A NEW METHOD FOR FINDING THE SOLUTION OF FUZZY PROJECT CRASHING PROBLEMS WITH DIFFERENT TYPE OF FUZZY PARAMETERS

PARMPREET KAUR

Abstract : Several authors have proposed different methods for solving fuzzy project crashing problems by representing the fuzzy activity times as fuzzy numbers. In this paper, a fuzzy project crashing problem, which cannot be solved by using any of the existing methods, is chosen and a new method is proposed to solve the same.

Keywords: Fuzzy project crashing problems, Linear programming, *L-R* fuzzy numbers.

1. Introduction : Management of complex projects that consist of a large numbers of interrelated activities poses problems involved in planning, scheduling, and control, especially when the project activities have to be performed in a specified technological sequence. With the help of the program evaluation and review technique (PERT) and the critical path method (CPM), the project manager can schedule project activities at appropriate times to conform with proper job sequences so that the project is completed as soon as possible.

In real-world applications, the time required to complete the various activities in a research and development project may be known only approximately due to insufficient information. To deal quantitatively with imprecise information, the concepts and techniques of probability could be employed. However, probability distribution requires a priori predictable regularity or a posteriori frequency determination to construct. It is a possible occurrence that a project can assume an activity it has never performed before. As an alternative, uncertain values can be represented by fuzzy sets [10].

The main advantages of methodologies based on fuzzy theory are that they do not require prior predictable regularities or posterior frequency distributions, and they can deal with imprecise input information containing feelings and emotions quantified based on the decision-makers subjective judgment. For finding the fuzzy critical path and to solve fuzzy project crashing problems [1-7, 9, 11] several approaches are proposed over the past years.

In this paper, a new method is proposed by modifying an existing method [6] to solve such fuzzy project crashing problems in which there is need to represent some or all the fuzzy activity times by different types of *L-R* fuzzy numbers.

2. Linear programming formulation of project crashing problems in fuzzy environment

In this section, linear programming formulation of project crashing problems in fuzzy environment is presented. Representing the activity times a_{ij} of project crashing problems by same type of *L-R* type fuzzy numbers $\tilde{a}_{ij}$, a project crashing problem of crashing the project, within the given additional budget, can be formulated into the following FLP (Fuzzy Linear Programming) problem:

Minimize $(y_n - y_1)$

subject to

$$y_j \ominus y_i \succcurlyeq \tilde{a}_{ij} \ominus t_{ij} \quad \forall \ (i,j) \in A \qquad (2.1)$$

$$\sum_{j:(i,j)\in A} C_{ij}\, t_{ij} \leq B \qquad (2.2)$$

$$t_{ij} \leq T_{ij} \qquad (2.3)$$

$t_{ij} \geq 0$, y_j is a real number.

where, A: Set of all activities (i,j),

a_{ij}: Time duration of the activity (i,j),

N: Set of nodes, n: Destination node,

y_j: Time of the event occurring corresponding to the node j,

t_{ij}: Time by which duration of activity (i,j) will be crashed,

T_{ij}: Maximum allowable crash time,

C_{ij}: Unit crash cost for t_{ij},

B: Additional budget for crashing the project.

By associating the dual variables x_{ij}, X and X_{ij} with the constraints (2.1), (2.2) and (2.3) respectively, the dual linear programming formulation of the project crashing problem, in fuzzy environment, can be written as follows:

Maximize

$$[\sum_{j:(i,j)\in A} \tilde{a}_{ij}\ x_{ij} \ominus BX \ominus \sum_{j:(i,j)\in A} T_{ij}\ X_{ij}]$$

Subject to

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = \begin{cases}1 & i=1\\ -1 & i=n\\ 0 & i\in N-\{1,n\}\end{cases}$$

$-x_{ij} + C_{ij}X + X_{ij} \geq 0$, $x_{ij} = 0$ or 1,

$X,\ X_{ij} \geq 0\ \forall\ (i,j) \in A$.

