ALMOST REGULAR SPACES INGENERALIZED TOPOLOGY

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Abstract: The purpose of the present paper is intended as a review, discussion and investigations of many more important results on generalized topology and is to obtain some characterizations of almost regular spaces in generalized topology. Moreover the new class of μ-semi regular spaces are discussed in generalized topology.

Keywords: weak θ-continuity, almost regular, almost μ-regular, μ-semi regular, μ-δ closed.

1. Introduction: The notion of continuity is one of the main ideas in mathematics. So much so, that in recent decades there has been the growing trend of speaking of two sorts of mathematics - continuous mathematics and discrete mathematics. In any attempt to generalize these basic objects must also provide a discussion of functions that corresponds to continuity. The literature contains many examples of properties closely related to the notion of continuity of a function in the (ordinary) topology. In many cases the property coincides with continuity if we alter the topology on either the domain or the range or both. Such a type of property is a continuity property and study these properties from this point of view.

In 1922, Blumberg defined what he meant by a real-valued function on Euclidean space being densely approached at a point in its domain. Continuous functions satisfy his condition at each point of their domains. Since then, and particularly in the past three decade, a large number of properties closely related to the notion of continuity of a function have been introduced. The number of properties is so large that different authors have used the same term for different concepts and other authors have resorted to exotic terms, sometimes because the natural term has already been preempted. It turns out that many of these concepts are not new in the sense that if one is willing to change the topology on the domain and / or the range then the class of functions satisfying a particular property often coincides with the class of continuous functions under the new topologies. From thispoint of view many of the results in the literature concerning such functions are essentially restatements in disguise of familiar propertiesof continuous functions. The main purpose of my paper is to make this more precise in (ordinary) topology and in generalized topology.

2. Preliminaries: Throughout this section X and Y denote topological spaces on which no separation axioms are assumed unless explicitly stated. In a subset A of X, the closure of A is the intersection of all closed sets containing A and the interior of A is the union of all open sets contained in A, denoted by cl (A) and int (A) respectively. A subset A of a space X is said to be regular open (resp.regular closed)if int(cl(A))=A (resp. cl(int A)=A). Throughout this paper we use the Bourbaki notation α(A) or α A denote int cl(A).

A point $x \in X$ is called δ-cluster (resp. θ-cluster) point of A if $A \cap \alpha (U) = \varphi$(resp. $A \cap cl (U) \neq \varphi$) for each open set U containing x. The set of all δ-cluster (resp. θ-cluster) points of a set A is called the δ-closure (resp.θ-closure) of A and is denoted by $cl_\delta(A)$ (resp. $cl_\theta(A)$). A subset A is said to be δ-closed (resp. θ-closed) if $cl_\delta(A)=A$ (resp. $cl_\theta(A)=A$).A subset is called δ-open (resp.θ-open) if its complement is δ-closed(resp.θ closed). The family of all δ-open (resp.θ-open) subsets of (X,τ) forms a topologyτ_δ (resp.τ_θ) on X.

We recall that a subset A of X is said to be (i) semi-open ifA $\subset$ cl(int(A)), denoted by so (X) (ii) preopen if A$\subset$ int(cl(A)), denoted by po (X), (iii) α-open if A$\subset$ int(cl(int(A))), denoted by τ^α (iv) β-open if A$\subset$ cl(int(cl(A))), denoted by τ^β.

Since the intersection of two regular open subsets of (X,τ) is regular open, the family of all regular open subsets of (X,τ) forms a base for a smaller topology $\tau_s$on X called the semi-regularization of τ. It turns out that $\tau_s=\tau_\delta$, and that $\tau_\theta\subset\tau_s\subset\tau$ for any topological space (X,τ). We recall that a space X is said to be semi regular, if for every closed set F and a point $x \notin F$, there exist disjoint semiopen sets A and B such that $x \in A$ and $F \subset B$. The space (X,τ) is semi-regular if and only if $\tau=\tau_s$.

The clopen subsets of a topological space (X,τ) form a base for a topology on X. This topology is called ultraregularization [16] of τ and is denoted by τ_u. A topological space (X,τ) is said to be ultra regular [12] if $\tau=\tau_u$.

