AN INTUTIONISTIC MULTIDIMENSIONAL ANALYSIS OF PREFERENCE IN VENDOR SELECTION PROBLEM

PRABJOT KAUR

Abstract : A vendor or a supplier is a supply chain management term meaning anyone who provides goods or services to a company or individuals. Vendor selection is a continuous and never-ending process for acquiring the necessary raw materials and components to support the final output of business organizations. The main objective of vendor selection is to choose the highly potential vendors through which all the goals of the organization can be achieved. Vendor selection problem is a multi-criteria decision making (MCDM) problem involving multiple criteria that can be both qualitative and quantitative. Many quantitative methods such as AHP, Topsis, SAW, LINMAP have proposed to solve the vendor selection problem. LINMAP can deal with problems in a fuzzy and crisp environment. In our paper a linear programming technique for multidimensional analysis of preference (LINMAP) under intuitionistic fuzzy environment in evaluating and selecting a vendor is proposed. A numerical example illustrates our methodology.

Keywords: Vendor Selection, LINMAP, Intuitionistic Fuzzy Sets(IFS),Multiattribute Decision Making (MADM)

Introduction : Vendor selection is a key element in purchasing and one of the major activities of most of professionals in buying firms. Buying firms select their appropriate vendors based on **Dickson's (1966)** criteria like cost, quality, delivery, service etc and a proper method of selection like mathematical programming(**Weber & Current(1993)**), data envelopment analysis(**Baker and Talluri (1997)**), AHP(**Narasimhan (1983)**), etc.This paper focuses on intuitionistic fuzzy multi-dimensional analysis of preference in vendor selection problem.The linear programming technique for multidimensional analysis of preference (LINMAP) was pioneered by **Srinivasan and Shocker (1973)** .The LINMAP is based on pair-wise comparisons of alternatives(vendors) given by the decision maker and generates the best compromise alternative as the solution that has the shortest distance to the PIS.In the LINMAP, all the decision data are known precisely or given as crisp values.However, under many conditions, crisp data are inadequate or insufficient to model real-life decision problems . Indeed, human judgments including preference information are vague or fuzzy in nature and as such it may not be appropriate to represent them by accurate numerical values. The single degree of freedom tells us nothing about the lack of knowledge . In real applications, however, the information A possible solution is to use the Atanassov's intuitionistic fuzzy (IF) set introduced by **Atanassov (1986, 1999)**, which is a generalization of the fuzzy set. Atanassov's IF sets seem to be better suited for expressing a very important factor which should be taken into account when trying to construct really adequate models and solutions of decision making problems, namely hesitation of the decision maker.There are very few articles regarding LINMAP in IF's environment.**Deng-Feng Li** [3] developed the IFS-LINMAP method for solving multi attribute decision making problems.**Li,Chen and Huang(2010)** developed a LP method for multiattribute group decision making using IF sets.This paper is a new application of IFS-LINMAP in solving vendor selection problem. .The organization of the paper is as follows: Section one deals with introduction to the problem and relevant literature review. Section two deals with basic definitions of intutionistic fuzzy sets and distance measure. Section three the methodology starting with construction of intutionistic fuzzy sets and LINMAP used for the study.Section four covers the case study under IFS LINMAP.Section five gives conclusions of the study.

Preliminaries : Intuitionistic fuzzy set theory is an extension of fuzzy set theory introduced by **Atanassov (1986)**, which is a suitable way to deal with vagueness.

Definition 1: An intuitionistic fuzzy set (IFS , for short) A on a universe U is defined as an object of the following form: A={(x, μ_A(x),v_A(x))/x ε U} where the functions μ_A:U→[0,1] and v_A :U→[0,1] define the degree of membership and the degree of non membership of the elements x ε U in A, respectively, and for every x ε U : $0 \le \mu_A(x) + v_A(x) \le 1$.

Definition 2: The value of $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ represents the degree of hesitation(or uncertainty) associated with the membership of elements xεU in IFS A. We call it intutionistic fuzzy index of A with respect of element u.

