ORDERING OF DECAGONAL FUZZY NUMBERS USING INCENTRE OF CENTROIDS

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Abstract : Ranking fuzzy numbers has become an important process in decision making. In this paper ranking method of decagonal fuzzy numbers based on area is proposed. To find out area given region is divided into nine triangles then centroids are calculated for each triangle. From centroids of triangles centroid of centroids and incentre of centroids are calculated from which ranking function is derived for doing ordering of decagonal fuzzy numbers. A numerical example is given to illustrate the proposed method.

***Keywords*:** Centriod, Decagonal fuzzy numbers, Incentre, Ranking function.

Introduction :A fuzzy number is a generalization of a regular, real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function. A fuzzy number is thus a special case of a convex, normalized fuzzy set of the real line. The various types of fuzzy numbers are triangular fuzzy numbers, trapezoidal fuzzy numbers, hexagonal fuzzy numbers etc. Here we study decagonal fuzzy numbers. The concept of fuzzy set theory was first given by Zadeh [1]. Jain [2] was the first to propose method of ranking fuzzy numbers for decision making in fuzzy situations. Yager [3] used the concept of centroids in ranking of fuzzy numbers. Thorani et al. [4] presented a procedure for ordering fuzzy numbers based on Area, Mode, Spreads and Weights of generalized fuzzy numbers. The area used in this method is obtained from the non normal trapezoidal fuzzy number. Dhanalakshmi and Felbin [6] gave ranking method for octagonal fuzzy numbers based on area between centroid point of an octagonal fuzzy number and the origin, sign distance and deviation. Rajarajeswari and Sudha [7] also introduced a method for ordering fuzzy numbers based on Area, Mode, Divergence, Spreads and Weights of generalized (non normal) hexagonal fuzzy numbers. Sudha [8] gave method for ranking of Fuzzy Numbers using incentre of centroids. In this paper, a method for ranking of decagonal fuzzy number is given which is based on centroid of centroids and incentre of centroids. In decagonal fuzzy number the given area is divided into nine triangles and the centroids of these triangles are calculated. After this centriods of triangles are calculated whose vertices are previous calculated centroids followed by centroid of centroids and incentre of centroids. In Section 2, some basic fuzzy definitions are given. In Section 3, definition of decagonal fuzzy numbers is given. Section 4, includes method for calculating centroid of centroids and incentre of centroids. In Section 5, proposed method is illustrated with the help of numerical example. Finally we conclude in Section 6.

2. Basic definitions

In this section, some basic definitions are presented [5].

Definition 2.1 Let X be universal set, the characteristic function of a crisp set A assigns a value either 0 and 1 to each individual in the universal set, so discriminating between members and non members of the crisp set. This function can be generalized such that the values assigned to the elements of the universal set fall within a specified range and indicate the membership grade of these elements in the set. Such a function is called membership function and the set defined by it a fuzzy set. The membership function of a fuzzy set A is denoted by μ_A; i.e. $\mu_A: X \rightarrow [0, 1]$ and the set $\{(x, \mu_A(x)): x$ lies in X$\}$ is called fuzzy set.

Definition 2.2 A fuzzy set A is called normal if $\text{Sup}\{\mu_A(x) : x \in X\} = 1$ and it is called subnormal if $\text{Sup}\{\mu_A(x) : x \in X\} < 1$.

Definition 2.3 A fuzzy set A in universal set X is called convex iff

$\mu_A(\gamma x_1 + (1-\gamma)x_2) \geq \min[\mu_A(x_1), \mu_A(x_2)]$ For all $x_1, x_2 \in$ X and $\gamma \in [0, 1]$.

Definition 2.4 A fuzzy set A defined on universal set X is said to be a fuzzy number if its membership function has the following characteristics:

i) A is convex.

ii) A is normal.

iii) μ_A is piecewise continuous.

Definition 2.5 A fuzzy number

$A = (m,n,\alpha,\beta)_{LR}$ is said to be an LR flat fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \leq m, \alpha > 0 \\ R\left(\frac{x-n}{\beta}\right), & x \geq n, \beta > 0 \\ 1, & m \leq x \leq n \end{cases}$$

If m = n then $A = (m,n,\alpha,\beta)_{LR}$ will convert into $(m,\alpha,\beta)_{LR}$ and is said to be an LR fuzzy number. L and R are called reference functions which are continuous, non increasing functions that defining the left and right shapes of $\mu_A(x)$ respectively and L(0) = R(0) = 1. Its special cases are triangular, trapezoidal, hexagonal, octagonal and decagonal fuzzy numbers.

3. Decagonal Fuzzy Number

Definition 3.1

A fuzzy number

$A = (m, n, \alpha', \alpha'', \alpha''', \beta, \beta', \beta'', \beta''')_{LR}$ is said to be an LR flat decagonal fuzzy number if its membership function $\mu_A(x)$ is given by

$$\mu_A(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right) & x \leq m, \alpha > 0 \\ L\left(\frac{m-x}{\alpha'}\right) & x \leq m, \alpha' > 0 \\ L\left(\frac{m-x}{\alpha''}\right) & x \leq m, \alpha'' > 0 \\ L\left(\frac{m-x}{\alpha'''}\right) & x \leq m, \alpha''' > 0 \\ L\left(\frac{x-n}{\beta}\right) & x \geq n, \beta > 0 \\ L\left(\frac{x-n}{\beta'}\right) & x \geq n, \beta' > 0 \\ L\left(\frac{x-n}{\beta''}\right) & x \geq n, \beta'' > 0 \\ L\left(\frac{x-n}{\beta'''}\right) & x \geq n, \beta''' > 0 \end{cases}$$