3. Generalized Topology (G.T)

1.Introduction in G.T: In 1963, Levine[14] introduced and investigated the semi open sets and semi continuous functions. In 1987, Bhattacharyya and Lahiri[1] used semi open sets to define and investigate the notion of semi generalized closed sets. The origin for the development in the field of strong generalized topological spaces (X,μ) is N.Levine's work of [14]. In topology weak forms of open sets play an important role in the generalization of various forms of continuity. Using various forms of open sets, many authors have introduced and studied various types of continuity. Generalized topology (X,μ) was first introduced and studied by A.Csaszar [2].

2. Preliminaries in G.T: Let X be a set. A subset μ of exp X is called a generalized topology on X and (X,μ) is called generalized topological spaces [2] (abbr. GTS) if μ has the following properties:

(i)$\varphi \in \mu$,

(ii) Any union of elements of μ belongs to μ.

Generalized topological spaces is an important generalization of topological spaces, and many interesting results have been obtained. Throughout this paper, a space (X,μ) or simply X for short, will always mean a strong generalized topological spaces with strong generalized topology μ unless otherwise explicitly stated. Here, a generalized topology μ is said to be strong [3] if $X\in\mu$.

For the space (X,μ), the elements of μ are called μ-open sets and the complement of μ-open sets are called μ-closed sets. For $A\subseteq X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, that is, the smallest μ- closed set containing A, and by $i_\mu(A)$, the union of all μ-open sets contained in A, that is, the largest μ-open set contained in A. Intensive research on the field of generalized topological space (X,μ) was done in the past ten years as the theory was developed by A. Csaszar [2], A.P.DhanaBalan [9]. It is easy to observe that i_μ and c_μ are idempotent and monotonic, where γ: exp X$\rightarrow$exp X is said to be idempotent if and only if $A\subseteq B\subseteq X$ implies $\gamma(\gamma(A)) = \gamma(A)$ and monotonic if and only if $A\subseteq B\subseteq X$ implies$\gamma(A)\subseteq\gamma(B)$. It is also well known that from [4, 5], that if μ is a generalized topology on X and $A\subseteq X$, $x\in X$ then $x\in c_\mu(A)$ if and only if $x\in M\in\mu \Rightarrow M\cap A \neq \phi$ and $c_\mu(X-A) = X - i_\mu(A)$.

Let $B\subset$expX and $\phi\in B$. Then B is called a base [3] for μ if $\{\cup B' : B'\subset B\}=\mu$.We also say that μ is generated by B. A pointx$\in$X is called a μ-cluster point of B if$U\cap(B-\{x\})\neq \phi$ for each $U\in\mu$ with $x\in U$. The set of all μ-cluster point of Bis denoted by d(B).

This paper is concerned with the adaptation of the change of topology approach from topological topics to aspects of the theory of generalized topological spaces. This shows that "change of generalized topology" exhibits some characteristic analogous to change of topology in the topological category. A general application of the change of generalized topology approach occurs when the spaces are ordinary topological spaces. In this case, the generalized topologies are families of distinguished subsets of a topological space which are not topologies but are generalized topologies. For background material, papers[9, 10, 11] may be perused.The end or omission of proof denoted by■.

Csaszar[7] has pointed out some common examples of generalized topologies that are associated with a given topological space.Consider the collection of all s.o, p.o, β-open, α-open sets in the (ordinary) topology(X,τ). Each collection is a generalized topology on X.In fact, the family of α-open set is a topology. But in general, the other three collections, namely, the family of s.o, p.o and β-open sets are not topologies on X.

Let (X,μ_1) and (Y,μ_2) be two generalized topologies. The functionf: $(X,\mu_1)\rightarrow(Y,\mu_2)$ is said to be generalized continuous or $(\mu_1$-$\mu_2)$ continuous or simply μ-continuous[9] if $f^{-1}(V)\in\mu_1$ for each$V\in\mu_2$.

The collection B is a base of γ and B generates γ if Bis an arbitrary subset of exp X, then the family $\gamma\subset$exp X composed of φ and all sets $M\subset X$ of the form $M = \bigcup_{i\in I} B_i$ where $B_i\in$Band I is a non-empty index set, is a generalized topology on X.It is enough to check generalized continuity for each member of a base for the co-domain generalized topology, just as is the case of continuity of functions between ordinary topological spaces.

