IMPLEMENTATION OF STOCHASTIC TREND ANALYSIS MODEL FOR PREDICTING THE FERTILIZERS CONSUMPTION

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Abstract : The variability of crops, soils, and pests within crop production fields has led to attempts to understand those variations and to manage crop production accordingly. The rapidly expanding research and development on the management and control of crop production according to in-field variations is reviewed. The various stages and components of this spatially-variable control are classified and presented along with a proposed terminology and notation. Advances in component technologies (sensors, actuators, locators, geostatistics, geographic information systems) and their integration should reduce crop production costs and be environmentally advantageous. Agriculture is back bone of Indian econmoy contribute the major part of grass domestic production in India. More than organic fertilizer, inorganice fertilizer like pottassium, nitrogen, phospate, etc., are consumed in large quantities in recent years. This study considers the consumption of fertlizers at national level from the period 1951-2011. This study is an attempt to compare stochastic trend for fertilizer consumption in India with Stochastic Spencer's 15 point trend.

Keywords: Stochastic, Fertilizers, Consumption, Spencer's 15 point trend, crop production.

Introduction : In India fertilizer consumption measures the quantity of plant nutrients used per unit of arable land. Fertilizer products cover Nitrogenous, Potassium, and Phosphate fertilizers (including ground rock phosphate). Traditional nutrients--animal and plant manures are not included. For the purpose of data dissemination, FAO has adopted the concept of a calendar year. Some countries compile fertilizer data on a calendar year basis, while others are on a split-year basis. A moving average is commonly used with time series data to smooth out short-term fluctuations and highlight longer-term trends or cycles. The threshold between short-term and long-term depends on the application, and the parameters of the moving average will be set accordingly. It is a very simple and flexible statistical method for measuring trend. It consists in

- Fitting a polynomial of power p(<m) to the first m values,
- Using the polynomial to estimate the value in the middle of the range, and
- Repeating the operation with m terms starting from 2^{nd}, 3^{rd}, etc., terms.

Thus, a moving average is characterized by two constants m and p and we write the moving average as [m,p]. This study compares the trend analysis for fertilizer consumption in India using ordinary linear regression trend line and Stochastic Spencer's Trend.

2.0 Methodology : It is obvious that moving average method is quite cumbersome and tedious to apply for large values of p and m and for long series. Literature on graduation provides a number of approximations to moving average. In the following sequences, we shall study briefly one of the most important of these approximations. Aman Malik (2012) studied the demand for inorganic fertilizer in India. Inorganic fertilizer has significantly supported global population growth, it has been estimated that almost half the people on the Earth are currently fed as a result of synthetic nitrogen fertilizer use was stated by Erisman et al. (2008). Tenkorang (2006) made forecasts based on economic optimization assumptions. A forecast analysis worldwide was made by Zhang et al. (2007) to make a review and forecast on fertilizers consumption worldwide in order to provide basal data for the decision-making of fertilizers production and for the environmental impact assessment of fertilizers application.

One of the most widely used trend line for forecasting purpose by the actuarians is Spencer's 15-Point Moving Average. A mean does not just "smooth" the data. A mean is a form of low-pass filter. The effects of the particular filter used should be understood in order to make an appropriate choice. In Spencer's trend we have the difference operator's E, Δ and δ as defined below:

$$E\, y_t = y_{t+1}, \Delta\, y_t = y_{I+1} - y_t$$

and

$$\delta\, y_t = y_{t+1/2} - y_{t-1/2} = (E^{\frac{1}{2}} - E^{-1/2})\, y_t => \delta \equiv (E^{1/2}\text{-}E^{-1/2}) => \delta^2 \equiv (E - 2 + E^{-1})$$

