventory.

The decay of items plays an important role in the inventory management. In reality, some of the items are either damaged or decayed or vaporized or affected by some other factors and is not in a perfect condition to satisfy the demand. Therefore, this deterioration effect fully depends on preserving facility and environmental condition of warehouse/storage. So, due to deterioration effect, the stock-level continuously decreases in addition to the demand. In the last few years, many researchers developed models in the area of deteriorating inventories taking various types of demand. Dave and Patel [15] were first to develop a no-shortage inventory model for decaying items with time proportional demand. Sachan [9] extended the scope of this model to cover the backlogging option. Bahari-Kashani [7] suggested a heuristic method to determine order quantities with constant rate of deterioration and time-dependent demand. Next, Haiping and Wang [17] developed the same type of model for prescribed finite time horizon without considering shortages. Goswami and Chaudhuri [2] developed an inventory model for deteriorating items with a linear trend in demand and shortages for the prescribed fnite time horizon. Bhunia and Maiti [1] developed an inventory model of deteriorating items with lot size dependent replenishment cost and a linear trend in demand. Giri, Chakrabarty andChaudhary [5] gave a study of deteriorating items with time-varying demands and shortages after it Skouri and papachristos [8] developed the inventory model for deteriorating items, time varying demand, linear replenishment cost, partially time varying backlogging. Sana and Chaudhary [12]determined the volume flexible production policy for a deteriorating item with time dependent demand and shortages in same year Ghosh and Chaudhary [11] established an order level inventory model for a deteriorating item with weibull distribution deterioration, time quadratic demand and shortages. Chakraborty, Giriand Chaudhary [13] introduced production lot sizing with process deterioration and machine breakdown under inspection schedule.Goyal, Gupta and Singh [3] developed an EOQ Model for Deteriorating Items with Stock Dependent Demand and Effect of Learning after it Goyal, Chauhan and Singh proposed an EPQ model with stock dependent demand and time varying deterioration with shortages under inflationary environment.In this present paper, authors developed a deterministic inventory model incorporating the constant rate of deterioration effect assuming the demand to be exponentially increasing function of time and allowing shortages for the prescribed finite time horizon. In the existing literature of inventory model for the fixed time horizon, the replenishment cost is constant in each cycle. However, the replenishment cost include clerical and administrative costs, telephone charges, telegrams, transportation costs, loading and unloading costs etc. Out of these costs, only transportation costs, loading and unloading costs are dependent on the lot-size. Again, due to time dependent exponentially increasing demand and constant cycle length for the finite time horizon, the optimal lot-size in each cycle cannot be the same. So, the replenishment cost should not be the same for the successive replenishments. As a result, this cost is linearly dependent on the lot-size which is included in the analysis of the proposed model. The results areillustrated with the help of numerical examples.

Assumption and Notation: The proposed inventory model is developed under the following assumptions and notations:

(i) The system operates for a prescribed planning horizon which is H time units. Inventory level is zero at times $t=0$ and at $t=H$.

(ii) The demand rate $f(t)$ at any instant t is an exponential function of t such that $f(t)=ae^{bt}$, $a,b \geq 0,\ 0 \leq t \leq H$.

(iii) Lead time is zero.

(iv)Shortages, if any, are backlogged and are cleared as soon as a fresh lot arrives.

(v) Replenishment rate is infinite. A variable lot size q_j raises the inventory level at the beginning of jth replenishment cycle to the level S_j after fulfilling the back order quantity $(q_j - s_j)$.

(vi) The on hand inventory deteriorates at a constant rate $\theta, (0 \leq \theta \leq 1)$ per time unit andthere is neither repair nor replacement of the deteriorated inventory during H.

(vii) The replenishment cost F_j for the jth cycle $j=1,2,----n$ is linearly dependent on lot-size on that cycle and is in the following form $F_j = A + pq_j$ where p the additional replenishment cost per unit of items is.

(viii) The inventory holding cost C_1 per unit per unit time, the shortage cost C_2 per unit per unit time and the unit cost C_3 are known and constant during the planning time horizon H.

The total time horizon H has been divided into n equal parts of length T so that $T = H/n$.

Therefore, the reorder times over the time horizon H will be $(j-1)T\,(j=1,2,----n)$ we assume that the period for which there is no-shortage in each interval $[(j-1)T, jT]$ is a fraction of the scheduling period T and is equal to $kT\,(0 \leq k \leq 1)$. Shortages occur at times $(k+j-1)T\,(j=1,2,---n-1)$ where $(j-1)T \leq (k+j-1)T \leq jT,\ j=1,2,---,n-1$. Last replenishment occurs at time $(n-1)T$ and shortages are not allowed in the last period $[(n-1)T, H]$. Our problem is to derive the optimal reorder and shortage points and hence to determine the optimal values of n and k which minimize the total cost over the time horizon $[0, H]$.

