CONTRA qI- CONTINUOUS FUNCTIONS IN IDEAL BITOPOLOGICAL SPACES

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Abstract: In this paper, we apply the notion of qI-open sets and qI-continuous functions to present and study a new class of functions called contra qI-continuous functions in ideal bitopological spaces.

Keywords: Ideal bitopological space, qI-open sets, contra qI-continuous functions, qI -irresolute functions.

Preliminaries : In 1961 Kelly [5] introduced the concept of bitopological spaces as an extension of topological spaces. A bitopological space (X, τ_1, τ_2) is a nonempty set X equipped with two topologies τ_1 and τ_2 [5]. The study of quasi open sets in bitopological spaces was initiated by Dutta [1] in 1971. In a bitopological space (X, τ_1, τ_2) a set A of X is said to be quasi open [1] if it is a union of a τ_1-open set and a τ_2-open set. Complement of a quasi open set is termed quasi closed. Every τ_1-open (resp. τ_2-open) set is quasi open but the converse may not be true. Any union of quasi open sets of X is quasi open in X. The intersection of all quasi closed sets which contains A is called quasi closure of A. It is denoted by qcl(A) [7]. The union of quasi open subsets of A is called quasi interior of A. It is denoted by qInt(A) [7].

The concept of ideal topological spaces was initiated Kuratowski [6] and Vaidyanathaswamy [8]. An Ideal I on a topological space (X, τ) is a non empty collection of subsets of X which satisfies: i)A $\in$ I and B $\subset$ A $\Rightarrow$ B $\in$ I and ii)A $\in$I and B $\in$ I $\Rightarrow$ A $\cup$ B $\in$ I If $\mathcal{P}$ (X) is the set of all subsets of X, in a topological space (X, τ) a set operator $(.)^*$:$\mathcal{P}$ (X) $\rightarrow$ $\mathcal{P}$ (X) called the local function [3] of A with respect to τ and I and is defined as follows:

$$A^*(\tau, I) = \{x\in X \mid U \cap A \notin I, \forall U\in \tau(x)\}, \text{ where } \tau(x) = U\in \tau \mid x\in U\}.$$

Given an ideal bitopological space (X,τ_1,τ_2I) the quasi local function [3] of A with respect to τ_1, τ_2 and I denoted by A^*_q (τ_1,τ_2, I) (in short A^*_q) is defined as follows:

$$A^*_q(\tau_1,\tau_2, I) = \{x\in X \mid U \cap A \notin I, \forall \text{ quasi open set U containing } x\}.$$

A subset A of an ideal bitopological space (X, τ_1, τ_2) is said to be qI- open [3] if A $\subset$ qInt A^*_q. A mapping f: (X,τ_1,τ_2I) $\rightarrow$ (Y,σ_1,σ_2) is called qI-continuous [3] if f^{-1}(V) is qI-open in X for every quasi open set V of Y .

In 1996 Dontchev [2] introduced a new class of functions called contra-continuous functions. A function f: X $\rightarrow$ Y to be contra continuous if the pre image of every open set of Y is closed in X.

Recently the author of this paper [4] defined qI-irresolute mappings in ideal bitopological spaces.

Definition1.1. [3] Given an ideal bitopological space (X, τ_1, τ_2 I) the quasi local mapping of A with respect to τ_1, τ_2 and I denoted by A^*_q(τ_1, τ_2, I) (more generally as A^*_q) is defined as follows: A^*_q(τ_1, τ_2 ,I) = {x$\in$X |U $\cap$ A $\notin$ I ,$\forall$ quasi -open set U containing x}

Definition1.2. [3] A subset A of an ideal bitopological space (X, τ_1, τ_2, I) is qI- open if A $\subset$ qInt(A^*_q). Complement of a qI- open set is qI-closed. If the set A is qI-open and qI-closed, then it is called qI-clopen

Definition1.3. [3] A mapping f: (X, τ_1, τ_2, I) $\rightarrow$ (Y, σ_1 ,σ_2) is called a qI- continuous if f^{-1}(V) is a qI-open set in X for every quasi open set V of Y.

Definition1.4. [4] A mapping f: (X,τ_1,τ_2,I) $\rightarrow$ (Y,σ_1,σ_2) is called qI- irresolute if f^{-1}(V) is a qI-open set in X for every quasi semi open set V of Y.

1. Contra qI-continuous functions

Definition 2.1. A function f: (X, τ_1, τ_2, I) $\rightarrow$ (Y, σ_1, σ_2) is called contra qI- continuous if f^{-1} (V) is qI-closed in X for each quasi open set V in Y.

Theorem 2.1. For a function f: (X, τ_1, τ_2, I) $\rightarrow$ (Y, σ_1, σ_2), the following are equivalent:

a) f is contra qI-continuous .

b) For every quasi closed subset F of Y, f^{-1} (F) is qI-open in X.

c) For each x $\in$ X and each quasi closed subset F of Y with f(x) $\in$ F, there exists a qI-open subset U of X with x $\in$ U such that f (U) $\subset$ F.