3. Applicability of the existing method

Liu [6] proposed a new method to find the solution of fuzzy project crashing problems. The existing method [6] can be used only to solve a particular type fuzzy project crashing problems in which all the fuzzy activity times are represented by same type of $L-R$ fuzzy numbers i.e., the existing method can be used only to solve the following type of fuzzy project crashing problems:

Minimize $(y_n - y_1)$

subject to

$$y_j \ominus y_i \succcurlyeq \tilde{a}_{ij} \ominus t_{ij} \quad \forall\ (i,j) \in A$$

$$\sum_{j:(i,j)\in A} C_{ij}\, t_{ij} \leq B$$

$t_{ij} \leq T_{ij}$, $t_{ij} \geq 0$, y_j is a real number.

where, $\tilde{a}_{ij} = (m_{ij}, \alpha_{ij}, \beta_{ij})_{L-R}$

Example 3.1: The existing fuzzy project crashing problem [6] can be formulated into the following FLP problem:

Minimize $(y_7 - y_1)$

subject to

$y_2 \ominus y_1 \succcurlyeq ((42,55,58)_{L-R}) \ominus t_{12}$,
$y_3 \ominus y_1 \succcurlyeq ((52,55,65)_{L-R}) \ominus t_{13}$,
$y_4 \ominus y_2 \succcurlyeq ((55,70,75)_{L-R}) \ominus t_{24}$,
$y_5 \ominus y_2 \succcurlyeq ((90,100,112)_{L-R}) \ominus t_{25}$,
$y_4 \ominus y_3 \succcurlyeq ((62,65,75)_{L-R}) \ominus t_{34}$,
$y_6 \ominus y_3 \succcurlyeq ((85,95,103)_{L-R}) \ominus t_{36}$,
$y_5 \ominus y_4 \succcurlyeq ((88,95,105)_{L-R}) \ominus t_{45}$,
$y_6 \ominus y_4 \succcurlyeq ((107,115,120)_{L-R}) \ominus t_{46}$,
$y_7 \ominus y_5 \succcurlyeq ((113,120,140)_{L-R}) \ominus t_{57}$,
$y_7 \ominus y_6 \succcurlyeq ((95,100,105)_{L-R}) \ominus t_{67}$,

$2000\, t_{12} + 1200\, t_{13} + 4000\, t_{24} + 1500\, t_{25} + 1000 t_{34} + 1000\, t_{36} + 2000\, t_{45} + 2000\, t_{46} + 2000\, t_{57} + 2000\, t_{67} \leq 160000$,

$t_{12} \leq 3$, $t_{13} \leq 17$, $t_{24} \leq 10$, $t_{25} \leq 12$, $t_{34} \leq 20$, $t_{36} \leq 21$, $t_{45} \leq 11$, $t_{46} \leq 10$, $t_{57} \leq 18$, $t_{67} \leq 10$,

$t_{ij} \geq 0$ for all $(i,j) \in A$,

y_j is a real number.

Where, $L(x) = R(x) = \text{maximum}\{0, 1-x\}$

4. Limitation of the existing method

In this section, the limitation of the existing method [6] is discussed.

Several authors [2, 3, 7, 11] have pointed out that in real life problems, it is not possible to represent the different uncertain parameters by same type of fuzzy numbers. There exist fuzzy project crashing problems in which there is need to represent all or some of the fuzzy activity times, fuzzy crash cost, maximum allowable fuzzy crash time and additional fuzzy budget by different types of *L-R* fuzzy numbers. But it is not possible to find the solution of such fuzzy project crashing problems by using the existing method [6] i.e., the existing method [6] cannot be used to find the solution of the following type of fuzzy project crashing problems:

Maximize

$$[\sum_{j:(i,j)\in A} \tilde{a}_{ij}\ x_{ij} \ominus \tilde{B}X \ominus \sum_{j:(i,j)\in A} \tilde{T}_{ij}\ X_{ij}]$$

Subject to

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = \begin{cases}1 & i=1\\ -1 & i=n\\ 0 & i\in N-\{1,n\}\end{cases}$$

$(\ominus\, x_{ij} \oplus \tilde{C}_{ij}X \oplus X_{ij}) \succcurlyeq 0$, $x_{ij} = 0$ or 1,

$X,\ X_{ij} \geq 0\ \forall\ (i,j) \in A$.