Let us write E $= e^{2i\emptyset}$, then

$$\delta^2 = e^{2i\emptyset} - 2 + e^{-2i\emptyset} = (e^{2i\emptyset} - e^{-i\emptyset})^2 = -4sin^2\emptyset \;\Rightarrow\; \delta^4 = 16sin^4\emptyset \qquad \dots (1)$$

$$\sum_{-k}^{k} y_t = \sum_{-k}^{k} E^t y_{0=\sum_{-k}^{k} e^{2i\emptyset\, t} y_o}$$

$$=[(e^{2ik\emptyset} + e^{-2ik\emptyset})\,(e^{2ik(\emptyset-1)} + e^{-2i\emptyset(k-1)} + \cdots + (e^{2i\emptyset} + e^{-2i\emptyset}) + 1]y_0$$

$$= (1+2\cos 2\emptyset + 2\cos 4\emptyset + \cdots + 2\, cos2k\emptyset)y_0$$

$$= [1+2\sum_{j=1}^{k} cos2j\emptyset]y_t$$

Let s $= \sum_{j=1}^{k} cos2i\emptyset$

$$\therefore \quad s.\sin\emptyset = \sum_{j=1}^{k} \cos 2\, j\, \emptyset\, .\sin\emptyset = \frac{1}{2}\sum_{j=1}^{k}[\sin(2j+1)\emptyset - \sin\,(2j-1)\emptyset] = \frac{1}{2}[\sin(2k+1)\,\emptyset - \sin\emptyset]$$

$$\Rightarrow \qquad S\sum_{j=1}^{k} \cos 2\, j\emptyset = \frac{\sin(2k+1)\emptyset - \sin\emptyset}{2\sin\emptyset}$$

Substituting in above formula we get

$$\sum_{-k}^{k} y_t \;=\; \left[1 + \frac{\sin 2k+1)\,\emptyset - \sin\emptyset}{\sin\emptyset}\right] y_o = \frac{\sin(2k+1)\,\emptyset}{\sin\emptyset} y_0$$

Thus, if m = (2k+1). Then writing $\frac{1}{m}[m]$ for the arithmetic mean of m terms, we get

$$\frac{1}{m}[m]y_0 = \frac{\sin m\emptyset}{m\, sin\emptyset} y_0$$

$$=\frac{1}{m\sin\emptyset}\left[m\sin\emptyset - \frac{m\,(m^2-1)}{3!}\, sin^3\,\emptyset + \frac{m(m^2-1^2)(m^2-3^2)}{5!} sin^5\,\emptyset - \cdots\right] y_0$$

$$= \left[1 - \frac{m^2-1}{3\,!} sin^2\emptyset + \frac{(m^2-1)\,(m^2-3^2)}{5\,!} sin^4\,\emptyset - \cdots\right] y_0$$

$$= \left[1 - \frac{m^2-1}{3\,!}\left(-\frac{\delta^2}{4}\right) + \frac{(m^2-1)(m^2-3^2)}{5\,!}.\frac{\delta^4}{16} - \cdots\right] y_0$$

If the series is approximated by polynomial of third degree so that fourth and higher differences vanish, we get

$$\frac{1}{m}[m]y_0 = \left(1 + \frac{m^2-1}{24}\delta^2\right) y_0$$

Similarly, for the iterated average, we have

$$\frac{1}{m_1 m_2}[\,m_1][\,m_2]\,y_0 = \frac{1}{m_2}[\,m_2].\frac{1}{m_1}[\,m_1]\,y_0$$

$$\frac{1}{m_2}[\,m_2].\left\{y_0 + \frac{{m_1}^2-1}{24}\delta^2\,y_0\right\} = \frac{1}{m_2}[\,m_2]\,y_0 + \frac{m1-1}{24}\delta^2.\frac{1}{m_2}[\,m_2]\,y_0$$

$$= y_0 + \frac{m_2^2-1}{24}\delta^2 y_0 + \frac{m_1^2-1}{24}\delta^2\left\{y_0 + \frac{m_2^2-1}{24}\delta^2 y_0\right\}$$

$$= y_0 + \frac{m_1^2+m_2^2-2}{24}\delta^2 y_0$$

a result, true up to third differences.

In general, we shall get (up to 3rd differences):

$$\frac{1}{m1.m2.mr}[\,m_1][\,m_2]\dots[\,m_r]\,y_0[\,y_0 + \frac{m_1^2+m_2^2+\cdots+m_r^2-r}{24}\delta^2\,y_0]$$

The above results give the simple average and iterated averages in terms of the middle value y_0 and its central differences, and form the basis of Spencer's 15 point which are widely used in actuarial sciences for 'graduating' a series, a process akin to fitting a trend line.