Model Formulation and Solution: Let $I_j(t)$ be the amount in inventory at time t during the jth cycle $[(j-1)T \leq t \leq jT,\ j=1,2,---,n]$. Then, the differential equations describing the system during the jth cycle are

$$dI_j(t)/dt + \theta I_j(t) + f(t) = 0, t \in [(j-1)T, (k+j-1)T]\ j=1,2,-----,(n-1), \quad (1)$$

$$dI_j(t)/dt + f(t) = 0, t \in [(k+j-1)T, jT]\ j=1,2,-----,(n-1), \quad (2)$$

and the differential equation governing the stock status for the last replenishment cycle [during $(n-1)T \le t \le H$] is

$dI_n(t)/dt + \theta I_n(t) + f(t) = 0,$ (3)

Using the boundary conditions $I_j(t) = 0$ at $t = (k+j-1)T,\ j = 1,2,-----,(n-1)$ and $I_n(t) = 0$ at $t = H$, the solutions of Eqs. (1)-(3) are

$$I_j(t) = e^{-\theta t}\int_t^{(k+j-1)t} ae^{bt+\theta t}dt \quad j = 1,2,---------(n-1),\ t \in [(j-1)T,(k+j-1)T]$$

$$I_j(t) = \int_t^{(k+j-1)T} ae^{bt}dt,\ j = 1,2,-----(n-1),\ t \in [(k+j-1)T, jT]\ (4)$$

$$I_n(t) = e^{-\theta t}\int_t^{H} ae^{bt+\theta t}dt,\ t \in [(n-1)T, H]\ (5)$$

The inventory level in the beginning of each cycle is the amount needed to satisfy the demand requirement for that cycle excluding the shortage period and the amount required to account for the deterioration in the no-shortage part of the staid period. Thus, if Q_j units be the inventory level in time $(j-1)T$, then

$$Q_j = I_j((j-1))T = \int_{(j-1)T}^{(k+j-1)T} ae^{bt}e^{\{t-(j-1)T\}}dt, j = 1,2,----(n-1).\ (6)$$

We also have

$$Q_n = I_n((n-1)T) = \int_{(n-1)T}^{H} ae^{bt}e^{\{\theta(t-(n-1)T)\}}dt\ (7)$$

Let $\bar{Q}_j$ be the number of units required in the interval $[(j-1)T, jT],(j = 1,2,----n-1)$ .when there is no deterioration.

Then $\bar{Q}_j = \int_{(j-1)T}^{(k+j-1)T} ae^{bt}dt = \frac{a}{b}e^{b(j-1)T}\left[e^{bkT}-1\right]$ (8)

$$\bar{Q}_n = \int_{(n-1)T}^{H} ae^{bt}dt = \frac{a}{b}\left[e^{bH} - e^{b(n-1)T}\right]\ (9)$$

Therefore, the number of deteriorated itemsin $[(j-1)T,(k+j-1)T],(j = 1,2,----(n-1))$ are given by

$$D_j = Q_j - \bar{Q}_j = ae^{bt(j-1)}\left[\frac{1}{b+\theta}\left(e^{kt(b+\theta)}-1\right) - \frac{1}{b}\left(e^{bkt}-1\right)\right] (10)$$

and similarly

$$D_n = Q_n - \bar{Q}_n = \frac{a}{b+\theta}\left[e^{(b+\theta)H-\theta(n-1)T} - e^{bT(n-1)}\right] - \frac{a}{b}\left[e^{bH} - e^{b(n-1)T}\right] (11)$$

The lot-size q_j for the jth cycle is given by

$$q_j = Q_j + \int_{(k+j-2)T}^{(j-1)T} ae^{bt}dt, j = 2,3,----n\ (12)$$

and $q_1 = Q_1$

The total inventory R_j held over the period $[(j-1)T, jT],[j = 1,2,----n]$ is given by

$$R_j = \int_{(j-1)T}^{(k+j-1)T} I_j(t)dt = \frac{a}{b\theta(b+\theta)}\left[be^{b(k+j-1)T}\left(1-e^{k\theta}\right)-\theta e^{b(j-1)T}\left(1-e^{bkT}\right)\right] \quad (13) \text{and}$$

$$R_n = \int_{(n-1)T}^{H} I_n(t)dt = \frac{a}{(b+\theta)^2}\left[\left\{e^{(b+\theta)H}\left((b+\theta)\left(H-(n-1)T\right)\right)-1\right\}+e^{(b+\theta)(n-1)T}\right] \quad (14)$$

Let S_j be the total amount of backlogging quantities over the period $\left[(k+j-1)T, jT\right](j=1,2,----,n-1)$.

Then $S_j = \int_{(k+j-1)T}^{jT} (jT-t)ae^{bt}dt = \frac{ae^{bjT}}{b^2}\left[1+e^{b(k-1)T}\left(bT(k-1)-1\right)\right]$ (15)

$j=1,2,--------,(n-1)$.