where, $\tilde{a}_{ij} = (m_{ij}, \alpha_{ij}, \beta_{ij})_{L_{ij}-R_{ij}}$ or
$\tilde{a}_{ij} = \left(m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}\right)_{L_{ij}-R_{ij}}$,
$\tilde{C}_{ij} = (m'_{ij}, \alpha'_{ij}, \beta'_{ij})_{L_{ij}-R_{ij}}$ or
$\tilde{C}_{ij} = \left(m'_{ij}, n'_{ij}, \alpha'_{ij}, \beta'_{ij}\right)_{L_{ij}-R_{ij}}$,
$\tilde{T}_{ij} = (m''_{ij}, \alpha''_{ij}, \beta''_{ij})_{L_{ij}-R_{ij}}$ or
$\tilde{T}_{ij} = \left(m''_{ij}, n''_{ij}, \alpha''_{ij}, \beta''_{ij}\right)_{L_{ij}-R_{ij}}$,
$\tilde{B} = (m''', \alpha''', \beta''')_{L-R}$ or
$\tilde{B} = (m''', n''', \alpha''', \beta''')_{L-R}$

To show the limitation of the existing method a fuzzy project crashing problem, which cannot be solved by using the existing method [6], is chosen in Example 4.1.

Example 4.1

Consider a project whose corresponding network is given in Figure 1. Suppose the project manager needs to crash the project within the additional total fuzzy budget Rs. $(150000, 170000, 30000, 30000)_{L-R}$,

Where, $L(x) = R(x) = \text{maximum}\{0, 1-x\}$ The fuzzy activity times $(\tilde{a}_{ij})$, the unit fuzzy crash cost $(\tilde{C}_{ij})$ and maximum allowable fuzzy crash time $(\tilde{T}_{ij})$ for activity $(i,j) \in A$, are shown in Table 1, Table 2 and Table 3.

Table 1

The fuzzy activity times $\tilde{a}_{ij}$, left shape function $L_{ij}(x)$, right shape function $R_{ij}(x)$ for the project crashing

Activ-ity (i,j)	$\tilde{a}_{ij} = (m_{ij}, n_{ij,} \alpha_{ij}, \beta_{ij})_{L_{ij}-R_{ij}}$	$L_{ij}(x)$	$R_{ij}(x)$
(1, 2)	$(50, 52, 5, 6)_{L_{12}-R_{12}}$	$\max\{o, 1-x^2\}$	$\max\{o, 1-x\}$
(1, 3)	$(55, 60, 3, 5)_{L_{13}-R_{13}}$	e^{-x}	$\max\{o, 1-x\}$
(2, 4)	$(60, 70, 5, 5)_{L_{24}-R_{24}}$	$\max\{o, 1-x^4\}$	$\max\{o, 1-x^2\}$
(2, 5)	$(95,100,5,12)_{L_{25}-R_{25}}$	e^{-x^2}	$\max\{o, 1-x^2\}$
(3, 4)	$(65, 69, 3, 6)_{L_{34}-R_{34}}$	$\max\{o, 1-x\}$	e^{-x}
(3, 6)	$(93, 98, 8, 5)_{L_{36}-R_{36}}$	$\max\{o, 1-x\}$	$\max\{o, 1-x\}$
(4, 5)	$(95,100,7,5)_{L_{45}-R_{45}}$	$\max\{o, 1-x^2\}$	e^{-x}
(4, 6)	$(110, 115,3,5)_{L_{46}-R_{46}}$	$\max\{o, 1-x\}$	e^{-x^2}
(5, 7)	$(120,130,7,10)_{L_{57}-R_{57}}$	$\max\{o, 1-x^2\}$	$\max\{o, 1-x^2\}$
(6, 7)	$(97, 103,2,2)_{L_{67}-R_{67}}$	$\max\{o, 1-x\}$	$\max\{o, 1-x^4\}$

Table 2

The fuzzy crash cost $\tilde{C}_{ij}$, left shape function $L_{ij}(x)$, right shape function $R_{ij}(x)$ for the project crashing