Spencer's 15 point formula requires 15 points to calculate one single trend value and hence the same. In above equation, taking m_1=4, m_2=4, m_3=5 and r=3 we get

$$\frac{1}{4.4.5}[4][4][5]y_0 = \left[1 + \frac{1}{24}(4^2+4^2+5^2-3)\delta^2\right] y_0 = \left(1 + \frac{9}{4}\delta^2\right) y_0$$

Operating inversely, the trend value of y_0 is given by:

$$y_0 = \left(1+\tfrac{9}{4}\delta^2\right)^{-1}\tfrac{1}{80}[4]^2[5]y_0 = \tfrac{1}{80}[4^2][5]\left(1+\tfrac{9}{4}\delta^2\right)^{-1}y_0 \;= \tfrac{1}{80}[4^2][5]\left(1-\tfrac{9}{4}\delta^2\right)y_0$$

(upto 3rd differences)

$$= \tfrac{1}{320}[4^2][5]\{4-9(E-2+E^{-1})\}y_0 = \tfrac{1}{320}[4^2][5]\{-9y_{-1}+22y_0-9y_{-1}\}$$

$$= \tfrac{1}{320}[4^2][5]\{-9,22,-9\}y_0$$

Therefore, add factors involving δ^4 or higher difference (since they are assumed to be zero) without affecting the accuracy of the result. This would help simplify the numerical coefficients to some extent. Adding the factor -3δ^4, we get the corresponding contribution as

$$-3\delta^4 y_0 = -3[E-2+E^{-1}]^2 y_0 = -3[E^2-4+4E^{-1}]y_0$$

$$= -3\left(y_{-2}-4y_{-1}+6y_0-4y_{1+y_2}\right) = [-3,12,-18,12,-3]y_0$$

(1) Simplifies to

$$y_0 = \tfrac{1}{320}[4^2][5]\{-3,12,-9,-18+22,12-9,-3\}$$

$$y_0 = \tfrac{1}{320}[4^2][5]\{-3,3,4,3,-3\}y_0$$

The calculation of the full weights of spencer's15-point formula is exhibited in the table-1.

Table 1 : Stochastic Spencer's 15-point

y_1	y_{-7}	y_{-6}	y_{-5}	y_{-4}	y_{-3}	y_{-2}	y_{-1}	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7
[4]	1	1	1	1											
		1	1	1	1										
			1	1	1	1									
				1	1	1	1								
$[4]^2$	1	2	3	4	3	2	1								
		1	2	3	4	3	2	1							
			1	2	3	4	3	2	1						
				1	2	3	4	3	2	1					
					1	2	3	4	3	2	1				
$[4]^2\{5\}$	1	3	6	10	13	14	13	10	6	3	1				
	-3	-9	-18	-30	-39	-42	-39	-30	-18	-9	-3				
		3	9	18	30	39	42	39	30	18	9	3			
			4	12	24	40	52	56	52	40	24	12	4		
				3	9	18	30	39	42	39	30	18	9	3	
					-3	-9	-18	-30	-39	-42	-39	-30	-18	-9	-3
$[4]^2[5]$ [-3, 3, 4, 3, -3]	-3	-6	-5	3	21	46	87	74	67	46	21	3	-5	-6	-3

The full weights for calculating a single trend value, are given by

$$\tfrac{1}{320}[-3,-6,-5,3,21,46,67,74,67,46,21,3,-5,-6,-3]y_0$$

Here also, it can be easily checked that the weights are symmetric about the mid-value and the sum of weights is unity.

The above table 2 gives the actual consumption of fertilizers in India over the years from 1950 to 2011 collected from the Reserve Bank of India. The predicted values using an ordinary linear trend are also provoided in the table 2.