Therefore, the total cost of the system for the entire time horizon H is

$$C = \sum_{j=1}^{n-1}\left(F_j + C_1R_j + C_3D_j + C_2S_j\right) + C_1R_n + C_3D_n + F_n$$

$$= nA + \sum_{j=1}^{(n-1)}\Big[\ \frac{pa}{(b+\theta)}\left\{e^{T(b(k+j-1)+\theta k)} - e^{bT(j-1)}\right\} +$$

$$\frac{C_1a}{b\theta(b+\theta)}\left\{be^{b(k+j-1)T}\left(1-e^{k\theta}\right)-\theta e^{b(j-1)T}\left(1-e^{bkT}\right)\right\} +$$

$$C_3ae^{bT(j-1)}\left\{\frac{1}{b+\theta}\left(e^{kT(b+\theta)}-1\right)-\frac{1}{b}\left(e^{bkT}-1\right)\right\} + \frac{C_2ae^{bjt}}{b^2}\left\{1+e^{b(k-1)T}\left(bT(k-1)-1\right)\right\}\ \Big]$$

$$+\frac{C_1a}{(b+\theta)^2}\left[\left\{e^{(b+\theta)H}\left((b+\theta)\left(H-(n-1)T\right)\right)-1\right\}+e^{(b+\theta)(n-1)T}\right]$$

$$+C_3\left[\frac{a}{b+\theta}\left\{e^{(b+\theta)H-\theta(n-1)T}-e^{bT(n-1)}\right\}-\frac{a}{b}\left\{e^{bH}-e^{b(n-1)T}\right\}\right]+\frac{pa}{(b+\theta)}\left[e^{(b+\theta)H-\theta(n-1)T}-e^{bT(n-1)}\right] \quad (16)$$

The above cost function $C(n,k)$ is a function of two variables n and k where n is a discrete and k is a continuous variable. Our problem is to determine the values of n and k which minimize C.

For a given value of $n(\geq 1)$, the optimal value of k for minimum total cost is the solution of $dC/dk = 0$ which gives

$$\frac{dC}{dk} = \sum_{j=1}^{(n-1)}\left[\frac{pa}{(b+\theta)}\left\{T(b+\theta)e^{T(b(k+j-1)+k\theta)}\right\}+\frac{C_1a}{b\theta(b+\theta)}\left\{b^2Te^{b(k+j-1)T}\left(1-e^{k\theta}\right)-b\theta e^{k\theta+b(k+j-1)T}+bT\theta e^{bkT+b(j-1)T}\right\}+C_3ae^{bT(j-1)}\left\{\frac{1}{b+\theta}\left((b+\theta)Te^{kT(b+\theta)}\right)-Te^{bkT}\right\}+\frac{C_2Tae^{bjT}}{b}\left\{e^{b(k-1)T}\left(bT(k-1)-1\right)+e^{b(k-1)T}\right\}\right]=0$$

(17)

For a given positive integer $n(\geq 1)$, the equation (17) can be easily solved for $k(0 \leq k \leq 1)$ by any iterative method when the other parameters are prescribed. The corresponding optimal value of C, say $C^*(n)$, can be evaluated from equation (16). Now putting $n = 2,3,4,-----$ one can easily calculate

$C^*(2)$, $C^*(3)$------

For n=1, the system reduces to a single period of finite time horizon without shortage system. In the case, the cost for the period H is independent of k and is given by

$$C^*(1) = \frac{C_1a}{(b+\theta)^2}\left[e^{(b+\theta)H}\left(H(b+\theta)-1\right)+1\right]+C_3\left[\frac{a}{(b+\theta)}\left\{e^{(b+\theta)H}-1\right\}-\frac{a}{b}\left\{e^{bH}-1\right\}\right]+\frac{pa}{(b+\theta)}\left[e^{(b+\theta)H}-1\right]$$

(18)

The minimum value of $C^*(1), C^*(2), C^*(3)$ would be the optimum total cost. The values of n and k for this minimum total cost are then taken as optimal values for n and k, respectively.

Numerical Example:

The proposed model is illustrated with the help of following example.

Example 1:

Let A = 100, p = 0.2, a = 20, b = .01, C_1 = 0.5, C_2 = 1.5, θ = .01, and H = 12 in appropriate units.

Equation (17) is solved for k (0<k<1) by NR method for different integral values of n. Then putting these values for n and k in the equation (16), the corresponding values of C is obtained. Among the total cost for various values of n, the minimum cost is 1392.00. The corresponding values of n, k and T are

n = 8, k = 0.2310 and T = 1.5

Concluding Remarks: In this paper, we have developed the inventory model for deteriorating items with exponential time dependent demand (increasing) and shortages over a finite time horizon. The replenishment cost is taken to be dependent on the lot-size of the current replenishment. One can ask why the replenishment cost is linear function of lot-size, the answer is, usually the big merchants purchase the items in terms of round lots i.e. rail wagon, full trucks etc. But, in small towns retailers purchase the lots depending upon the capital available to them. After that they joined together to hire a truck/ small transporting vehicle to transport the items to the selling place. Studying this scenario, we have formulated this problem. Further this present model is applicable for seasonal products like seasonal fruits, Cold drinks, ice cream etc. as the demand of these products increases rapidly for a fixed time horizon i.e. during the season.

A similar analysis can be done for a generalized f(t). This model can be generalized by extending more realistic situations, such as, multi-items and quantity discount policies.

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