Activ-ity (i,j)	$\tilde{C}_{ij} = (m'_{ij}, n'_{ij,} \alpha'_{ij}, \beta'_{ij})_{L_{ij}-R_{ij}}$	$L_{ij}(x)$	$R_{ij}(x)$
(1, 2)	$(1500, 2500, 500, 500)_{L_{12}-R_{12}}$	$\max\{o, 1-x\}$	$\max\{o, 1-x^2\}$
(1, 3)	$(1200, 1400, 600, 200)_{L_{13}-R_{13}}$	e^{-x}	$\max\{o, 1-x\}$
(2, 4)	$(3500, 4500, 1500,1500)_{L_{24}-R_{24}}$	e^{-x}	$\max\{o, 1-x^2\}$
(2, 5)	$(1000,2000, 500,500)_{L_{25}-R_{25}}$	$\max\{o, 1-x^4\}$	$\max\{o, 1-x\}$
(3, 4)	$(800,1200, 400, 400)_{L_{34}-R_{34}}$	$\max\{o, 1-x\}$	e^{-x}
(3, 6)	$(700,1100, 200, 600)_{L_{36}-R_{36}}$	$\max\{o, 1-x^2\}$	$\max\{o, 1-x\}$
(4, 5)	$(1500,2500, 500,500)_{L_{45}-R_{45}}$	e^{-x}	$\max\{o, 1-x^2\}$
(4, 6)	$(2000,2000, 500,500)_{L_{46}-R_{46}}$	$\max\{o, 1-x\}$	$\max\{o, 1-x^2\}$
(5, 7)	$1700,2400, 400,200)_{L_{57}-R_{57}}$	$\max\{o, 1-x^4\}$	$\max\{o, 1-x\}$
(6, 7)	$(2000, 2000, 500,500)_{L_{67}-R_{67}}$	$\max\{o, 1-x\}$	$\max\{o, 1-x^2\}$

Table 3

The maximum allowable fuzzy crash time $\tilde{T}_{ij}$, left shape function $L_{ij}(x)$, right shape function $R_{ij}(x)$ for the project crashing

Activ-ity (i,j)	$\tilde{T}_{ij} = (m''_{ij}, n''_{ij,} \alpha''_{ij}, \beta''_{ij})_{L_{ij}-R_{ij}}$	$L_{ij}(x)$	$R_{ij}(x)$
(1, 2)	$(3, 3,2,2)_{L_{12}-R_{12}}$	$\max\{o, 1-x^2\}$	$\max\{o, 1-x\}$
(1, 3)	$(14, 20, 4, 4)_{L_{13}-R_{13}}$	$\max\{o, 1-x\}$	$\max\{o, 1-x^4\}$
(2, 4)	$(8, 12, 4, 4)_{L_{24}-R_{24}}$	e^{-x}	$\max\{o, 1-x^2\}$
(2, 5)	$(10,14, 1,1)_{L_{25}-R_{25}}$	$\max\{o, 1-x^4\}$	$\max\{o, 1-x\}$
(3, 4)	$(15, 25, 5, 5)_{L_{34}-R_{34}}$	$\max\{o, 1-x\}$	e^{-x}
(3, 6)	$(16, 26, 5, 5)_{L_{36}-R_{36}}$	$\max\{o, 1-x^2\}$	$\max\{o, 1-x^4\}$
(4, 5)	$(8,12, 4, 8)_{L_{45}-R_{45}}$	e^{-x}	$\max\{o, 1-x\}$
(4, 6)	$(7, 11, 2, 6)_{L_{46}-R_{46}}$	$\max\{o, 1-x^4\}$	$\max\{o, 1-x^2\}$
(5, 7)	$(20,20, 10, 2)_{L_{57}-R_{57}}$	e^{-x}	$\max\{o, 1-x\}$
(6, 7)	$(8, 12, 4, 4)_{L_{67}-R_{67}}$	$\max\{o, 1-x^2\}$	e^{-x}

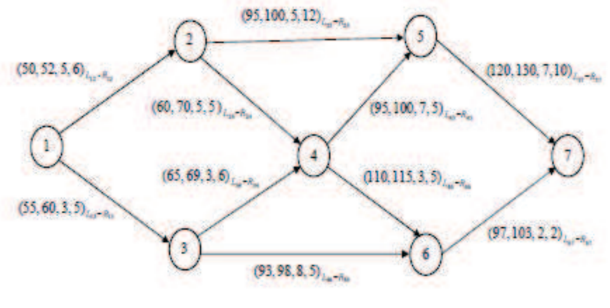

Figure 1. Project network with fuzzy activity times

5. Proposed method

In this section, to overcome the limitation of the existing method, pointed out in Section 4, a new method is proposed by modifying an existing method [6] to find the optimal solution of fuzzy project crashing problems. The fuzzy crash time for the project can be obtained by using the following steps of proposed method:

Step 1

Find the fuzzy critical path and total initial fuzzy completion time of the project by using the following steps of the existing method [3].