Table 2 : Consumption of Fertilizers and Predicted Trend Values

(Quantity in lakh Tonnes)

Year	Consumption	trend value	Year	Consumption	trend value
1950-51	0.69	-44.064	1980-81	55.16	80.736
1951-52	0.66	-39.904	1981-82	60.64	84.896
1952-53	0.66	-35.744	1982-83	63.88	89.056
1953-54	1.05	-31.584	1983-84	77.10	93.216
1954-55	1.21	-27.424	1984-85	82.11	97.376
1955-56	1.31	-23.264	1985-86	84.74	101.536
1956-57	1.54	-19.104	1986-87	86.45	105.696
1957-58	1.84	-14.944	1987-88	87.84	109.856
1958-59	2.24	-10.784	1988-89	110.40	114.016
1959-60	3.05	-6.624	1989-90	115.68	118.176
1960-61	2.92	-2.464	1990-91	125.46	122.336
1961-62	3.38	1.696	1991-92	127.28	126.496
1962-63	4.52	5.856	1992-93	121.55	130.656
1963-64	5.43	10.016	1993-94	123.66	134.816
1964-65	7.73	14.176	1994-95	135.64	138.976
1965-66	7.85	18.336	1995-96	138.76	143.136
1966-67	11.01	22.496	1996-97	143.08	147.296
1967-68	15.39	26.656	1997-98	161.88	151.456
1968-69	17.61	30.816	1998-99	167.98	155.616
1969-70	19.82	34.976	1999-00	180.70	159.776
1970-71	21.77	39.136	2000-01	167.02	163.936
1971-72	26.57	43.296	2001-02	173.60	168.096
1972-73	27.68	47.456	2002-03	160.94	172.256
1973-74	28.39	51.616	2003-04	167.99	176.416
1974-75	25.73	55.776	2004-05	183.98	180.576

1975-76	28.94	59.936	2005-06	203.40	184.736
1976-77	34.11	64.096	2006-07	216.51	188.896
1977-78	42.86	68.256	2007-08	225.70	193.056
1978-79	51.17	72.416	2008-09	249.09	197.216
1979-80	52.55	76.576	2009-10	264.86	201.376
			2010-11	281.22	205.536

Applying Spencer's trend is a little copmplicated work than an ordinary linear trend. The calculation has to be carefully examined step by step.

Table 3 : Spencer trend Prediction

Year	Observed	[4]t	y_{-1}	y_0	y_1	Spencer trend
1950-51	0.69					0.00
1951-52	0.66					0.00
1952-53	0.66	3.06				0.00
1953-54	1.05	3.58				0.00
1954-55	1.21	4.23	15.98			0.00
1955-56	1.31	5.11	18.82	115.13		0.00
1956-57	1.54	5.90	22.17	136.39		0.00
1957-58	1.84	6.93	26.61	161.75	602.30	1.88
1958-59	2.24	8.67	31.55	191.34	705.96	2.21
1959-60	3.05	10.05	37.24	227.50	825.91	2.58
1960-61	2.92	11.59	44.18	272.66	969.89	3.03
1961-62	3.38	13.87	51.76	330.28	1157.71	3.62
1962-63	4.52	16.25	62.77	406.69	1418.86	4.43
1963-64	5.43	21.06	76.71	506.32	1787.86	5.59
1964-65	7.73	25.53	94.86	633.24	2288.28	7.15
1965-66	7.85	32.02	120.59	788.79	2937.60	9.18
1966-67	11.01	41.98	151.39	969.98	3718.91	11.62
1967-68	15.39	51.86	189.69	1169.42	4607.15	14.40
1968-69	17.61	63.83	232.26	1378.64	5551.58	17.35
1969-70	19.82	74.59	276.05	1583.34	6467.10	20.21
1970-71	21.77	85.77	320.03	1770.44	7252.19	22.66
1971-72	26.57	95.84	360.61	1935.08	7833.56	24.48
1972-73	27.68	104.41	394.39	2082.97	8249.23	25.78
1973-74	28.39	108.37	419.36	2238.99	8644.26	27.01
1974-75	25.73	110.74	440.69	2431.18	9237.76	28.87
1975-76	28.94	117.17	467.92	2682.97	10209.67	31.91
1976-77	34.11	131.64	516.63	3001.31	11579.69	36.19
1977-78	42.86	157.08	586.58	3367.57	13208.26	41.28
1978-79	51.17	180.69	671.15	3761.21	14915.36	46.61
1979-80	52.55	201.74	759.03	4166.89	16589.35	51.84
1980-81	55.16	219.52	834.18	4576.31	18235.52	56.99
1981-82	60.64	232.23	910.27	4996.02	19896.75	62.18