Proof: (a) ⇒ (b) and (b) ⇒ (c) are obvious.
(c) ⇒ (b) Let F be any quasi closed subset of Y. If $x \in f^{-1}(F)$ then $f(x) \in F$, and there exists a qI-open subset U_x of X with $x \in U_x$ such that $f(U_x) \subset F$. Therefore, $f^{-1}(F) = \cup\{U_x : x \in f^{-1}(F)\}$. Hence we get $f^{-1}(F)$ is qI-open. [3]

Remark 2.1. The concepts of qI -continuity and contra qI-continuity are independent of each other

Theorem 2.2. If a function f: $(X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$ is contra qI-continuous and Y is regular, then f is qI-continuous

Proof: Let $x \in X$ and let V be a quasi open subset of Y with $f(x) \in V$ Since Y is regular, there exists an quasi open set W in Y such that $f(x) \in W \subset Cl(W) \subset V$. (by Theorem 2.1) f is contra qI-continuous, and hence there exists a qI-open set U in X with $x \in U$ such that $f(U) \subseteq Cl(W)$. Then $f(U) \subseteq Cl(W) \subseteq V$. Hence f is qI-continuous [3].

Definition 2.2. A topological space (X, τ_1, τ_2, I) is said to be qI -connected if X is not the union of two disjoint non-empty qI-open subsets of X.

Theorem 2.3. If f: $(X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$ is a contra qI-continuous function from a qI-connected space X onto any space Y , then Y is not a discrete space.

Proof: Suppose that Y is discrete. Let A be a proper non-empty quasi clopen set in Y. Then $f^{-1}(A)$ is a proper non-empty qI- clopen subset of X, which contradicts the fact that X is qI-connected.

Theorem 2.4. A contra qI-continuous image of a qI-connected space is connected.

Proof: Let f: $(X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$ be a contra qI- continuous function from a qI-connected space X onto a space Y . Assume that Y is disconnected. Then $Y = A \cup B$, where A and B are non-empty quasi clopen sets in Y with $A \cap B = \emptyset$. Since f is contra qI-continuous , we have that $f^{-1}(A)$ and $f^{-1}(B)$ are qI-open non-empty sets in X with $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(A \cup B) = f^{-1}(Y) = X$ and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\emptyset) = \emptyset$. This means that X is not semi-I-connected, which is a contradiction. Then Y is connected.

Definition 2.3. A mapping f: $(X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$ is called contra qI- irresolute if $f^{-1}(V)$ is a qI-closed set in X for every quasi semi open set V of Y.

Remark 2.2. Contra qI- irresoluteness and qI -irresoluteness are independent

Definition 2.4. A function f: $(X, \tau_1, \tau_2, I_1) \to (Y, \sigma_1, \sigma_2, I_2)$ is called quasi- irresolute if $f^{-1}(V)$ is qI_1- open in X for each qI_2- open set V of Y.

Definition 2.5. A function f: $(X, \tau_1, \tau_2, I_1) \to (Y, \sigma_1, \sigma_2, I_2)$ is called contra quasi- irresolute if $f^{-1}(V)$ is qI_1- closed in X for each qI_2- open set V of Y.

The following two remarks are evident from the definition:

Remark 2.3. Contra quasi- irresoluteness and quasi- irresoluteness are independent

Remark 2.4. Contra quasi- irresolute function is contra qI-continuous, but the converse is not true.

Theorem 2.5. A function f: f: $(X, \tau_1, \tau_2, I_1) \to (Y, \sigma_1, \sigma_2, I_2)$ is quasi-irresolute if and only if the inverse image of each qI_2-closed set in Y is qI_1-open in X.

Proof: Obvious

Theorem 2.6. Let f: $(X, \tau_1, \tau_2, I_1) \to (Y, \sigma_1, \sigma_2, I_2)$ and g: $(Y, \sigma_1, \sigma_2, I_2) \to (Z, \rho_1, \rho_2, I_3)$ Then,

i. gof is contra quasi- irresolute if g is quasi -irresolute and f is contra quasi- irresolute.
ii. gof is contra quasi- irresolute if g is contra quasi-irresolute and f is quasi -irresolute.

Proof: Obvious

Theorem 2.7. Let f: $(X, \tau_1, \tau_2, I_1) \to (Y, \sigma_1, \sigma_2, I_2)$ and g: $(Y, \sigma_1, \sigma_2, I_2) \to (Z, \rho_1, \rho_2, I_3)$ Then,

i. gof is contra qI- continuous if g is continuous and f is contra qI- continuous.
ii. gof is contra qI- continuous if g is qI-continuous and f is contra quasi- irresolute

Proof: Obvious.

Conclusion : Ideal bitopological spaces is an extension for both ideal topological spaces and bitopological spaces. It has opened new areas of research in topology and in the study of topological concepts via Fuzzy ideals in ideal bitopological spaces. The application of the results obtained would be remarkable in other branches of science too.

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