We recall that a subset A of (X, μ) is μ-regular open (abbr μ-r.o) if A = $i_\mu c_\mu$(A). A space (X,μ) is said to be μ-semi regular,if for every μ-closed set F and each point $x \notin F$, there exists disjoint μ-semiopen sets A and B such that $x \in A$ and $F \subset B$. A subset A of X is μ-dense in X, if c_μ(A) = X. Clearly, X is μ-dense in X and in fact X is the only μ-closed set dense in (x, μ). A $\subset$ X is nowhere μ-dense if $i_\mu c_\mu(A) = \phi$.

Lemma 3.1[7]: A function f: $(X,\mu_1)\rightarrow(Y,\mu_2)$ is generalized continuous if and only if $f^{-1}(V) \in \mu_1$ for each member V of a baseBof μ_2. ■

Lemma 3.2: Let $A\subset(X,\mu)$. Suppose A is μ-open or μ-dense then

(i) μ-r.o(A,μ/A)={V∩A/V ∈μ-r.o(X,μ)}

(ii) $(\mu/A)_s=\mu_s/A$.

Proof: Proof of (ii) follows immediately from (i), if we prove (i).

Let A be μ-open and let W ∈μ-r.o(A,μ/A).

Then $(\mu/A)c_\mu(W)=c_\mu(W) \cap A$.

Note that α(W) denote $i_\mu c_\mu(W)$.

Hence (μ/A) α(W)=(μα(W)) ∩A.

= V∩ A,

where V=μ-α(W) ∈μ-r.o(X,μ).

Conversely, let V ∈μ-r.o(X,τ) and W=V∩A.

Then (μ/A) α(W)=α(W).

$=i_\mu(c_\mu(V\cap A)\cap A)=i_\mu(c_\mu(V)\cap A)$

=μ-α(V)∩A=V∩A=W.

Hence W ∈μ-r.o(A,μ/A).

Now, let A be μ-dense in X. By property ofchange of topology, the result follows from the proof of theorem 4.1 in [17]. ■

The last result we require is the fact that the process of μ-semi regularizing a generalized topological space is an idempotent operation. This proof depends on the observation that the family of all μ-regularly open subsets of (X,μ_s) coincides with the collection of all μ-regularly open subsets of (X,μ). (By using section 4 of [11]).

We recall that a generalized topological space (X,μ), is called μ-Hausdorff if every two disjoint points in X can be separated by disjoint μ-open sets. Every μ-Hausdorff space is μ-semi Hausdorff but the reverse is not necessarily true.

Lemma 3.3: For any generalized topological space (X,μ), $(\mu_s)_s=\mu_s$. ■

Lemma 3.4: (X,μ) is μ-Hausdorff if and only if (X,μ_s) is μ-Hausdorff. ■

Definition 3.5: Call a property Pof a functions between generalized topological spaces a μ-continuity property if to each pair (X, μ_1) and (Y, μ_2) of generalized topological spaces there correspond new generalized topologies μ_1' on X and μ_2' on Y such that f: $(X, \mu_1) \rightarrow (Y, \mu_2)$ has property Pif and only iff: $(X, \mu_1') \rightarrow (Y, \mu_2')$ is continuous. As formulated, the definition allows each of μ_1' andμ_2' to depend on bothμ_1 and μ_2.

Definition3.6: Call a propertyPof functions between generalized topological spaces an i-continuity (resp.φ-continuity, μ-continuity) property in generalized topology if there is a function αwhich assigns to each generalized topology μ a generalized topology $\alpha(\mu)$on the same underlying set such that

f: $(X, \mu_1) \rightarrow (Y, \mu_2)$ has property Pif and only iff:

$(X, \alpha(\mu_1)) \rightarrow (Y, \mu_2)$

[resp.f:$(X, \mu_1) \rightarrow (Y,\alpha(\mu_2))$;

f: $(X,\alpha(\mu_1))\rightarrow (Y,\alpha(\mu_2))$] is continuous.