Definition3: Distance between two intuitionistic fuzzy sets is defined as :

$$\mathbf{d(A,B)} = \sqrt{\frac{1}{2}\sum_{l=1}^{n}[(\mu_A(x_l) - \mu_B(x_l))^2 + ((\vartheta_A(x_l) - v_B(x_l))^2 + ((\pi_A(x_l) - \pi_B(x_l))^2]}$$

Methodology

3.1 CONSTRUCTION OF IFS FOR QUALITATIVE ATTRIBUTES

Assume that a_{ij} be a value of alternative A_j ε A on qualitative attributes x_i ε X. The formulae for relative degrees of membership and relative degrees of non-membership are chosen as follows:

$$\mu_{ij} = \begin{cases} \alpha_i \frac{a_{ij}}{a_i^{max}} \ (i\ \varepsilon F^1) \\ \delta_i \frac{a_i^{min}}{a_{ij}} \ (i\ \varepsilon\ F^2) \end{cases} \quad \ldots(1)$$

$$v_{ij} = \begin{cases} \beta_i \frac{a_{ij}}{a_i^{max}} \ (i\ \varepsilon\ F^1) \\ \gamma_i \frac{a_i^{min}}{a_{ij}} \ (i\ \varepsilon\ F^2) \end{cases} \quad \ldots(2)$$

Respectively, where F^1 and F^2 are the set of benefit qualitative attributes and cost quantitative attributes respectively and

$$a_i^{max} = \max\{a_{ij}\}, 1 \le j \le n ,$$

$$a_i^{min} = \min\{a_{ij}\}, 1 \le j \le n$$

and α_i ε[0,1] , β_i ε [0,1], δ_i ε[0,1] γ_i ε[0,1] satisfying conditions $0 \le \alpha_i + \beta_i \le 1$ and $0 \le \delta_i + \gamma_i \le 1$.Values of parameters α_i , β_i , δ_i , γ_i are chosen a prior by the decision maker according to characteristics and needs in real- life situations.

3.2 The LINMAP model under IFS environment [3]

LINMAP is a method on the concept of pairwise comparison of alternatives given by decision makers and then generates the best compromise alternative as the solution that has the shortest distance to the positive ideal solution (**Ross (1995)**).LINMAP can deal with deterministic data only but fuzziness is inherent in real life situation problems. Besides fuzzy LINMAP,LINMAP has been extended for solving MADM problems under Atanassov's IF environments. In this methodology, Atanassov's IF sets are used to describe fuzziness in decision information and decision making processes by means of an Atanassov's IF decision matrix (3).The algorithm of steps of LINMAP is as follows:The decision maker identifies the m alternatives with n attributes for evaluation. Let the prference relation between alternatives be denoted by Ω={*(k, j)* |*Ak* preferred *Aj* , *(k, j* = 1, 2, . . . , *n)*}.Construct the IFS decision matrix of alternatives and attributes by converting crisp data to iFS form. The following LPP model is constructed which is as follows

:

$$\text{Max}\left\{\sum_{(k,j)\in\Omega} \lambda_{kj}\right\}$$

$$\sum_{i=1}^{m}\omega_i \sum_{(k,j)\in\Omega}[(\mu_{ij}^2 - \mu_{ik}^2) + \nu_{ij}^2 - \nu_{ik}^2) + (\pi_{ij}^2 - \pi_{ik}^2) + 2(\mu_{ij} - \mu_{ik}) + 2(\nu_{ij} - \nu_{ik})] - \sum_{i=1}^{m} u_i \sum_{(k,j)\in\Omega} [4(\mu_{ij} - \mu_{ik}) + 2(v_{ij} - v_{ik})] - \sum_{i=1}^{m} v_i \sum_{(k,j)\in\Omega}[2(\mu_{ij} - \mu_{ik}) + 4(v_{ij} - v_{ik})] \ge 2h$$

$$\sum_{i=1}^{m}\omega_i\ [\mu_{ik}^2 - \mu_{ij}^2) + (\nu_{ik}^2 - \nu_{ij}^2) + (\pi_{ik}^2 - \pi_{ij}^2) + 2(\mu_{ik} - \mu_{ij}) + 2(\nu_{ik} - \nu_{ij})] - \sum_{i=1}^{m} u_i\ [4(\mu_{ik} - \mu_{ij}) + 2(v_{ik} - v_{ij})] - \sum_{i=1}^{m} v_i\ [2(\mu_{ik} - \mu_{ij}) + 4(v_{ik} - v_{ij})] + 2\lambda_{kj} \ge 0\ ((k,j) \in \Omega))$$