3.1 Ranking of Decagonal Fuzzy Numbers

The ranking of decagonal fuzzy number is defined as

$A > B$ if $R(A) > R(B)$

$A < B$ if $R(A) < R(B)$

$A \approx B$ if $R(A) = R(B)$

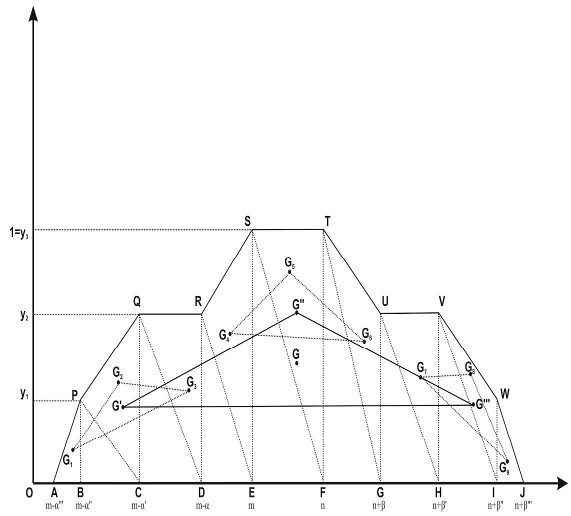
4. Proposed Ranking Method

Find the centriod of given figure, it is considered as balancing point of the given area. For this divide given area into nine triangles namely ΔPAB, ΔCPQ, ΔDQR, Find centriod of each of these triangles namely G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9. Now find centroid of triangles whose vertices are above calculated centroids i.e., find the centroids of centroids

$G' = \left(\frac{9m-(2\alpha+3\,\alpha'+3\alpha''+\alpha''')}{9}, \frac{2y_1+3y_2}{9}\right)$

$G'' = \left(\frac{5n+4m+2\beta-\alpha}{9}, \frac{2y_2+4y_3}{9}\right)$

$G''' = \left(\frac{9n+(\beta+3\,\beta'+4\beta''+\beta''')}{9}, \frac{2y_1+3y_2}{9}\right)$



Here equation of the line $G'G'''$ is $y = \frac{2y_1+3y_2}{9}$ and G'' does not lie on this line so the points G', G'', G''' are non-collinear and therefore they form a triangle. Now

$$\left(\frac{13m+14n-(3\alpha+3\,\alpha'+3\alpha''+\alpha''')+(3\beta+3\,\beta'+4\beta''+\beta''')}{9}, \frac{4y_1+8y_2+4y_3}{9}\right)$$

is centroid of triangle with vertices G', G'', G'''. Let it be G. Now define the incentre I_A of the triangle with vertices G', G'', G''' of decagonal fuzzy number as of this triangle

$$\left(a\left(\frac{9m-(2\alpha+3\,\alpha'+3\alpha''+\alpha''')}{9}\right)+b\left(\frac{5n+4m+2\beta-\alpha}{9}\right)+c\left(\frac{9n+(\beta+3\,\beta'+4\beta''+\beta''')}{9}\right), a\left(\frac{2y_1+3y_2}{9}\right)+b\left(\frac{2y_2+4y_3}{9}\right)+c\left(\frac{2y_1+3y_2}{9}\right)\right)$$

Where

$$a = \sqrt{\frac{(4n-4m+\alpha-\beta+3\beta'+4\beta''+\beta''')^2+(2y_1+y_2-4y_3)^2}{9}}$$

$$b = \frac{9n-9m+(\alpha'''+3\alpha''+3\alpha'+2\alpha)+(\beta+3\beta'+4\beta''+\beta''')}{9}$$

$$c = \frac{\sqrt{(5n-5m+2\beta+\alpha+3\alpha'+3\alpha''+\alpha''')^2+(4y_3-2y_1-y_2)^2}}{9}$$

Here the ranking function of the decagonal fuzzy number is defined as

$$R(A) = \sqrt{x^2+y^2}$$

5. Numerical Example

Consider

A=(0.1,0.15,0.2,0.3,0.35,0.4,0.45,0.5,0.6,0.7,0.3,0.7,1) and B=(0.1,0.2,0.3,0.4,0.5,0.6,0.65,0.7,0.75,0.8,0.35,0.65,1) be two decagonal fuzzy numbers.

Now

$a_A = 0.3487$, $b_A = 0.3667$, $c_A = 0.3545$

and $I_A(x,y) = (0.40726, 0.39028)$

$\Rightarrow R(A) = \sqrt{x^2+y^2}$

$= \sqrt{0.40726^2+0.39028^2}$

$= 0.564$

Also $a_B = 0.3411$, $b_B = 0.4611$, $c_B = 0.4125$

And $I_B(x,y) = (0.64738, 1.02089)$

$\Rightarrow R(B) = \sqrt{x^2+y^2}$

$= \sqrt{0.64738^2+1.02089^2}$

$= 1.208$

Since R (A) < R (B) so A < B

6. Conclusion

In this paper, a new method is proposed for ranking of decagonal fuzzy numbers. Proposed Ranking method is based on incentre of centroids. Method is illustrated with the help of numerical example. This ranking procedure can be used in various fields.

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