Loosely speaking, a μ-continuity property arises through imposing the wrong generalized topology on the domain and/or the range (cf.[15] 2 by change of topology) whereas a non-continuity property is something quite new. That is outside the category of generalized topological spaces and μ-continuous functions. Many properties of functions related to μ continuity turn out to be equivalent to demanding that the inverse image of a certain class of sets be μ-open. Such properties are φ-continuity properties, in generalized topology the following theorem shows:

Theorem 3.7: Let Pbe a property of functions between the generalized topological spaces and suppose that β is a function which assigns to each generalized topology μ on a set Y a family $\beta(\mu)$of subsets of Y such that f: $(X,\mu_1) \rightarrow (Y,\mu_2)$ has property P if and only if$f^{-1}(V) \in \mu_1$ for all $V \in \beta(\mu_2)$. Then P is aφ-continuity property in generalized topology.

Proof: For any generalized topology μ_2 on Y, let $\alpha(\mu_2)$be that topology having β(μ)as a sub-basis. Then it is well known that

$f^{-1}(V) \in \mu_1$for all $V \in \beta(\mu_2)$if and only if

f: $(X,\mu_1) \rightarrow (Y,\alpha(\mu))$ is continuous, so Pis aφ-continuity property in the generalized topology Type equation here.

The usual situation has $\beta(\mu) \subset \mu$, in which case the propertyPwill be weaker than μ-continuity itself.

Lemma 3.8[13]: Let X = {a, b, c}, Y = {a, b} and define f, g, h: X →Y by f(a) = f(b) = a, f(c) = b, g(a) = b, g(b) = g(c) = a; h(a) = h(c) = b, h(b) = a. Then for any topologies on X and Y for which the function f and g are continuous, the function h is also continuous.

Its proof follows from:

$h^{-1}(a) = f^{-1}(a) \cap g^{-1}(a)$ and

$h^{-1}(b) = f^{-1}(b) \cup g^{-1}(b)$ ■

Definition 3.9[]: A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called weakly θ-continuous if for each point x ∈ X and each neighbourhood V of f(x) there is an open neighbourhood U of x such that f(α(U)) ⊂ cl V.

4. Almost Regularity in Generalized Topology

Cammaroto and Noiri[8] point out with their examples 3.2 and 3.3 that the class of weakly θ-continuous functions lies strictly between the class of θ-continuous functions and the class of weakly continuous functions. In [13] we observe that there are many properties closely related to the notion of continuity which in fact coincide with continuity if the topology on either the domain or the range or both is changed. Such a property is called a continuity property of functions in topological space. Such properties satisfied in generalized topological space with generalized topology μ. Here we discuss some properties which are both in (ordinary) topology and in generalized topology. Similar argument follows if we replace μ-continuity in the place of continuity.

Lemma 4.1[18]: weak θ-continuity is not a continuity property.

Proof: Lemma(2) of [8]. It is shown in the proof of theorem(2) of [8] that f and g are θ-continuous and hence they are weakly θ-continuous, where h is not weakly continuous and hence not weakly θ-continuous. So the result follows from lemma (2) of [8]. ■

Remark 4.2: Weak continuity is not a continuity property. Recall that a subset A of X is regular open if A = int cl (A) if $\tau_s = \tau$.

Definition 4.3: A space X is semiregular [18] if for each open set G and x ∈ G, there exists a regular open set H such that x ∈ H ⊂ G.

Example: Let X = {a, b, c, d} with topology τ = {φ, X, {a, b}, {a, b, c}} and Y = X with the topology σ = {φ, Y, {a, b}, {a, b, d}}.

Define f: X → Y is the identity function.

Then f is neither continuous nor open.

Lemma 4.4: For any topological space (X,τ), $(\tau_s)_s = \tau_s$. ■

Lemma 4.5: (X,τ) is Hausdorff if and only if (X, τ_s) is Hausdorff. ■

Result 4.6: Any regular space is semi-regular.

Definition 4.7: A space (X,τ) is almost regular if for each τ-regular closed subset A of X and each point x ∈ X -A there are disjoint τ-open set U and V such that A ⊂ U and x ∈ V.

Theorem 4.8 [15]: Regularity is not a semi-regular property, but almost regularity is .

Theorem 4.9: For a subset A of a topological space X, the following hold:

(i) A is semi-open then s-cl(A) is semi-regular.

(ii) A is semi-regular set then it is semi θ-open. ■

The family of all μ-regular open sets constitute a base for a generalized topology μ_s on X. This topology μ_s is known as the semi regularization of μ. Note that $\mu_s \subset \mu$ and θ-open in generalized topology ⊂ δ -open in generalized topology ⊂ generalized topology.