$\lambda_{kj} \ge 0\ ((k,j) \in \Omega))$

$u_i \ge 0, v_i \ge 0$ (i=1,2,...,m)

$u_i + v_i \le w_i$ (i=1,2,...,m)

$w_i \ge \varepsilon$ (i=1,2,...,m)

$\sum_{i=1}^{m} \omega_i$ ...(3)

where:

$$\begin{cases} u_i = \omega_i \mu_i^+ \\ v_i = \omega_i v_i^+ \end{cases} \dots . (4)$$

$F_i^+ = \{\langle \mu_i^+, v_i^+ \rangle\}$ $(i = 1,2,3, \dots \dots \dots \dots . m)$ is an intuitionistic fuzzy set on attribute x_i. Therefore we obtain $0 \leq \mu_i^+ + v_i^+ \leq 1$ $(i = 1,2,3, \dots \dots \dots \dots . m)$.

Thus the following inequality is obtained:

$$u_i + v_i \leq \omega_i \ (i = 1,2,3, \dots \dots \dots \dots . m)$$

ω_i, u_i *and* $v_i (i = 1,2,3, \dots \dots \dots \dots . m)$ can be obtained by solving the above linear programming using the Simplex method. Then, the best value of $\mu_i^+ and\ v_i^+$ $(i = 1,2,3, \dots \dots \dots \dots . m)$ are calculated using the above relation (4) and are denoted as the Atanassov's IF set.

$F_i = \{\langle \mu_i^+, v_i^+ \rangle\}$ $(i = 1,2,3, \dots \dots \dots \dots . m)$ and the IFPIS $F^+ = (F_1^+, F_2^+ \dots \dots \dots F_m^+)$, i.e., $F^+ = (\langle \mu_1^+, v_1^+ \rangle, \langle \mu_2^+, v_2^+ \rangle \dots \dots \dots \dots . \langle \mu_m^+, v_m^+ \rangle)$. The ranking order of the alternative set $A = \{A_1, A_2, \dots . A_n\}$ is generated based on the increasing order of distances $S_j (j = 1, ,2, \dots \dots n)$.

Numerical Example

Assume that the management of a JIT manufacturer decides to choose their best suppliers and assign their optimum order quantities to maximize the TVP[1]. The main criteria for supplier selection are cost, quality and service. According to the corporate strategies the quality includes defects and process capability while service involves on-time delivery, response to changes and process flexibility. Four suppliers are included in the evaluation process and their cost, quality, On time delivery and capacities are presented in Table 1. The demand is 1000 units and the maximum acceptable defect rate is 0.02.

Table 1: The crisp data for the various suppliers

Supplier	Cost	Quality	one time delivery	Capacity
A_1	30	.03	.95	400
A_2	40	.05	.98	700
A_3	50	.01	.85	600
A_4	45	.06	.92	500

Step-1: The relative degrees of membership and relative degrees of non-membership (Table 2) for table 1 are calculated using equations 1 and 2.

Table 2: The intuitionistic fuzzy data for various suppliers

Supplier	Cost	Quality	one time delivery	Capacity
A_1	(.65,.30)	(.425,.025)	(.85,.1)	(.42,.11)
A_2	(.48,.22)	(.75,.041)	(.85,.1)	(.75,.2)
A_3	(.39,.18)	(.84,.008)	(.76,.089)	(.64,.17)
A_4	(.42,.19)	(.85,.05)	(.82,.096)	(.53,.14)

The preferences provided by the decision maker based on his discretion are as follows

Ω = {(1,2), (2,4), (1,3)}.

Step 2: The fuzzy decision matrix after transformation into Atanassov's IF decision matrix F is as follows:

$$\begin{matrix} x_1 & \langle 0.65, 0.30 \rangle & \langle 0.425, 0.025 \rangle & \langle 0.85, 0.1 \rangle & \langle 0.42, 0.11 \rangle \\ x_2 & \langle 0.48, 0.22 \rangle & \langle 0.75.0.041 \rangle & \langle 0.85, 0.1 \rangle & \langle 0.75,\ \ 0.2 \rangle \\ x_3 & \langle 0.39, 0.18 \rangle & \langle 0.84, 0.008 \rangle & \langle 0.76, 0.089 \rangle & \langle 0.64, 0.17 \rangle \\ x_4 & \langle 0.42, 0.19 \rangle & \langle 0.85, 0.05 \rangle & \langle 0.82, 0.096 \rangle & \langle 0.53, 0.14 \rangle \end{matrix}$$