Step (1a)

Represent all the fuzzy activity times $\tilde{a}_{ij}$ by different types of *L-R* fuzzy numbers i.e., $\tilde{a}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{L_{ij-R_{ij}}}$ and use the Yager's ranking approach, to find the Yager's ranking index $I(\tilde{a}_{ij})$ for all$(i,j) \in A$.

Step (1b)

Formulate the chosen fuzzy critical path problem into the following FLP problem:

$$\text{Maximize} \sum_{j:(i,j)\in A} \widetilde{(a_{ij})}\ x_{ij}$$

Subject to

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = \begin{cases} 1 & i = 1 \\ -1 & i = n \\ 0 & i \in N - \{1, n\} \end{cases}$$

x_{ij} = 0 or 1 $\forall\ (i,j) \in A$.

Step (1c)

Convert the FLP problem, obtained in Step (1b), into the following CLP(Crisp Linear Programming) problem:

$$\text{Maximize} \sum_{j:(i,j)\in A} I\widetilde{(a_{ij})}\ x_{ij}$$

Subject to

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = \begin{cases} 1 & i = 1 \\ -1 & i = n \\ 0 & i \in N - \{1, n\} \end{cases}$$

x_{ij} = 0 or 1 $\forall\ (i,j) \in A$.

Step (1d)

Solve CLP problem, obtained in Step (1c), to find the fuzzy critical path and the optimal value, representing the Yager's ranking index corresponding to total initial fuzzy completion time of the project.

Step 2

Formulate the chosen fuzzy project crashing problem into the following FLP problem:

Maximize

$$[\sum_{j:(i,j)\in A} \tilde{a}_{ij}\ x_{ij} \ominus \tilde{B}X \ominus \sum_{j:(i,j)\in A} \tilde{T}_{ij}\ X_{ij}]$$

Subject to

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = \begin{cases} 1 & i = 1 \\ -1 & i = n \\ 0 & i \in N - \{1, n\} \end{cases}$$

$(\ominus\, x_{ij}\ \oplus \tilde{C}_{ij}X \oplus X_{ij}) \succcurlyeq 0$, x_{ij} = 0 or 1,

$X,\ X_{ij}\ \geq 0\ \forall\ (i,j)\ \in A$.

Step 3

Suppose the chosen fuzzy project crashing problem, obtained in Step 2, have h feasible solutions and $\{x_{ij,}^{w} X_{ij}^{w}, X^{w}\}$ is the w^{th} feasible solution then the aim is to be find the feasible solution with the largest objective value i.e., $\underset{1\le w\le h}{\text{maximum}}\ [\sum_{j:(i,j)\in A} \tilde{a}_{ij}\ x_{ij} \ominus \tilde{B}X \ominus \sum_{j:(i,j)\in A} \tilde{T}_{ij}\ X_{ij}]$

Although, till now many ways has been defined to compare fuzzy numbers but in the existing method [6] it is assumed that if

$$\underset{1\le w\le h}{\text{maximum}} \left\{ \sum_{j:(i,j)\in A} I(\tilde{a}_{ij}) x_{ij}^{w} - I(\tilde{B}) X^{w} - \sum_{j:(i,j)\in A} I(\tilde{T}_{ij}) X_{ij}^{w} \right\}$$

is $\sum_{j:(i,j)\in A} I(\tilde{a}_{ij}) x_{ij}^{\eta} - I(\tilde{B}) X^{\eta} - \sum_{j:(i,j)\in A} I(\tilde{T}_{ij}) X_{ij}^{\eta}$ then $\underset{1\le w\le h}{\text{maximum}}\ [\sum_{j:(i,j)\in A} \tilde{a}_{ij}\ x_{ij} \ominus \tilde{B}X \ominus \sum_{j:(i,j)\in A} \tilde{T}_{ij}\ X_{ij}]$ will also be

$$\sum_{j:(i,j)\in A} \tilde{a}_{ij}\ x_{ij} \ominus \tilde{B}X \ominus \sum_{j:(i,j)\in A} \tilde{T}_{ij}\ X_{ij}$$

where, $I(\tilde{A})$ represents the Yager ranking index [8] of an *L-R* flat fuzzy number $\tilde{A}$.