We recall that a subset A of the space (X,μ) is μ-θ-open in X, if for all x ∈ A, there exists an open set U such that x ∈ U ⊂ c_μ(A) ⊂ A.

Definition 4.10: The space (X,μ) is said to be μ-semi regular if $\mu_s = \mu$.

Notation: $i_\mu c_\mu(A)$ is denoted by $\alpha(A)$ or αA

Definition 4.11: Let f: $(X,\mu_1) \rightarrow (Y,\mu_2)$ be a function. Then

(i) f is said to be μ-δ continuous if for each x ∈ X and each μ-open neighbourhood f(x) there is an μ-open neighbourhood U of x such that f($\alpha\mu_2$) ⊂ αV.

(ii) f is said to be almost μ-continuous if for each x ∈ X and each μ-open neighbourhood U of x such that

f(U) ⊂ αV.

Definition 4.12: A function f is said to μ-δ closed if f: $(X,\mu_{1s}) \rightarrow (Y,\mu_{2s})$ is closed.f is μ-regularly open whenever U is μ-regularly open.

Result 4.13: μ-θ-open set in generalized topology ⊂ μ-δ-open set in generalized topology ⊂ μ-open in generalized topology.

Definition 4.14: (X, μ) is almost μ-regular if for each μ-regular closed subset A of X and each point x ∉ A there are disjoint μ_s-open sets U and V such that A ⊂ U and x ∈ V.

Lemma 4.15: If A and B are non-empty disjoint μ-open sets in (X, μ), then $\mu\alpha$(A) and $\mu\alpha$(B) are disjoint μ-open sets in (X, μ_s) containing A and B respectively. ■

Theorem 4.16:(X, μ) is almost μ-regular if and only if (X, μ_s) is μ-regular.

Proof:Step 1: Suppose (X, μ) is almost μ-regular.Let C be aμ_s-closed subset of X and x $\notin$C. Let I be an indexed set. We have

C = $\bigcap_{i\in I} C_i$, where C_i is aμ-regularly closed set for each i. Hence there is some i$\in$Isuch that x$\notin$ C_j.But (X, μ) is almost μ-regular. Therefore, there exists disjoint μ-open sets U and V such that C $\subset C_j \subset$ U and x$\in$V. By the Lemma 4.15, there are disjoint μ_s-open sets U′ and V′ such that C$\subset$U $\subset$U′ and

x$\in$ V $\subset$V′. Thus, (X, μ_s) is μ-regular.

Step 2: Suppose that (X, μ_s) is μ-regular. Let C be aμ-regularly closed set and x $\notin$C. But (X, μ_s) isμ-regular. Consequently, there are disjoint μ_s-open sets U and V such that C$\subset$U and x$\in$ V. But $\mu_s \subset \mu$.Hence U, V$\in$ μ.

Hence (X, μ) is almost μ-regular.■

Theorem 4.17:(X, μ) is almost μ-regular if and only if for each x$\in$X and each μ-regular open neighbourhood U of x, there exists aμ-regularly open set V such that x$\in$V$\subset c_\mu(V) \subset$U.

Proof: Step1: Suppose that X is an almost μ-regular space. Let x$\in$X and let U be μ-regular open set containing x. But then X-U is μ-regularly closed set with x $\notin$ X-U. But X is almost μ-regular. Hence there are disjointμ-open sets V and W such that x$\in$V and X-U$\subset$W. But then V$\subset$X-W and X-W$\subset$U.Therefore, x$\in$V$\subset$ $c_\mu(V) \subset$ c_μ(X-W).Accordingly, x$\in$V$\subset c_\mu(V) \subset$U.

Step 2: Suppose that for each x$\in$X and for each μ-regular open neighbourhood U of x, there is aμ-regular open set V such thatx$\in$V$\subset c_\mu(V) \subset$ U.Let F be a μ-regular closed set in X and let x$\notin$ F.Then there exists a μ-regularopenset V such that

x$\in$V$\subset c_\mu(V) \subset$X-F.

$\Rightarrow$x$\in$V and F$\subset$X-$c_\mu(V)$.