Step 3: Using equation (4) we construct the linear programming problem.

$$-0.3625\omega_1 + 0.801\omega_2 + 0.693\omega_3 + 0.55\omega_4 + 0.45u_1 - 1.14u_2 - 1.135u_3 - 0.876u_4 + 0.81v_1 - 0.36v_2 - 0.258v_3 - 0.222v_4 \geq 2$$

$$-1.346\omega_1 + 1.221\omega_2 + 2.173\omega_3 + 1.65\omega_4 + 1.9u_1 - 1.862u_2 - 2.591u_3 - 2.316u_4 + 2.36v_1 - 0.184v_2 - 0.682v_3 - 0.822v_4 \geq 0$$

$$-0.2765\omega_1 + 1.116\omega_2 + 0.287\omega_3 - 0.115\omega_4 + 0.3u_1 - 1.458u_2 - 0.659u_3 - 0.224u_4 + 0.48v_1 - 0.996v_2 - 0.506v_3 + 0.058v_4 \geq 0$$

$$-0.1425\omega_1 + 1.667\omega_2 + 1.489\omega_3 + 1.486\omega_4 + 0.05u_1 - 2.38u_2 - 2.433u_3 - 2.288u_4 + 1.21v_1 - 0.62v_2 - 0.634v_3 - 0.646v_4 \geq 0$$

$$-0.51\omega_1 + 0.21\omega_2 + 0.74\omega_3 + 0.48\omega_4 + 0.725u_1 - 0.361u_2 - 0.728u_3 - 0.72u_4 + 0.775v_1 + 0.088v_2 - 0.212v_3 - 0.3v_4 \geq 0$$

$$0.0428\omega_1 + 0.157\omega_2 - 0.203\omega_3 - 0.332\omega_4 - 0.075u_1 - 0.159u_2 + 0.238u_3 + 0.55u_4 - 0.165v_1 - 0.318v_2 - 0.124v_3 + 0.14v_4 \geq 0$$

$$0.11\omega_1 + 0.433\omega_2 + 0.398\omega_3 + 0.468\omega_4 - 0.2u_1 - 0.62u_2 - 0.649u_3 - 0.706u_4 + 0.2v_1 - 0.13v_2 - 0.188v_3 - 0.212v_4 \geq 0$$

$$u_i \geq 0, v_i \geq 0 \ \ (i = 1,2, \dots 4)$$

$$u_i + v_i \leq \omega_i \ (i = 1,2, \dots 4)$$

$$\omega_i \geq \varepsilon \ (i = 1,2, \dots 4)$$

$$\sum_{i=1}^{4} \omega_i = 1 \qquad \dots(5)$$

Step 4: Taking h = 1 and ε = 0.01. Solving Eq.5 using the existing Simplex method software, the optimal solutions can be obtained as follows

$$\omega = (\omega_1, \omega_2, \omega_3, \omega_4)^T = (0.01,0.97,0.01,0.01)$$

$$u = (u_1, u_2, u_3, u_4)^T = (0.00,0.00,0.00,0.03)$$

$$v = (v_1, v_2, v_3, v_4)^T = (0.97,0.00,0.00,0.0)$$

$$S_1 = 0.5(\omega_1((\mu_{13} - \mu_1^+)^2 + (\upsilon_{13} - \upsilon_1^+)^2 + (\pi_{13} - \pi_1^+)^2) + \omega_2((\mu_{13} - \mu_2^+)^2 + (\upsilon_{23} - \upsilon_2^+)^2 + (\pi_{23} - \pi_2^+)^2) + \omega_3((\mu_{33} - \mu_3^+)^2 + (\upsilon_{33} - \upsilon_3^+)^2 + (\pi_{33} - \pi_3^+)^2) + \omega_4((\mu_{43} - \mu_4^+)^2 + (\upsilon_{43} - \upsilon_4^+)^2 + (\pi_{43} - \pi_4^+)^2)$$