1982-83	63.88	256.78	992.26	5424.94	21611.65	67.54
1983-84	77.10	283.73	1080.57	5863.47	23331.39	72.91
1984-85	82.11	307.83	1178.74	6312.55	25038.01	78.24
1985-86	84.74	330.40	1263.10	6782.30	26840.12	83.88
1986-87	86.45	341.14	1348.80	7291.56	28903.53	90.32
1987-88	87.84	369.43	1441.34	7837.00	31275.19	97.73
1988-89	110.40	400.37	1550.32	8394.32	33817.58	105.68
1989-90	115.68	439.38	1688.00	8927.85	36246.87	113.27
1990-91	125.46	478.82	1808.54	9393.19	38154.25	119.23
1991-92	127.28	489.97	1906.12	9772.02	39447.03	123.27
1992-93	121.55	497.95	1974.87	10111.72	40350.46	126.10
1993-94	123.66	508.13	2015.66	10457.41	41328.88	129.15
1994-95	135.64	519.61	2066.83	10868.38	42814.42	133.80
1995-96	138.76	541.14	2148.24	11375.00	45069.29	140.84
1996-97	143.08	579.36	2251.81	11940.39	47880.78	149.63
1997-98	161.88	611.70	2385.84	12494.93	50650.55	158.28
1998-99	167.98	653.64	2522.28	12961.81	52813.81	165.04
1999-00	180.70	677.58	2632.22	13303.59	53978.91	168.68
2000-01	167.02	689.30	2702.78	13535.94	54264.57	169.58
2001-02	173.60	682.26	2718.69	13747.97	54207.56	169.40
2002-03	160.94	669.55	2727.62	14049.48	54659.88	170.81
2003-04	167.99	686.51	2754.63	14543.27	56237.54	175.74
2004-05	183.98	716.31	2844.25	15267.98		
2005-06	203.40	771.88	3004.29	16214.67		
2006-07	216.51	829.59	3212.48			
2007-08	225.70	894.70	3452.33			
2008-09	249.09	956.16	3701.32			
2009-10	264.86	1020.87				
2010-11	281.22					

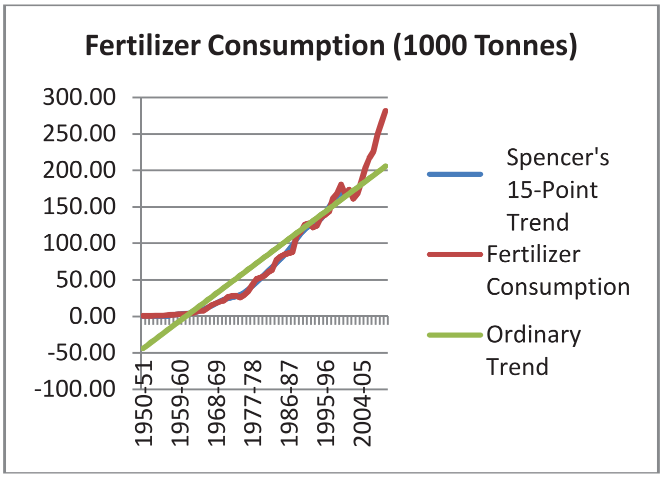

Figure 1 : Comparison of Ordinary Linear Trend and Spencer's Trend

Table 4: Correlation between ordinary value and Spencer trend value

Pearson coefficient	Observed value	Ordinary Trend	Spencer Trend
Observed value	1	0.954	0.998
Ordinary Trend	0.954	1	0.961
Spencer Trend	0.998	0.961	1

The comparison of ordinary linear trend and Spencer's trend is illustrated using figure 1. Further Karl Pearson's correlation analysis was made to know the amount of relationship between original value, ordinary linear trend and Spencer's trend. It is clear that the Spencer's trend is more approipriate than ordinary linear trend and has a correlation value 0.998 while the ordinary linear trend shows much less coefficient of 0.961.

Conclusion : This study finds that predicition of consumption of fertilizers by using ordinary linear trend is not as beneficiary as the Spencer's 15-point trend. Hence this spencer's trend can be applied in an efficient way to predict the time series then any other regression method and cost can be optimized for any consumption work

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