$\Rightarrow$the sets V and X-$c_\mu(V)$ are μ-regular open sets with V$\subset c_\mu(V)$.

$\Rightarrow$V and X-V are disjoint.

$\Rightarrow$X is almostμ-regular ■

Theorem 4.18: Let X and Y be non-empty generalized topological spaces. The product XxY is almostμ-regular if and only if both X and Y are almostμ-regular.

Proof:Step 1: Suppose XxY is almostμ-regular. Let $x_o \in$X and let U be any μ-open set of x_o. Let y_o $\in$Y. Then UxY is a μ-regular open set of (x_o,y_o). But XxY is almostμ-regular. Hence there exists a μ-regular open set W in XxY such that (x_o,y_o)$\in$W $\subset c_\mu(W) \subset$UxY. Let V_1xV_2 be a basis μ-regular open subset ofXxY Such that (x_o,y_o)$\in V_1$x$V_2 \subset$W.

$\Rightarrow$(x_o,y_o)$\in V_1$xV_2 $\subset c_\mu(V_1)$ x $c_\mu(V_2)$ $\subset$UxY.$\Rightarrow x_o \in$ $V_1 \subset c_\mu(V_1) \subset$U.

$\Rightarrow$X is almost μ-regular.

Similarly we can show that Y is almost μ-regular.

Step 2: Suppose that X and Y are almost μ-regular.Let (x_o,y_o)$\in$XxY.Let W be a μ-regularly open neighbourhood of (x_o,y_o).Then there exists basic μ-regularly open set U_1 and V_1 such that x_o $\in U_1 \subset c_\mu(U_1) \subset$U and

$y_o \in V_1 \subset c_\mu(V_1)$ $\subset$V.Hence (x_o,y_o) $\in$ U_1x$V_1 \subset c_\mu(U_1)$ x $c_\mu(V_1)$ $\subset$UxV$\subset$W.

$\Rightarrow$(x_o,y_o)$\in U_1$x$V_2 \subset c_\mu(U_1$x$V_2) \subset$UxV$\subset$W.$\Rightarrow$XxY is almost μ-regular■

Lemma 4.19: If A and B are subsets of (X,μ), where A and B are μ-regular open and A$\subset$B then c_μ(A) $\subset c_\mu i_\mu c_\mu$(B).

Proof: Let A be a subset of B.

Then A $\subset c_\mu$(B).

$\Rightarrow$A$\subset i_\mu c_\mu$(B), because A is μ-regularopen.$\Rightarrow$ c_μ(A) $\subset c_\mu i_\mu c_\mu$(A). ■

Theorem 4.20: Let f: (X,μ_1)$\rightarrow$(Y,μ_2) be a μ-regular open, μ-δ-continuous, μ-α-closed surjection. If (X, μ) is almost μ-regular then(Y,μ_2) is almost μ-regular.

Proof: Let y $\in$Y. Choose x$\in$Y such that f(x) = y. Let U be a μ-regularly open neighbourhood of y. Since f isμ-δ continuous, it follows that $f^{-1}(U)$ is μ-regularly open. But X is almost μ-regular. Hence there exists a μ-regular open set V in X such that x$\in$V$\subset c_\mu$(V) $\subset$ $f^{-1}(U)$.

$\Rightarrow$y$\in$f(V)$\subset$f(c_μ(V))$\subset$U.

$\Rightarrow$y$\in$f(V)$\subset c_\mu(f$(V)) $\subset$U$\rightarrow$ (1)

Since f is μ-regular open, we have f(V) is μ-regular open.Also f(c_μ(V)) isμ-α-closed, because f is an μ-α-closed function.

Take A = f(V) and B = f(c_μ(V)) in the lemma [4.19] we have c_μ(A)$\subset c_\mu i_\mu c_\mu$(B). That is c_μ(f(V))$\subset c_\mu i_\mu c_\mu$[f($c_\mu$(V))]$\rightarrow$ (2)

Substituting (2) in (1), we obtain

y$\in$f(V) $\subset c_\mu i_\mu c_\mu$[f($c_\mu$(V))] $\subset$U.

$\Rightarrow$y$\in$f(V) $\subset c_\mu$f(V) $\subset$U,because f(c_μ(V)) $\subset c_\mu$f(V).

$\Rightarrow$Y is almost μ-regular. ■

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