In other words, the existing method [6] have assumed that the optimal solution of the fuzzy project crashing problem can be obtained by solving the following crisp linear programming problem:

Example	Yager ranking index corresponding to total fuzzy completion time (per unit time)			
	Existing method		Proposed method	
	Without Additional budget	With Additional budget	Without Additional budget	With Additional budget
3.1	342.5	302	342.5	302
4.1	Not applicable	Not applicable	350.17	291.22

$$\text{Maximize}\left[\sum_{j:(i,j)\in A} I(\tilde{a}_{ij})x_{ij} - I(\tilde{B})X \;-\; \sum_{j:(i,j)\in A} I(\tilde{T}_{ij})X_{ij}\right]$$

Subject to

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = \begin{cases} 1 & i=1 \\ -1 & i=n \\ 0 & i\in N-\{1,n\} \end{cases}$$

$(-x_{ij} + I(\tilde{C}_{ij})X + X_{ij}) \geq 0$, x_{ij} = 0 or 1,

$X,\ X_{ij} \geq 0\ \forall\ (i,j) \in A$.

Step 4

Solve CLP problem, obtained in Step 3, to find the Yager ranking index corresponding to fuzzy completion time and the fuzzy critical path of the project with additional fuzzy budget.

6. Advantage of proposed method over existing method

In this section, advantage of proposed method over existing method [6] is discussed.

The main advantage of proposed method over existing method [6] is that proposed method can be used to solve both type of fuzzy project crashing problems i.e., fuzzy project crashing problems in which all the fuzzy activity times, fuzzy crash cost, maximum allowable fuzzy crash time and additional fuzzy budget, are represented by either the same type of *L-R* fuzzy numbers or some or all the fuzzy activity times, fuzzy crash cost, maximum allowable fuzzy crash time and additional fuzzy budget are represented by different types of *L-R* fuzzy numbers.

To show the advantage of proposed method over the existing method the results of fuzzy project crashing problem, chosen in Example 3.1 [6] and Example 4.1, obtained by using the existing method [6] and proposed method are shown in Table 4. Also the membership function of the *L-R* fuzzy number, representing the total fuzzy completion time of the project network without additional budget and with additional budget, is shown in Figure 2 and Figure 3 respectively.

Table 4: Results of the existing and proposed methods

It is obvious from the results, shown in Table 4, that the value of Yager's ranking index corresponding to total fuzzy completion time with additional budget is 291.22 and the value of Yager's ranking index corresponding to total fuzzy completion time without additional budget is 350.17. Since the difference of both the values is 58.95, so on the basis of this value it can be concluded that by adding the given additional fuzzy budget i.e., Rs. (150000, 170000, 30000, 30000) the total fuzzy completion time of the project can be reduced by approximately 58.95 unit of time.

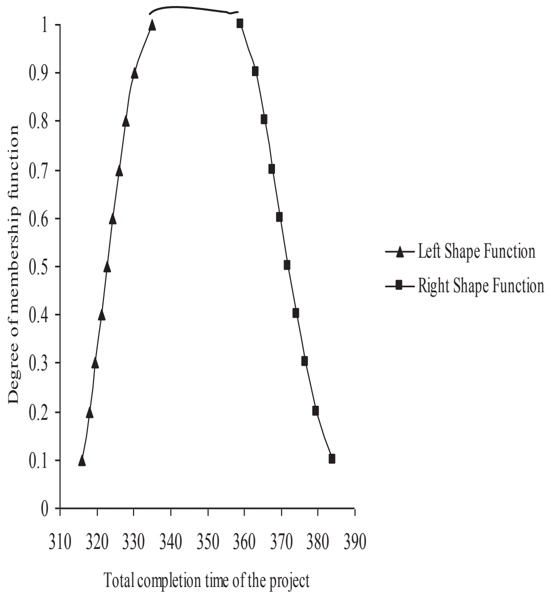

Figure 2. Membership function of *L-R* fuzzy number representing the total maximum completion time of the project without additional budget