$$S_2 = 0.5(\omega_1((\mu_{14} - \mu_1^+)^2 + (\upsilon_{14} - \upsilon_1^+)^2 + (\pi_{14} - \pi_1^+)^2) + \omega_2((\mu_{24} - \mu_2^+)^2 + (\upsilon_{24} - \upsilon_2^+)^2 + (\pi_{24} - \pi_2^+)^2) + \omega_1((\mu_{34} - \mu_3^+)^2 + (\upsilon_{34} - \upsilon_3^+)^2 + (\pi_{34} - \pi_3^+)^2) + \omega_1((\mu_{44} - \mu_4^+)^2 + (\upsilon_{44} - \upsilon_4^+)^2 + (\pi_{44} - \pi_4^+)^2)$$

$$S_3 = 0.5(\omega_1((\mu_{14} - \mu_1^+)^2 + (\upsilon_{14} - \upsilon_1^+)^2 + (\pi_{14} - \pi_1^+)^2) + \omega_2((\mu_{24} - \mu_2^+)^2 + (\upsilon_{24} - \upsilon_2^+)^2 + (\pi_{24} - \pi_2^+)^2) + \omega_1((\mu_{34} - \mu_3^+)^2 + (\upsilon_{34} - \upsilon_3^+)^2 + (\pi_{34} - \pi_3^+)^2) + \omega_1((\mu_{44} - \mu_4^+)^2 + (\upsilon_{44} - \upsilon_4^+)^2 + (\pi_{44} - \pi_4^+)^2)$$

Step 5: According to the increasing order of the distances *Sj (j* = 1, 2,*3,4)*,the best alternative from the alternative set *A* is determined and the ranking order of all alternatives is generated.

S1=.9312

S2=.9397

S3=.9347

S4=.9382

S1>S3>S4>S2

Conclusions : In this paper of MADM–IFS approach for selection of vendor we considered a linear dimensional analysis of preference for selection of right vendor. We observe that vendor 1 has the smallest values among vendors 2,3,4.The lower value of the distance for an alternative indicates that the alternative is closer to the Atanassov's IFPIS.also vendor 1 has a low cost and high quality wich make it all the more suitable for selection and placing an order for the company.

References

1. Ghodsypour,S.H. and Brien,C.O.(1998).A DSS for supplier selection using an integrated AHP and Linear Programming, International J. of Production Economics, 56-57,199-212.
2. Susheel Kumar, Kirandeep Bala,Pradeep Kumar Pandey, Logarithmic Order of Entire Monogenic Functions; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 175-179
3. Atanassov,K.(1986). Intuitionistic fuzzy sets.Fuzzy Sets and Systems,79,403-405.

4. Deng-Feng Li (2008).Extension of the LINMAP for multiattribute decision-making under Atanassov's intuitionistic fuzzy environment, Fuzzy Optimization Decision Making ,7,17–34.
5. Ross,T.J.(1995) .Fuzzy logic with engineering applications .New York:McGraw-Hill.
6. *M. Vidya Bhalchandra,* Estimation of Log X Based on Regression Technique; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 606-609
7. Atanassov, K. T. (1999). *Intuitionistic Fuzzy Sets*. Heidelberg: Springer-Verlag.
8. Deng-Feng Li , Guo-Hong Chen, Zhi-Gang Huang(2010).Linear programming method for multiattribute group decision making using IF sets,Information Sciences 180 (2010) 1591–1609.
9. Narsimhan R. (1983). An analytical approach to supplier selection, J Purch Mater Manag, Winter, pp. 27–32. (1983)
10. *H. M. Nasir,* A New Class of Multicomplex Algebra With Applications; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 169-174
11. Baker, R.C., Talluri, S., (1997). A closer look at the use of DEA for technology selection. Computers and Industrial Engineering 32 (1), 101–108.
12. Weber, C.A. and Current, J.R. (1993).A multiobjective approach to vendor selection, European J. of Operational Research, vol 68, 173-184.
13. V. Srinivasan, A.D. Shocker(1973).Linear programming techniques for multidimensional analysis of preference, Psychometrica, 38 , 337–342.
14. *S. Karthikeyan , S.Rama,* Wave Propagation in Magnetopyroelectric Plates; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 600-605
15. Dickson, G.W. (1966).An analysis of vendor selection systems and decisions, *J. of Purchasing*, vol 2(1), 5-17.

Prabjot Kaur*
Department of Mathematics, Birla Institute of Technology,
Mesra , Ranchi, Jharkhand, India
Email of corresponding author:tinderbox_fuzzy@yahoo.com