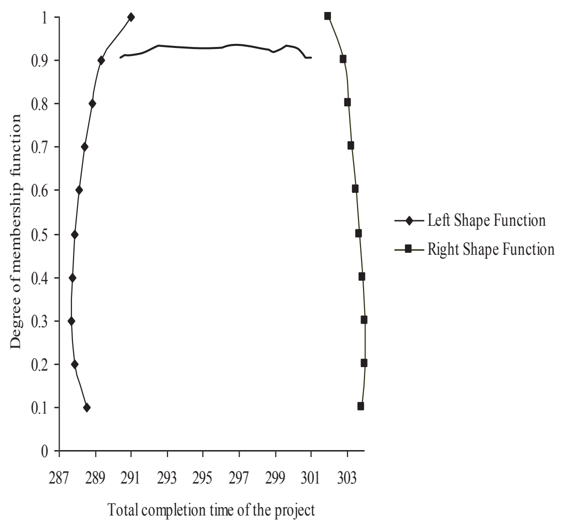

Figure 3. Membership function of *L-R* fuzzy number representing the total maximum completion time of the project with additional budget

7. Conclusion :In this paper, limitation of an existing method for solving fuzzy project crashing problems is discussed and to overcome this limitation a new method is proposed. The advantage of proposed method over existing method is discussed and also to illustrate proposed method a fuzzy project crashing problem is solved. By comparing the results of the existing and proposed method it is shown that it is better to use proposed method instead of existing method.

References:

1. J. Chao-guang, J. Zhuo-shang, L. Yan, Z. Yuan-min, H. Zhen-dong, Research on the fully fuzzy time-cost trade-off based on genetic algorithms, Journal of Marine Science and Application, 4 (2005) 18-23.
2. S.P. Chen, Analysis of critical paths in a project network with fuzzy activity times, European Journal of Operational Research, 183 (2007) 442-459.
3. S.P. Chen, Y.J. Hsueh, A simple approach to fuzzy critical path analysis in project networks, Applied Mathematical Modelling, 32 (2008) 1289-1297.
4. V. Chandrasekar ,B.Sathishkumar, Bounded Solution of Third Order Nonlinear; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 219-224
5. A. Kaufmann, M.M. Gupta, Fuzzy Mathematical Models in Engineering and Management Science, Amsterdam Netherland, Elsevier, 1988.
6. F.T. Lin, Fuzzy crashing problem on project management based on confidence-interval estimates, Proceedings 8th International Conference on Intelligent Systems Design and Applications, 2 (2008) 164-169.
7. *Alveera Khan, A. M. Quraishi, Shrish Joshi, M. Ayaz Ahmad,* Synthesis and Characterization of Chemically Treated Fibre; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 673-676
8. S.T. Liu, Fuzzy activity times in critical path and project crashing problems, Cybernetics and Systems, 34 (2003) 161-172.
9. D.L. Mon, C.H. Cheng, H.C. Lu, Application of fuzzy distributions on project management, Fuzzy Sets and Systems, 73 (1995) 227-234.
10. R.R. Yager, A procedure for ordering fuzzy subsets of the unit interval, Information Sciences, 24 (1981) 143-161.
11. *Kamal Lochan Mahanta , Binay Kumar Bishi,* Kaluza Klein Cosmological Models With Perfect Fluid ; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 659-661
12. J.S. Yao, F.T. Lin, Fuzzy critical path method based on signed distance ranking of fuzzy numbers, IEEE Transactions on Systems, Man and Cybernetics-Part A:SystemsandHumans, 30 (2000) 76-82.
13. L.A. Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.
14. A. Zareei, F. Zaerpour, M. Bagherpour, A.A. Noora, A.H. Vencheh, A new approach for solving fuzzy critical path problemusing analysisofevents,ExpertSystemswithApplications, 38 (2011) 87-93.

Parmpreet Kaur,Assistant Professor, Khalsa College Patiala (Punjab)
Email: parmpreetsidhu@gmail.com